Testing Quantum Devices

Stephanie Wehner

Based on:

- Capacity estimation with arbitrarily correlated errors, 2016
  C. Pfister, A. Rol, A. Mantri, M. Tomamichel, S. Wehner
- Randomized benchmarking for many qubits using few samples, 2016
  J. Helsen, J. Wallman, S. Flammia, S. Wehner
- On representations of the Clifford group, 2016
  J. Helsen, J. Wallman, S. Wehner

People

Capacity estimation

Corsin Pfister

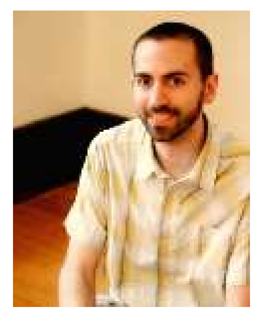
M. Adriaan Rol
(DiCarlo Group)

Atul Mantri

Marco
Tomamichel

Benchmarking

Jonas Helsen



Steve
Flammia

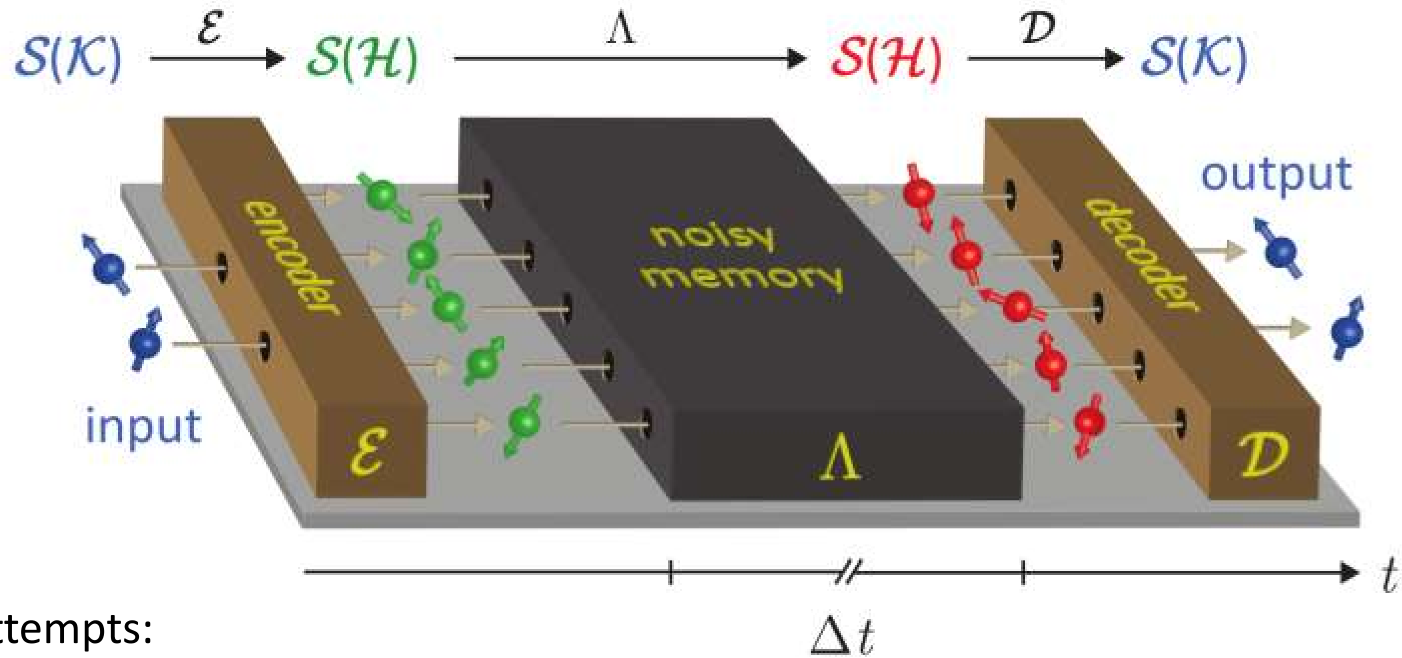
Joel
Wallman

Menu

- How good is a quantum memory or communication channel?
  - New Procedure: Capacity estimation and verification

- How good is the fidelity of quantum gates?
  - Analysis: Reducing the number of samples to perform randomized benchmarking.

Problem 1:
How well can we store (or transmit) quantum information ?

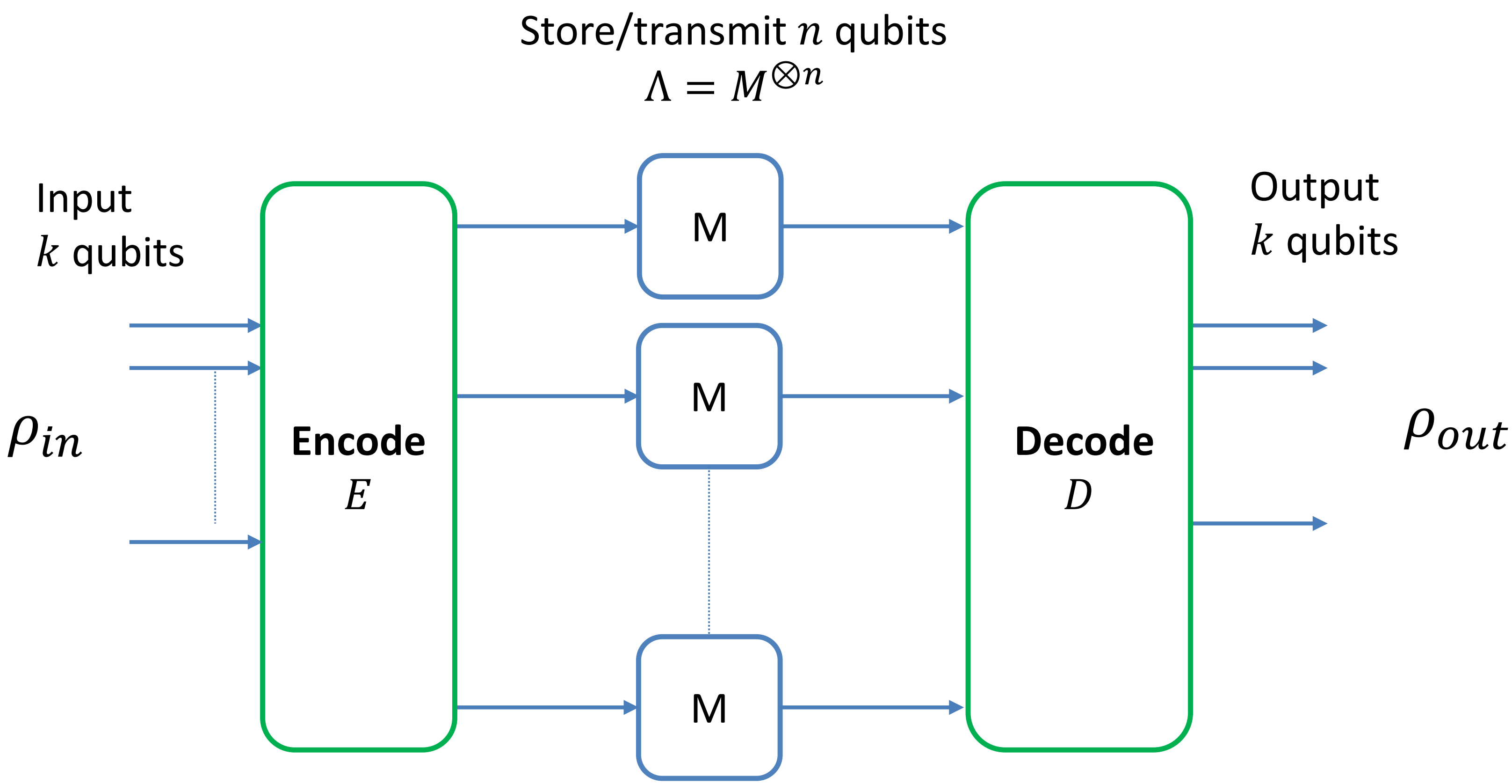


Some attempts:

- Let's implement an error correcting code!
- Let's fully characterize the device!
- Well… then let's assume $\Lambda = M^{\otimes N}$ and then characterize!
  - Noise is almost never of that form.
  - Even if we knew M, some capacities are unknown.

Goal:
estimate the quantum capacity directly, for any device, using only simple operations

What is the capacity?



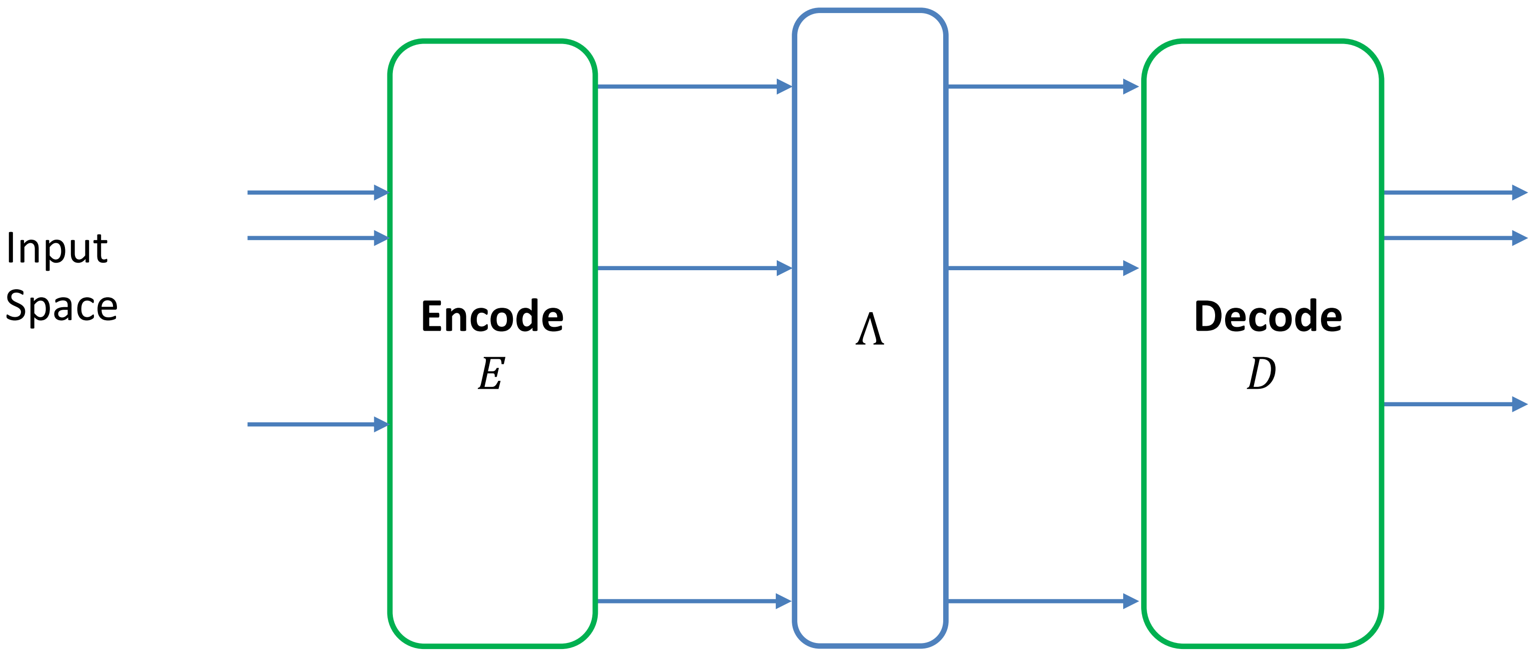
Rate $R = \frac{k}{n}$

Capacity: maximum rate

Single-shot capacity

- No structure – arbitrarily correlated errors
- Finite number of channel uses

Store/transmit n qubits



Definition

$$Q^{\epsilon}(\Lambda) = \max\{\log m \mid F_{min}(\Lambda, m) \geq 1 - \epsilon\}$$

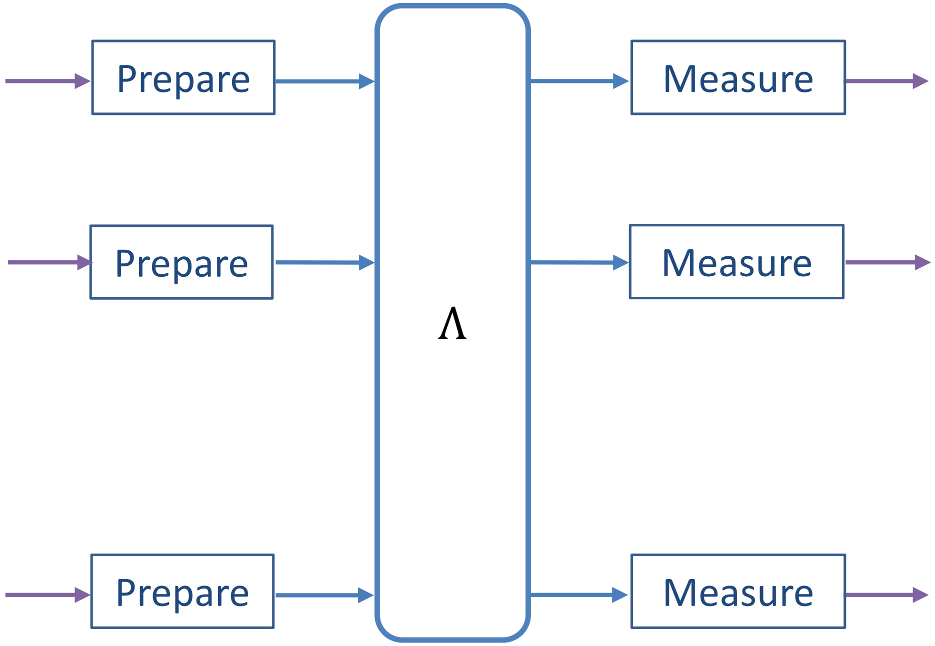
$$F_{min}(\Lambda, m) = \max_{\substack{H_{in} \\ \dim(H_{in})=m}} \max_{D,E} \min_{|\Phi\rangle \in H_{in}} \langle\Phi|(D \circ \Lambda \circ E(\Phi)|\Phi\rangle$$

F. Buscemi and N. Datta, IEEE Trans. Inf. Theory, 56(3), 2010

Goal

- Estimate $Q^{\epsilon}(\Lambda) \geq ?$
- Using only
  - Single qubit preparations and measurements
- Two flavors
  - Capacity estimation of all qubits used
  - Capacity verification of data qubits

Capacity estimation with correlated errors



Goal

$$Q^{\epsilon}(\Lambda) \geq \ f(measured\ data)$$

A simple protocol for capacity estimation

- Choose $s \in \{0,1\}^N$ and $b \in \{X,Z\}^N$ s.t. X, Z occur $\frac{N}{2}$ times in b
- For each $i = 1, \dots, N$
  - Prepare qubit in state $|s_i\rangle_{b_i}$ and send through channel
  - Measure qubit in basis b_i to obtain outcome s_i'
  - Estimate error rates

$$e_X = \frac{\sum_{i \in I_X} s_i \oplus s_i'}{|I_x|} \qquad e_Z = \frac{\sum_{i \in I_Z} s_i \oplus s_i'}{|I_Z|}$$

$$I_X = \{i \mid b_i = X\} \qquad I_Z = \{i \mid b_i = Z\}$$

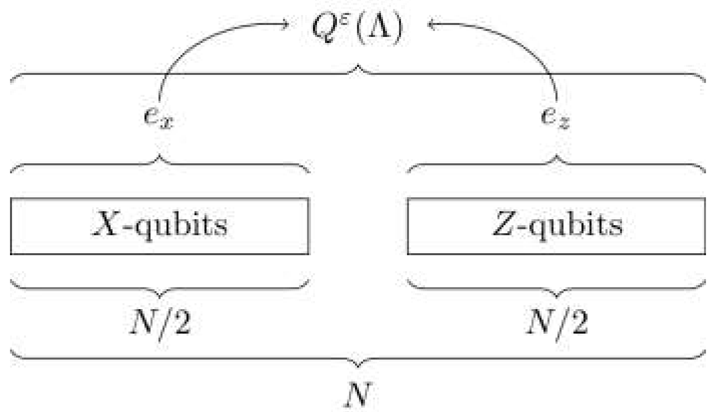
Conclude

$$Q^\epsilon(\Lambda) \gtrapprox N(q - h(e_X) - h(e_Z))$$

$$q = -\log \max_{i,j \in \{0,1\}} |\langle i_X | j_Z \rangle| = 1 \ \ (Preparation\ quality)$$

$$h(p) = -p\log(p) - (1-p)\log(1-p)$$

Capacity estimation: a more precise statement



Theorem 1 : *Let $N \in \mathbb{N}_+$ be an even number, let e_x and e_z be error rates determined in a run of the Estimation Protocol where the used bases X and Z had a preparation quality of q. Then, for every $\varepsilon > 0$ and for every $p \in [0,1)$, it holds that*

- *either, the probability that at least one error rate exceeds e_x or e_z, respectively, was higher than p,*
- *or the one-shot quantum capacity of the N-qubit channel Λ is bounded by*

$$Q^\varepsilon(\Lambda) \geq \sup_{\eta \in \left(0, \sqrt{\varepsilon/2}\right)} \left[N\Big(q - h\left(e_x + \mu\right) - h\left(e_z + \mu\right)\Big) - 2\log\left(\kappa\right) - 4\log\left(\frac{1}{\eta}\right) - 2\right], \tag{1}$$

where h is the binary entropy function

$$h(x) := -x\log(x) - (1-x)\log(1-x) \tag{2}$$

and μ and κ are given by

$$\mu = \sqrt{\frac{N+2}{N^2}\ln\left(\frac{3+\frac{5}{\sqrt{1-p}}}{\sqrt{\varepsilon/2}-\eta}\right)}, \quad \kappa = 2\left(\frac{3+\frac{5}{\sqrt{1-p}}}{\sqrt{\varepsilon/2}-\eta}\right)^2. \tag{3}$$

What is this parameter p?

- *either, the probability that at least one error rate exceeds e_x or e_z, respectively, was higher than p,*

Example: Fully depolarizing channel on N qubits

$$\Lambda(\rho^N) = \frac{I}{2^N}$$

Channel has zero capacity, yet with probability $p = \frac{1}{2^N}$ we have $e_x = e_Z = 0$

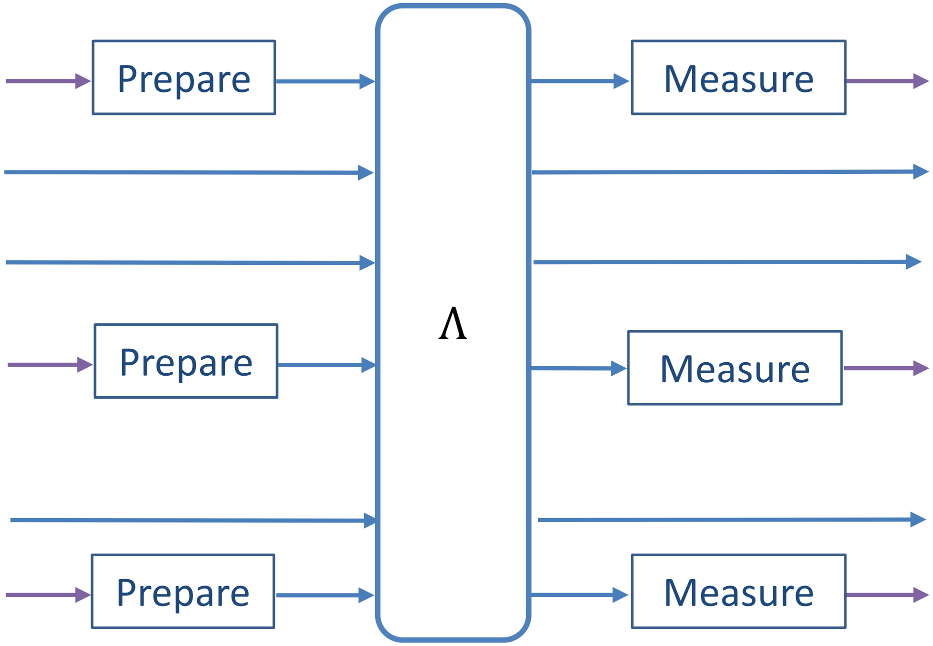
Let's say we observe $e_x = e_Z = 0$ which is highly untypical. We have

- Either probability the error rate exceeds 0 was actually higher than $p = 1/2^N$
- Or the capacity bound applies

In practice: Pick any constant p

Already for moderately sized N, the estimate is essentially independent of any constant p. Example: $p = 1/2$.

Capacity verification with correlated errors



Goal

$$Q^{\epsilon}(\Lambda \text{ on data qubits}) \geq \; f(measured\ data)$$

A simple protocol for capacity verification

- Decide on maximum acceptable error rates e_x and e_Z
- Choose $s \in \{0,1\}^{3N}$ and $b \in \{X, Z, D\}^{3N}$ s.t. X, Z, D occur N times in b
- For each $i = 1, \dots, 3N$
  - If $b_i = D$ send data!
  - else
    - Prepare qubit in state $|s_i\rangle_{b_i}$ and send through channel
    - Measure qubit in basis b_i to obtain outcome s_i'
    - Estimate error rates

$$\gamma = \frac{\sum_{i \in I_X} s_i \oplus s_i'}{|I_X|} \qquad \lambda = \frac{\sum_{i \in I_Z} s_i \oplus s_i'}{|I_Z|}$$

$$I_X = \{i \mid b_i = X\} \qquad I_Z = \{i \mid b_i = Z\}$$

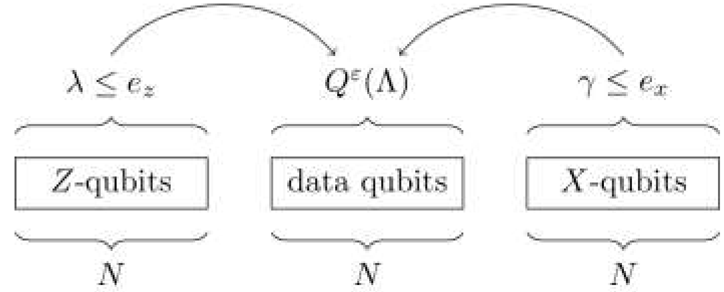
If $\gamma > e_x$ and $\lambda > e_Z$ abort, else conclude

$$Q^{\epsilon}(\Lambda) \gtrapprox N(q - h(e_X) - h(e_Z))$$

Λ is channel on data qubits only!

QUTech

Capacity verification
a more precise statement



Theorem 2 : *Let $N \in \mathbb{N}_+$, let e_x, $e_z \in [0,1]$. Assume that the Verification Protocol is run successfully without abortion, where the used bases X and Z had a preparation quality of q. Then, for every $\varepsilon > 0$ and for every $p \in [0,1)$, it holds that*

- *either, the probability that the protocol aborts was higher than p,*
- *or the one-shot quantum capacity of the channel Λ on the N data qubits is bounded by*

$$Q^{\varepsilon}(\Lambda) \geq \sup_{\eta \in \left(0, \sqrt{\varepsilon/2}\right)} \left[N \left(q - h\left(e_x + \mu\right) - h\left(e_z + \mu\right) \right) - 2\log\left(\kappa\right) - 4\log\left(\frac{1}{\eta}\right) - 2 \right], \tag{4}$$

where κ and μ are given by

$$\mu = \sqrt{\frac{2(N+1)}{N^2} \ln\left(\frac{3 + \frac{5}{\sqrt{1-p}}}{\sqrt{\varepsilon/2} - \eta} \right)}, \quad \kappa = 2\left(\frac{3 + \frac{5}{\sqrt{1-p}}}{\sqrt{\varepsilon/2} - \eta} \right)^2. \tag{5}$$

How can this be proven?

Already know (Barnum, Knill, Nielsen (2000) and Buscemi, Datta (2010))

$$Q^{\epsilon}(\Lambda) \geq \sup_{\eta \in (0, \sqrt{\frac{\epsilon}{2}})} \left(H_{min}^{\frac{\sqrt{\epsilon}}{4} - \eta}(A|E)_{\rho} - 4 \log \frac{1}{\eta} - 1 \right) - 1$$

$$H_{min}^{\delta}(A|E)_{\rho} = \max_{\rho' \in B^{\delta}(\rho)} H_{min}(A|E)$$

$$H_{min}(A|E) = -\log[|A| Dec(A|E)]$$

$$Dec(A|E) = \max_{\Lambda_{E \to A}} \mathrm{F}(\Phi_{AA'}, I_A \otimes \Lambda_{E \to A}(\rho_{AE}))$$

Measure how entangled E has become with A!

If only A was classical.....

Using a tripartite uncertainty relation (Tomamichel, Renner PRL 2011)

$$H_{min}(X|E) + H_{\max}\ (Z|B) \succcurlyeq q$$

Not qubits?
Change this to extend!

Using a number of properties of the min and max entropies

$$H_{min}(A|E) \geq Nq\ - \left(H_{max}\left(X^N|B\right)_\rho\ + H_{max}\left(Z^N|B\right)_\rho\right) - f(\epsilon)$$

Estimate using error rates as in QKD!

How well does this work?

Example: Capacity estimation, i.i.d. dephasing noise

$$\Lambda = D^{\otimes N} \quad \text{with } D(\rho) = (1-r)\rho + r\ Z\rho Z$$

What happens?

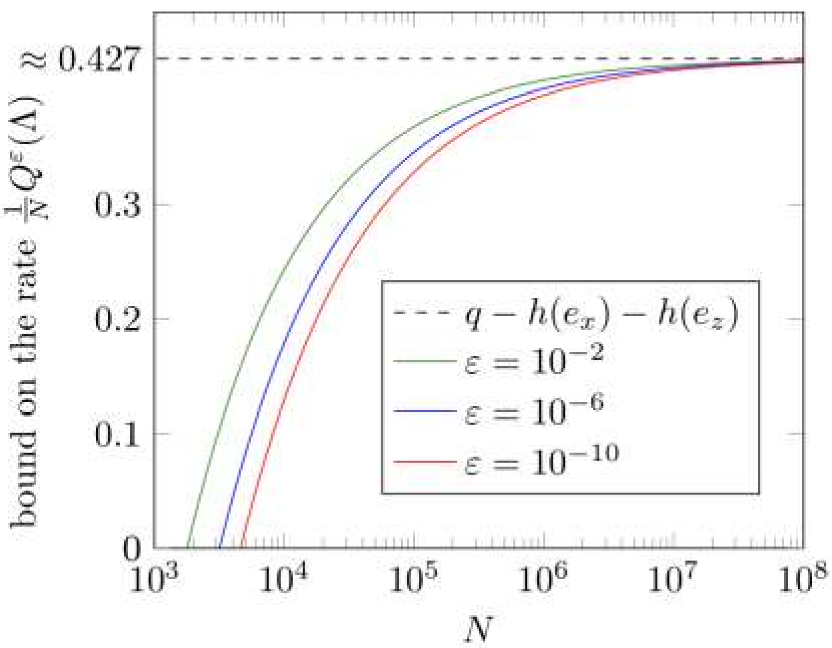
- Z basis left invariant: $e_Z = 0$
- X basis flipped with probability r: $e_X = r$ (asymptotically)
- Asymptotically bound is $q - h(0) - h(r) = 1 - h(r)$

This is the quantum capacity of dephasing noise.

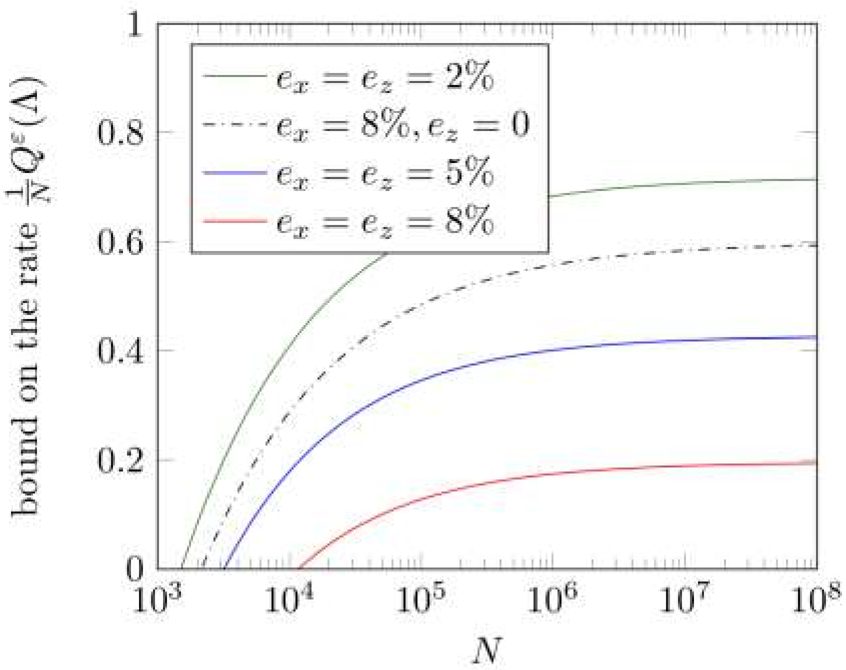
Asymptotically optimal!

Scaling

Finite size



Dependence on error rate



Remark:

- Same finite size effects in QKD
- Capacities are in fact much smaller for finite N

(W. Matthews, S. Wehner, IEEE Trans. IT 2012,
M. Berta, J. Renes, M. Tomamichel, Nat. Comm. 2016, ….)

What is this “X” and “Z”?

Example: Capacity estimation, i.i.d. dephasing noise

$$\Lambda = D^{\otimes N} \quad \text{with } D(\rho) = (1-r)\rho + r\ Z\rho Z$$

Wait! Doesn’t this depend on the noise being aligned with the bases used??

What happens?

- Z basis left invariant: $e_Z = 0$
- X basis flipped with probability r: $e_X = r$ (asymptotically)
- Asymptotically bound is $q - h(0) - h(r) = 1 - h(r)$

Of course ☺

In practice:

- Any choice of basis gives a bound.
- Rotate to minimize error rate ahead of time.
- Best way to do so: open question!

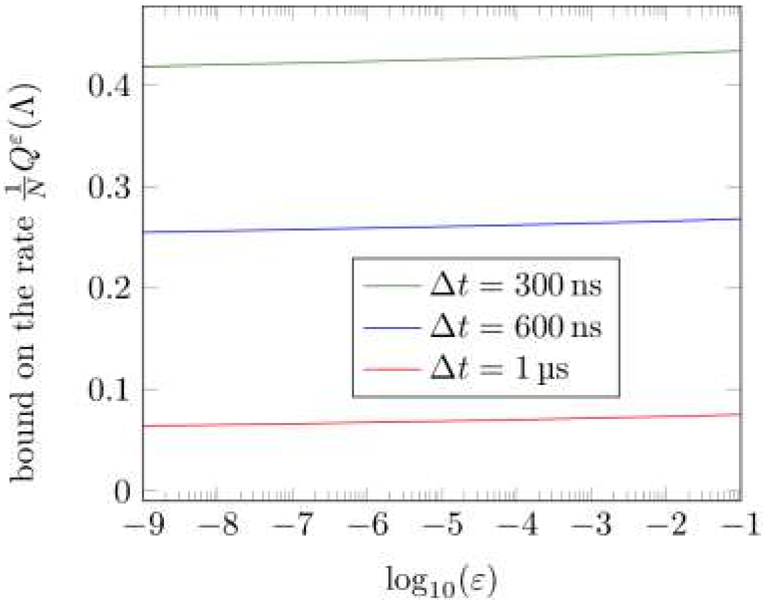
Test in experiment

M. Adriaan Rol

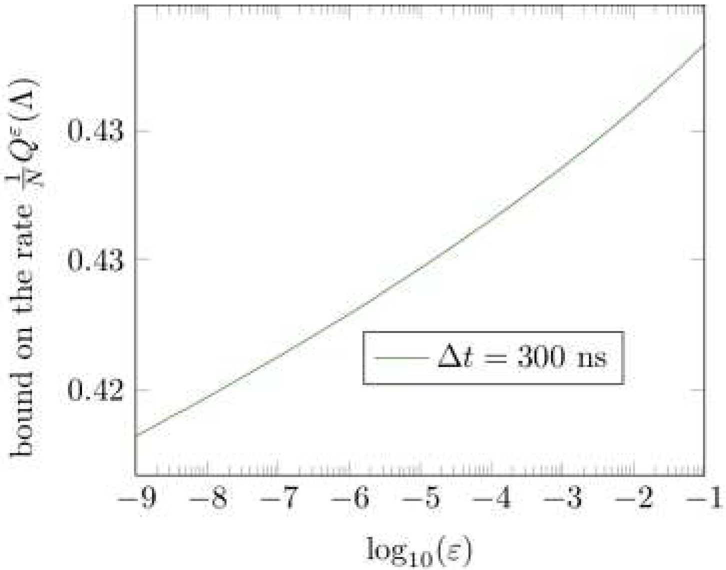
Transmon qubit (Leo DiCarlo group, QuTech), $N = 1.04 \times 10^6$, ~1.5 hours
Take: $q = 0.9, p = 0.5$
Estimate the capacity of the idling operation $I(\Delta t)$

- Generate 8000 pairs of random numbers b,s
- For each element
  - Rotate $|0\rangle$ to the right state
  - Wait time Δt
  - Rotate if measuring X
  - Measure Z
- Repeat 130 times

Dependence on ϵ



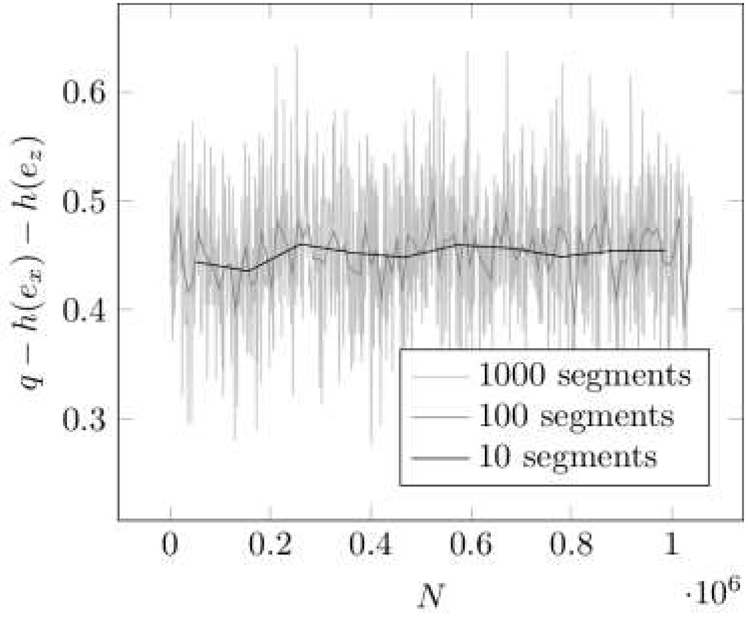
Zoomed in



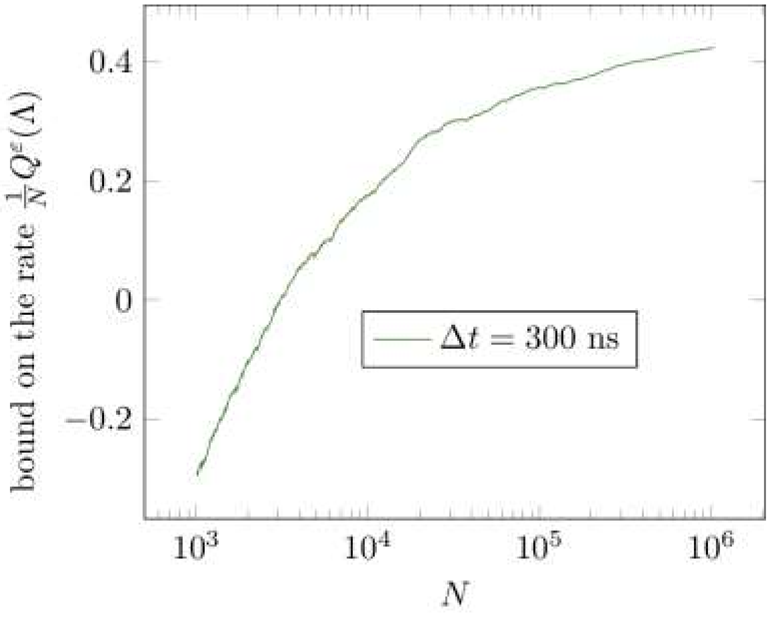
Errors over time

$$N = 1.04 \times 10^6$$
$$\epsilon = 10^{-6}$$
$$p = \frac{1}{2}$$
$$q = 0.9$$

Fluctuations asymptotic bound



Cumulative bound

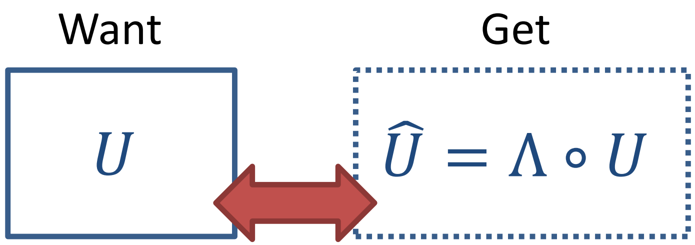


Menu

- How good is a quantum memory or communication channel?
  - New Procedure: Capacity estimation and verification

- How good is the fidelity of quantum gates?
  - Analysis: Reducing the number of samples to perform randomized benchmarking.

Testing quantum gates



Average fidelity

$$F_{avg}(\Lambda, U) = \int d\phi \, F\left(U(\phi), \hat{U}(\phi)\right) = \int d\phi \, \langle\phi|\Lambda(\phi)|\phi\rangle$$

Entanglement fidelity

$$f(\Lambda) = \langle\Phi|I_A \otimes \Lambda(\Phi)|\Phi\rangle$$

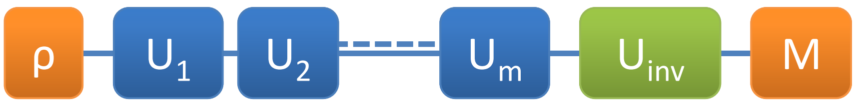
$$|\Phi\rangle = \frac{1}{\sqrt{d}} \sum_{j=1}^{d} |j\rangle_A |j\rangle_B$$

Equivalent estimate

$$f(\Lambda) = \frac{d \, F_{avg}(\Lambda, \mathrm{U}) - 1}{d \; - 1}$$

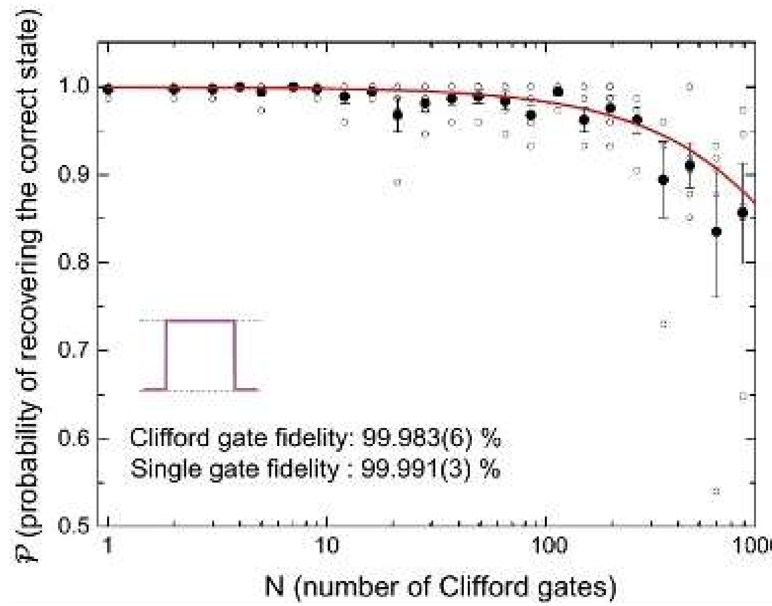
Randomized Benchmarking

What is Randomized Benchmarking?



1. Initialize state ρ
2. Apply (noisy) gates $U_1,\ldots,U_m$
3. Apply inversion gate U_{inv}
4. Measure output state
5. Repeat for many (N) random $U_1,\ldots,U_m$
6. Average over measurement results
7. Repeat for many values of m
8. Plot results and find decay

Apply strings of random gates: errors accumulate exponentially (on average)



Decay constant gives average fidelity of gate set.

Muhonen et al., Journal of Physics Condensed Matter, 27(15):154205, Apr.16,2015.

Randomized Benchmarking: what we do

Let's be a little more precise:

We perform RB by sampling m gates from the Clifford group $\mathcal{C}$

Average over ALL possible strings of m gates → Impossible to do in practice!

Fit P_m to function $Af^m + B$ → Yields estimate for f

f Is related to the Average Gate Fidelity F of average gate $\mathcal{C}$

Question:

How many (N) strings $U_1, \ldots, U_m$ do I need to average over to get correct results?

Solution: Establish confidence bounds

For a given ϵ, δ, we want N to be as small as possible

$$\mathrm{Prob}\left(|P_m - P_{m,N}| \geq \epsilon\right) \leq \delta$$

P_m is the true average over all possible strings

$P_{m,N}$ is the empirical average over a subset of strings

Study the probability distribution that arises from applying random strings of Clifford gates

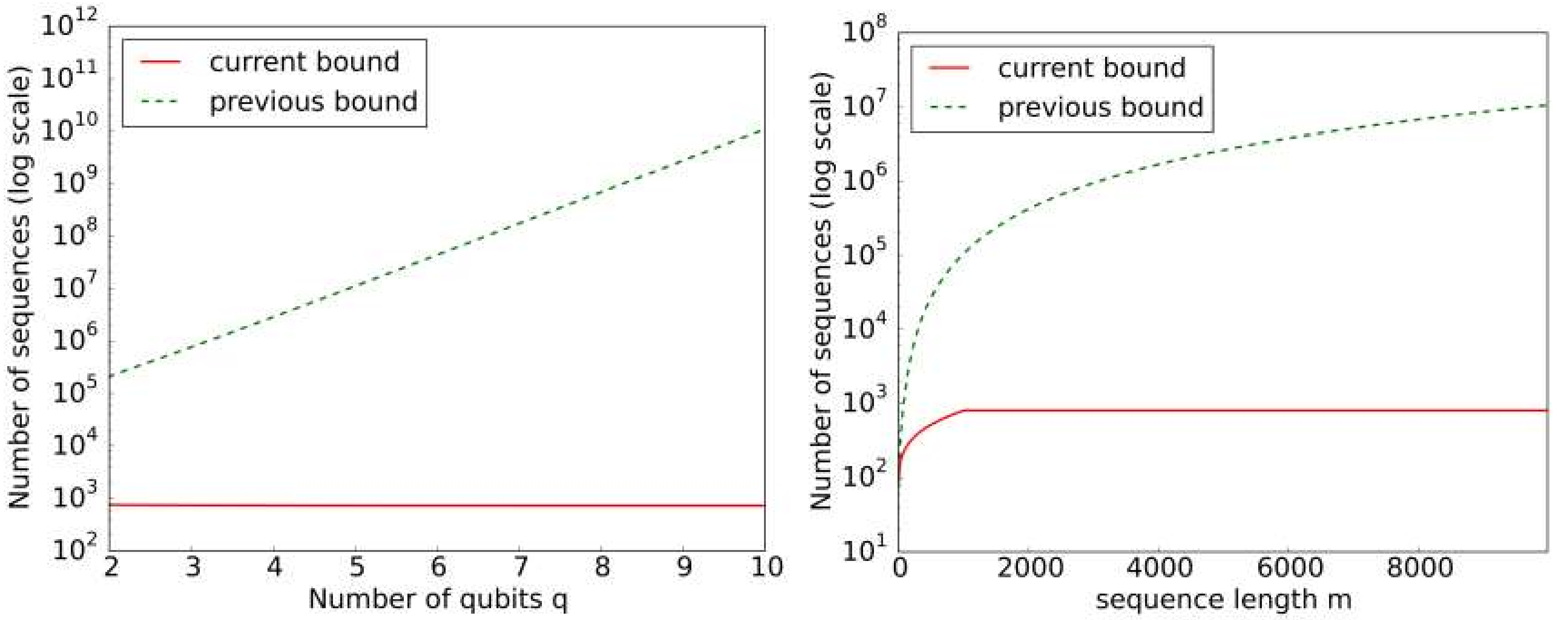
Try to upper bound variance of this distribution

Clifford group is a group → Use REPRESENTATION THEORY

Jonas Helsen - PhD student

Results on randomized benchmarking

Crucial ingredient: Analysis of the representations of the Clifford group!



Summary and open questions

- First procedure for direct capacity estimation and verification
    - Using only simple preparations and measurements
    - Asymptotically optimal for dephasing noise
    - Tested in experiment: bad qubits? They may still be useful ☺
- Open questions
    - How about non-qubits? Change uncertainty relation!
    - What is a good way to calibrate the bases before or during the protocol?
    - Better method?

- Randomized benchmarking
    - Significantly less samples!
    - How about correlated forms of noise?

http://blog.qutech.nl

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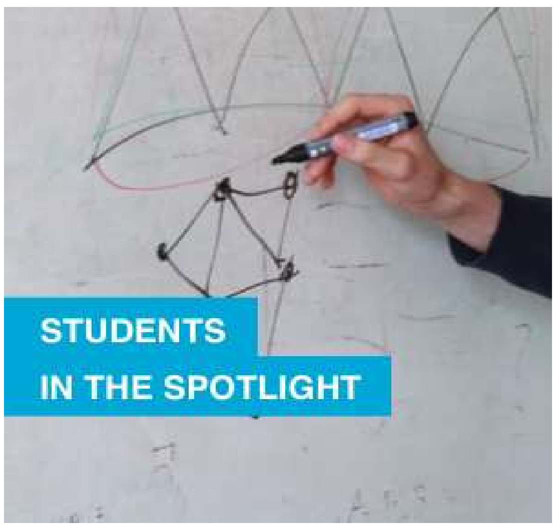

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