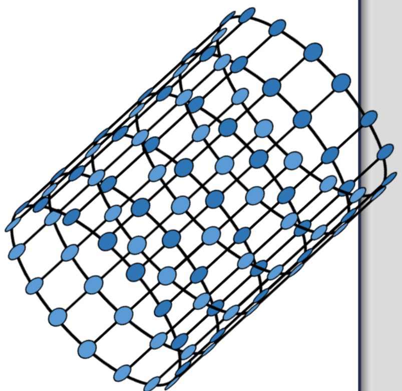
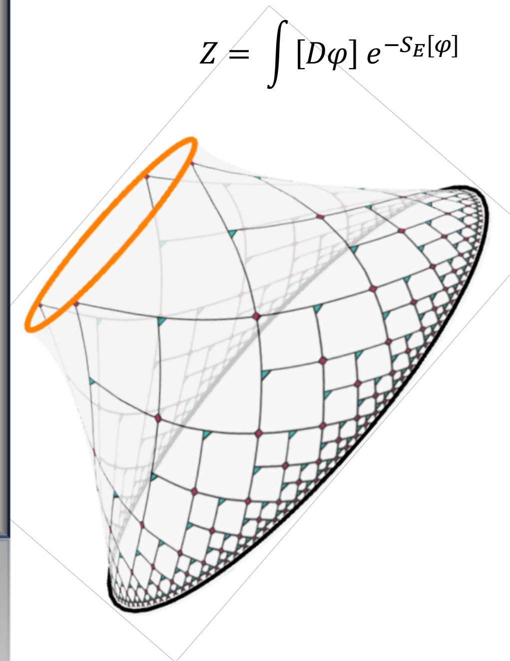
Sydney Quantum Information Theory Workshop

Coogee Bay Hotel, Sydney Feb 8th 2019



Tensor networks, geometry, and AdS/CFT



Guifre Vidal

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

compute canada | calcul canada

TENSOR NETWORKS INITIATIVE

SIMONS FOUNDATION

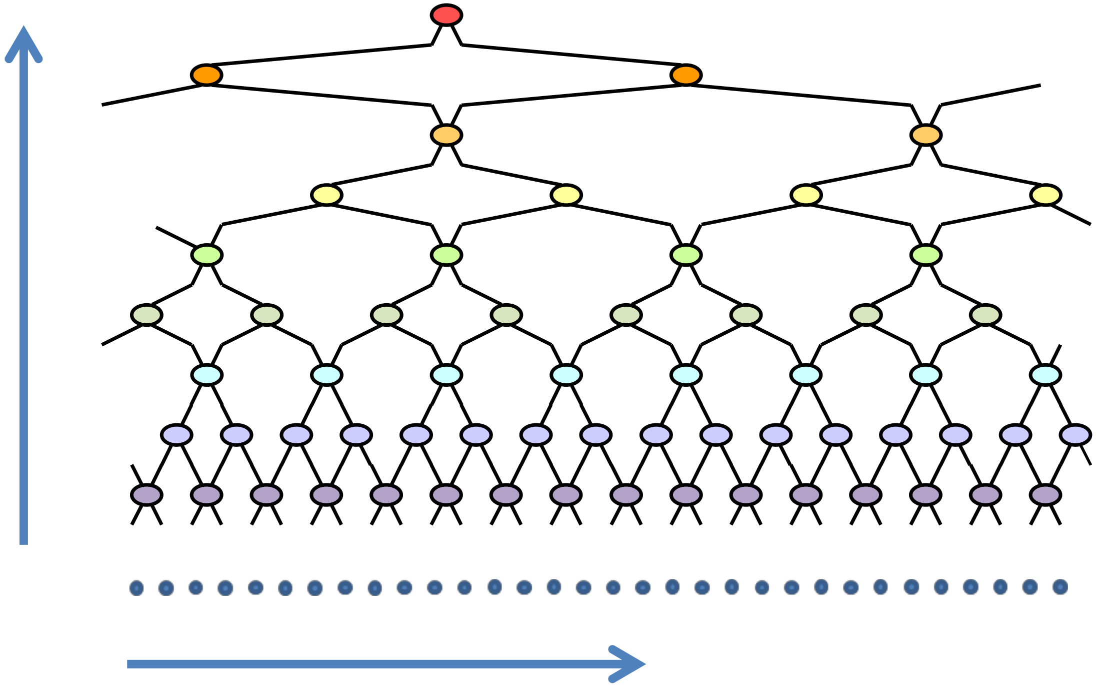
NSERC

MERA

Multi-scale entanglement renormalization ansatz

Tensor network = sparse, efficient representation of many-body wavefunctions

scale



spin chain

space

N spins

$|\Psi\rangle \longrightarrow |\Psi\rangle = \sum_{i_1 i_2 \cdots i_N} \Psi_{i_1 i_2 \cdots i_n} |i_1 i_2 \cdots i_N\rangle$ $\quad 2^N$ coefficients

$\searrow$ tensor network $\quad O(N)$ tensors $\quad O(N)$ coefficients

G. Vidal, PRL 99 (22), 220405 (2007)

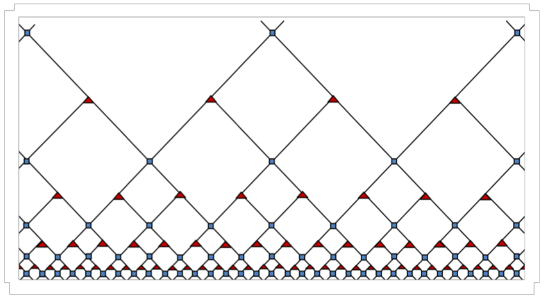
G. Vidal, PRL 101 (11), 110501 (2008)

G. Evenbly, Vidal, PRB 79 (14), 144108 (2009)

Glen Evenbly
Georgia Tech

Outline

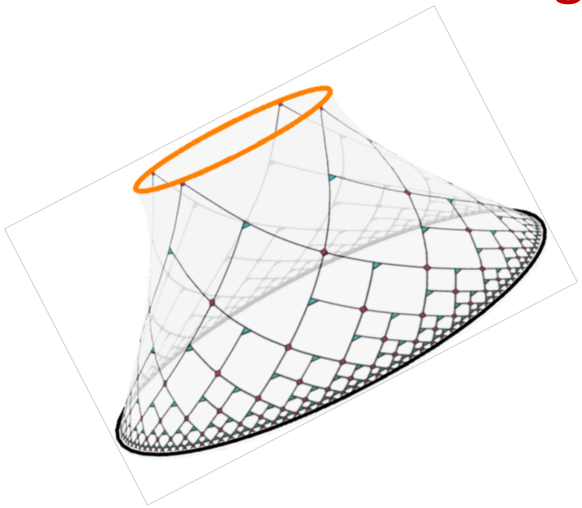
MERA tensor network



1) quantum circuit

2) renormalization group

Tensor network as geometry



3) path integral

4) light cone

5) euclidean / lorentzian MERA

quantum information

condensed matter

holography

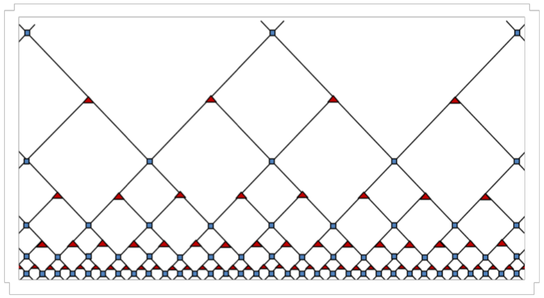
cosmology

Glen Evenbly
Georgia Tech

Ash Milsted
Perimeter Institute

Outline

MERA tensor network



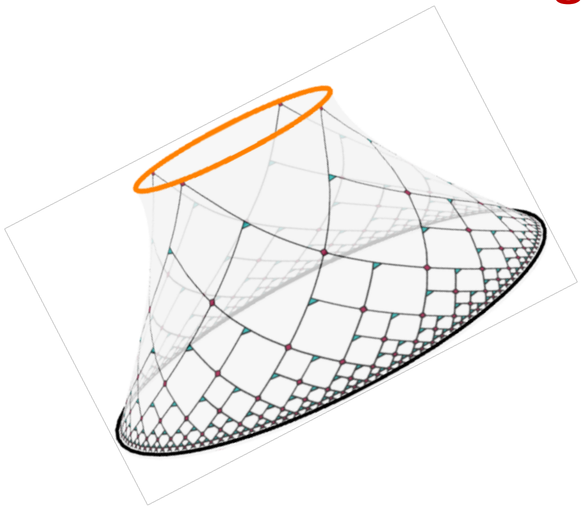
1) quantum circuit

2) renormalization group

quantum information

condensed matter

Tensor network as geometry



3) path integral

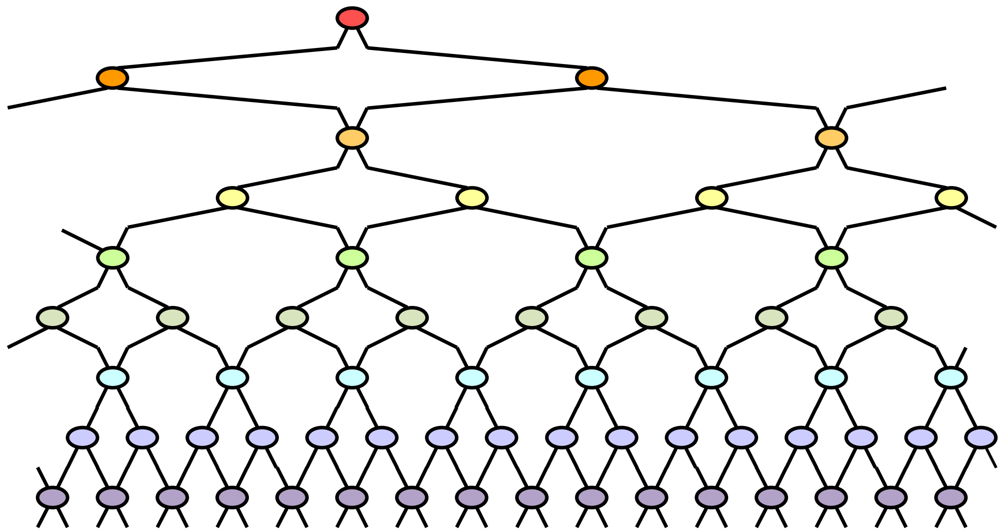
4) light cone

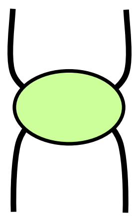
5) euclidean / lorentzian MERA

holography

cosmology

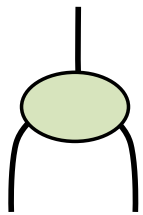
MERA as a quantum circuit:





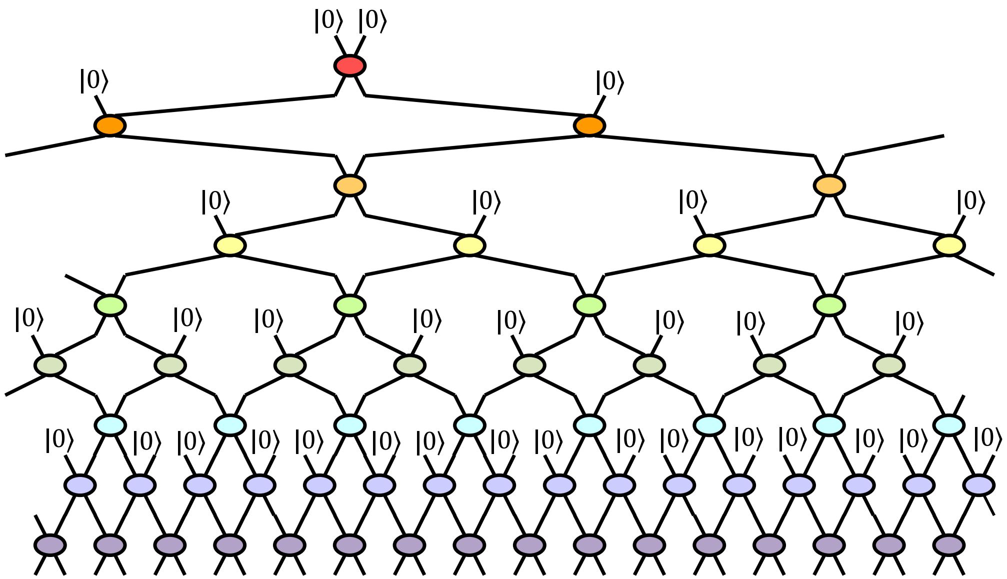
disentangler

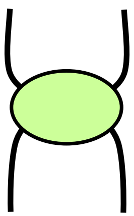
two-body unitary gate



isometry

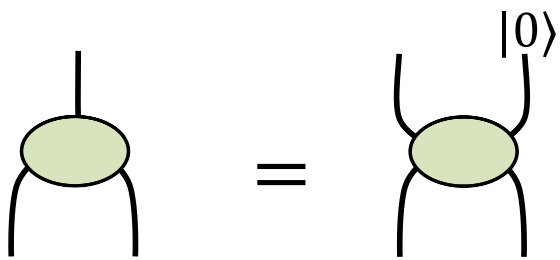
MERA as a quantum circuit:





disentangler

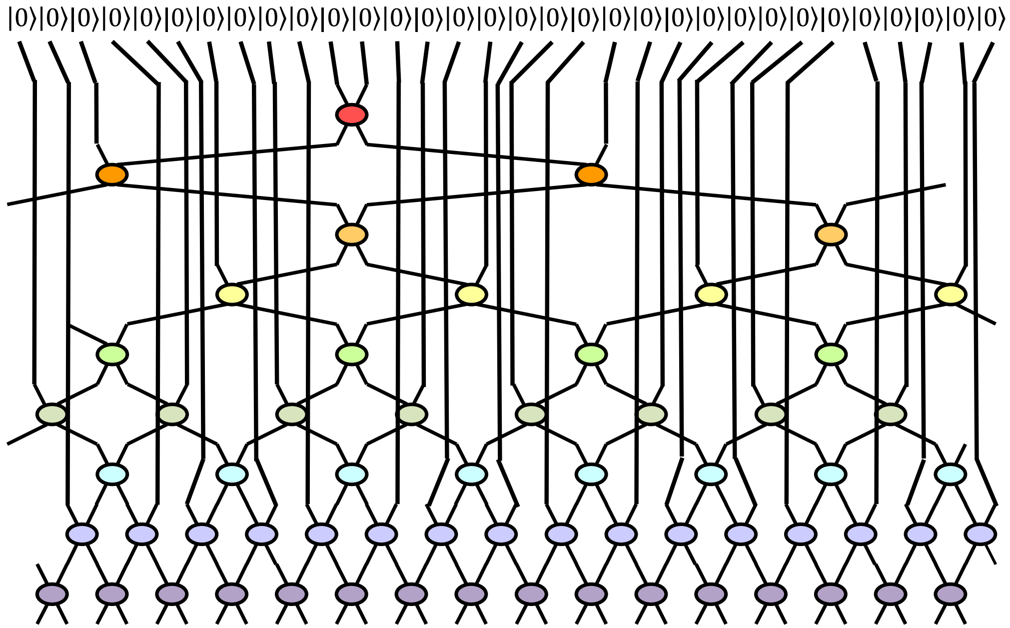
two-body unitary gate

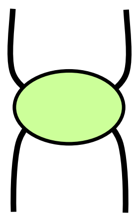


isometry

also a
two-body unitary gate

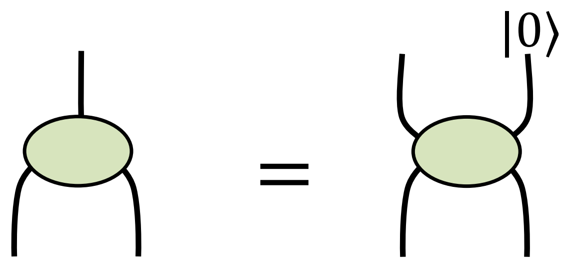
MERA as a quantum circuit:





disentangler

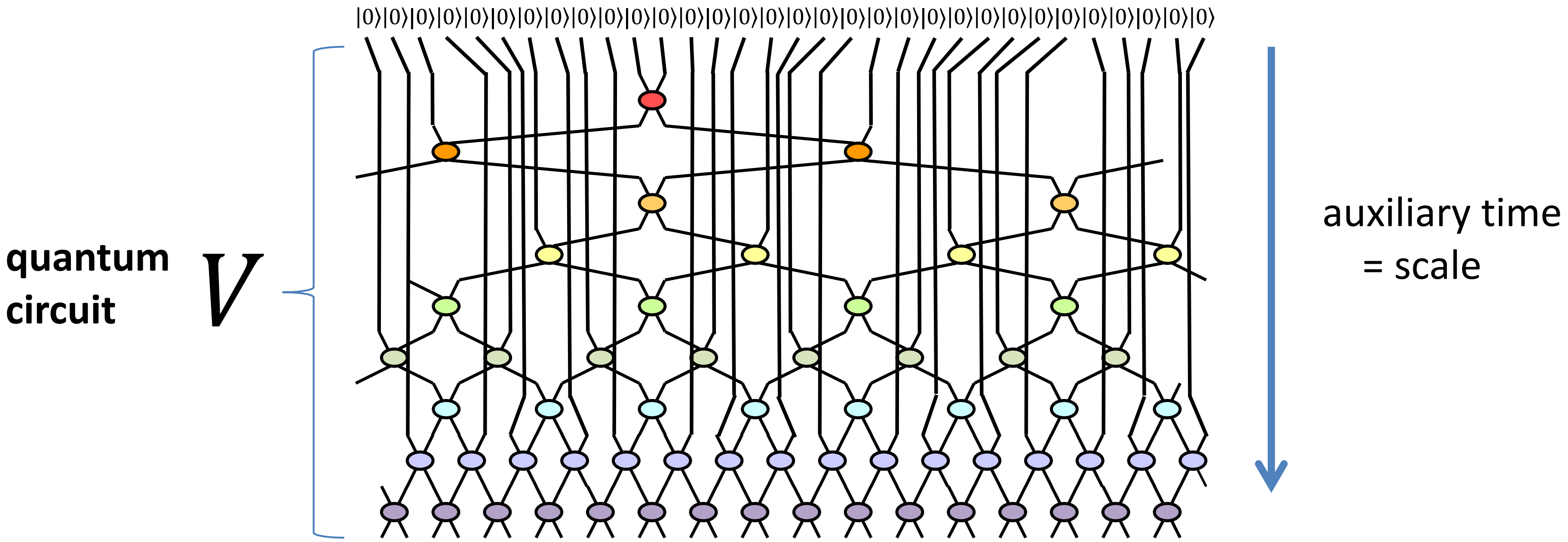
two-body unitary gate



isometry

also a
two-body unitary gate

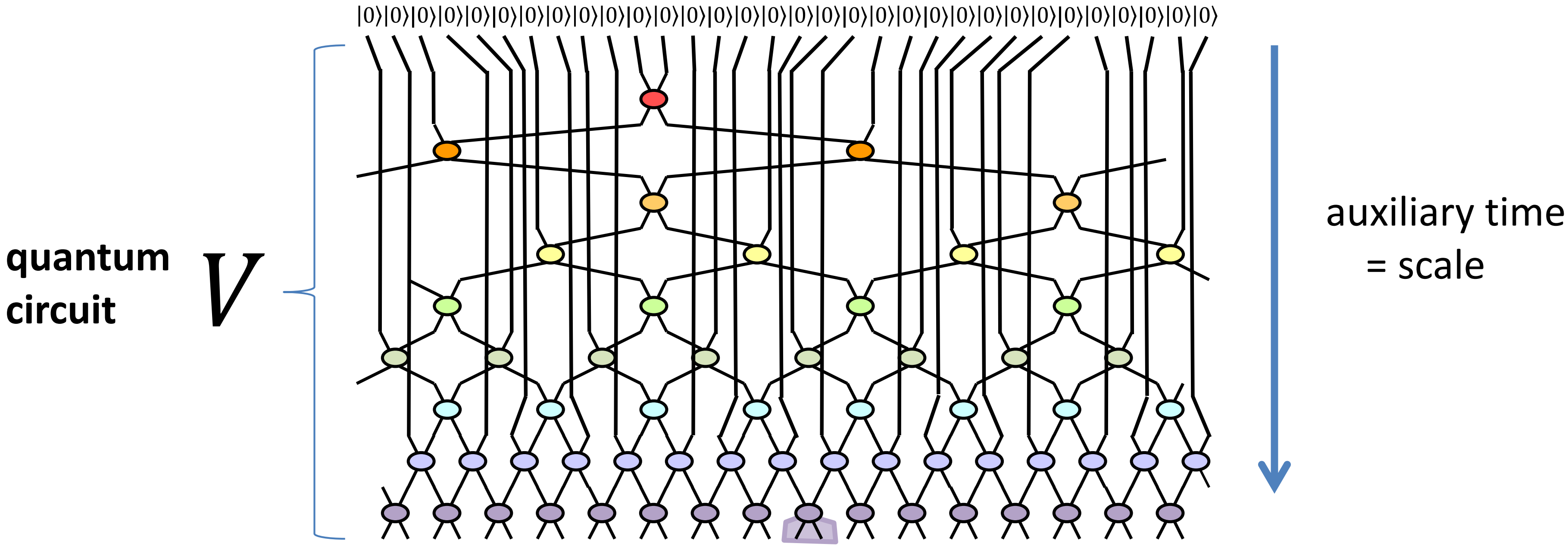
MERA as a quantum circuit:



ground state $|\Psi\rangle = V\ |0\rangle^{\otimes N}$

entanglement introduced sequentially at different length scales

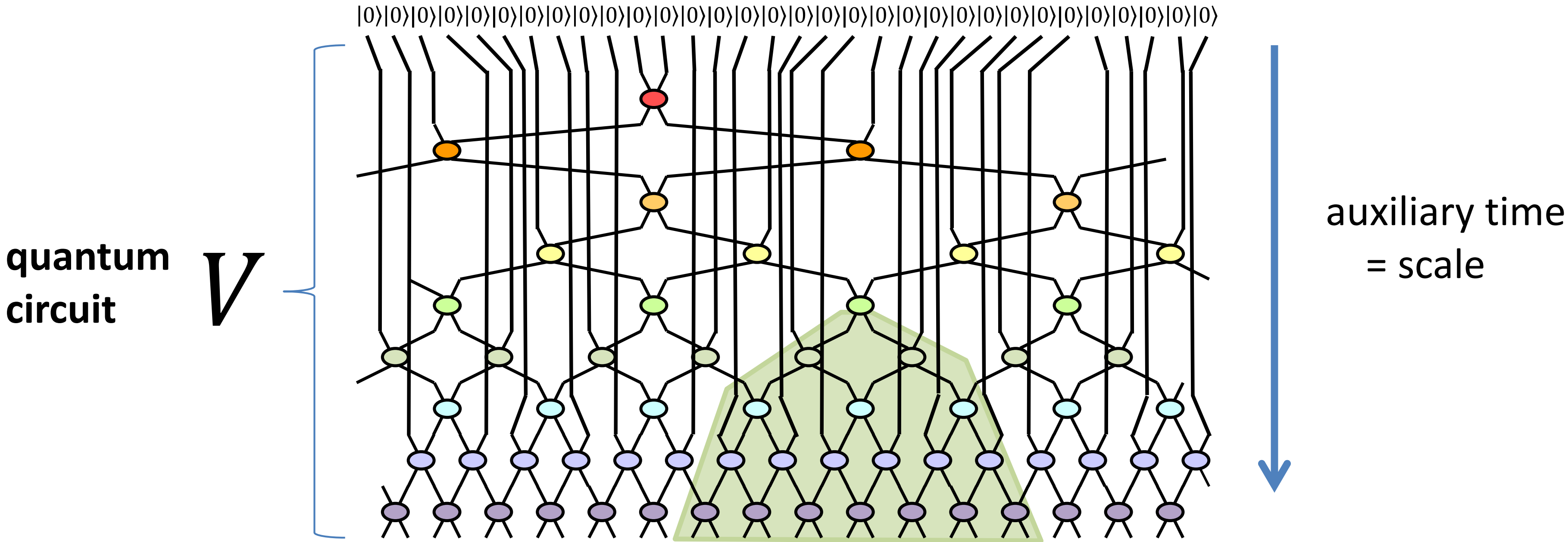
MERA as a quantum circuit:



ground state $|\Psi\rangle = V\ |0\rangle^{\otimes N}$

entanglement introduced sequentially at different length scales

MERA as a quantum circuit:

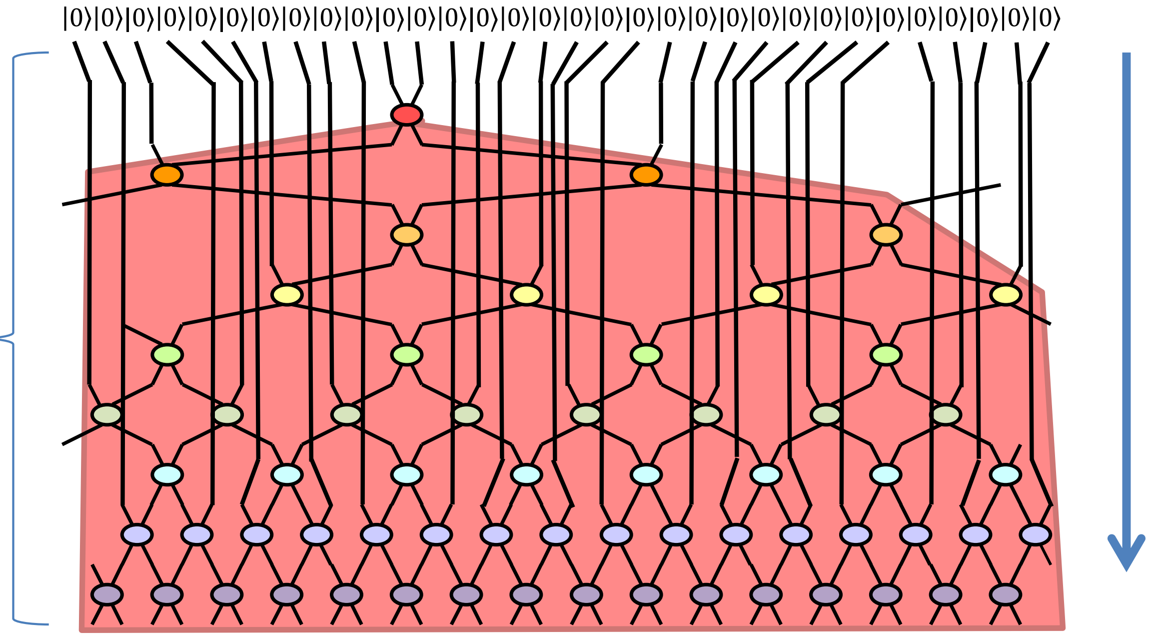


ground state $|\Psi\rangle = V\ |0\rangle^{\otimes N}$

entanglement introduced sequentially at different length scales

MERA as a quantum circuit:

quantum circuit V



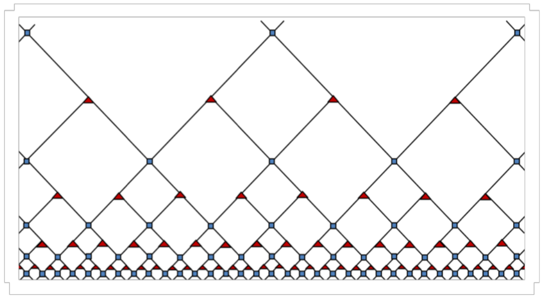
auxiliary time
= scale

ground state $|\Psi\rangle = V\ |0\rangle^{\otimes N}$

entanglement introduced sequentially at different length scales

Outline

MERA tensor network



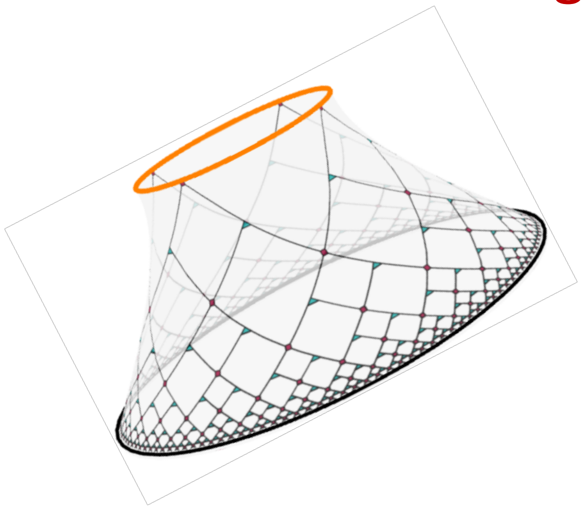
1) quantum circuit

quantum information

2) renormalization group

condensed matter

Tensor network as geometry



3) path integral

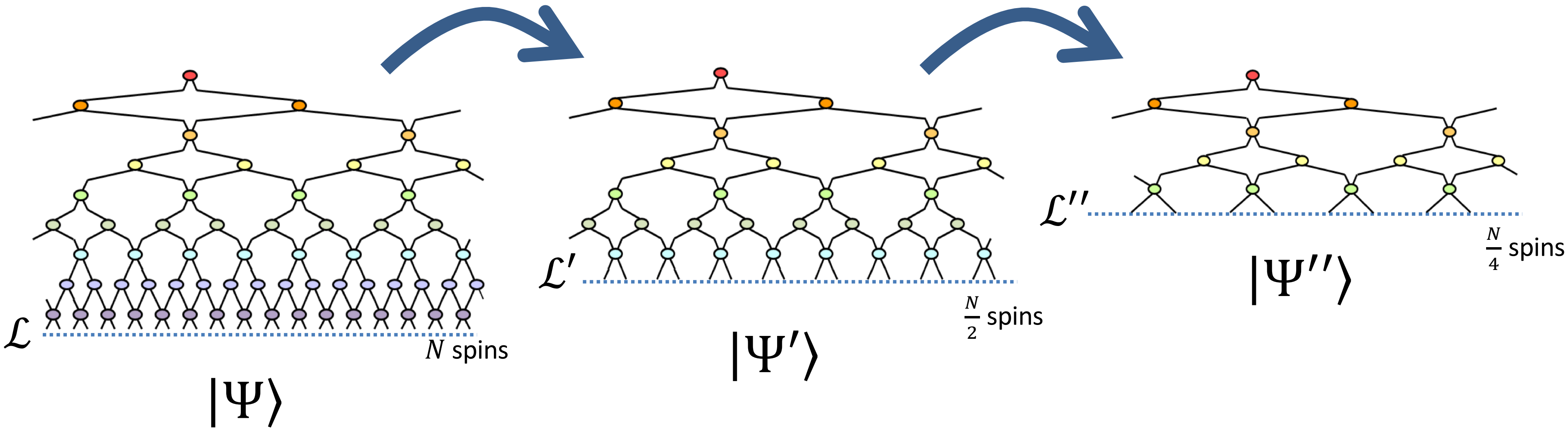
holography

4) light cone

cosmology

5) euclidean / lorentzian MERA

MERA as a coarse-graining transformation:



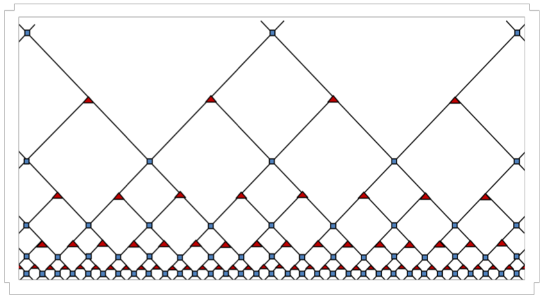
Wilson's **RG**

$$|\Psi\rangle \rightarrow |\Psi'\rangle \rightarrow |\Psi''\rangle \rightarrow \cdots$$

(for wavefunctions,
in real space)

Outline

MERA tensor network



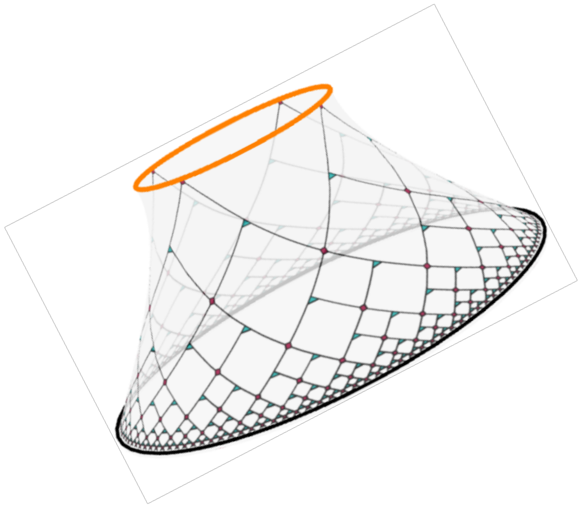
1) quantum circuit

2) renormalization group

quantum information

condensed matter

Tensor network as geometry



3) path integral

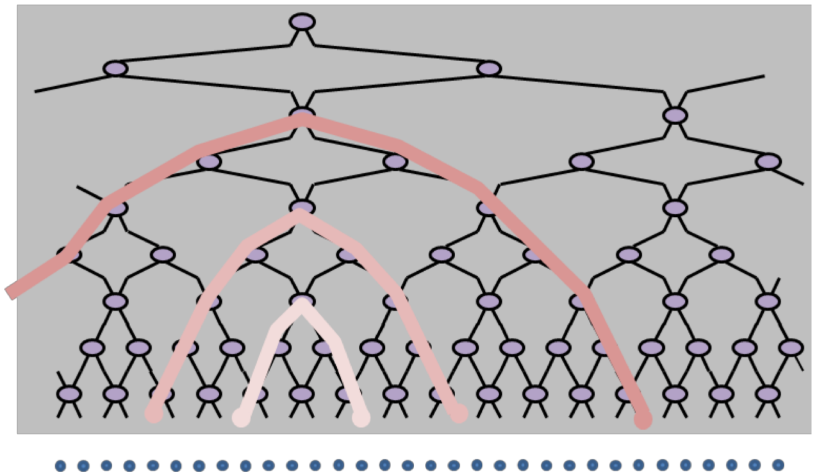
4) light cone

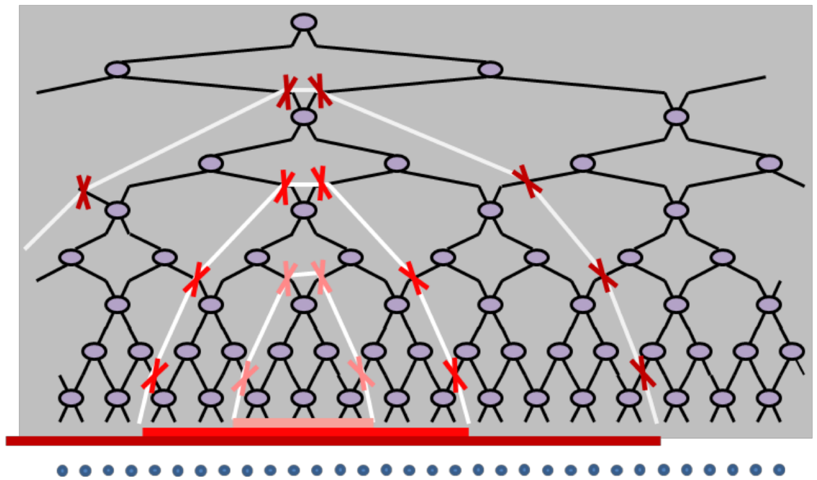
5) euclidean / lorentzian MERA

holography

cosmology

MERA as a variational ansatz:





correlations

entanglement entropy

Accurate representation of **ground states** of

critical systems

↓

In continuum:

conformal field theories

CFTs

$$\langle \phi(0)\phi(L)\rangle = \frac{1}{L^{2\Delta_\phi}}$$

$$S(L) = \frac{c}{3}\log\left(L/\epsilon\right)$$

minimal distance within the network

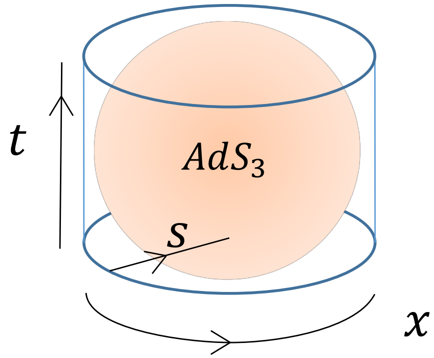
geometry?

minimal # of cuts within the network

MERA as (toy model for) holography & cosmology

AdS/CFT correspondence, J. Maldacena, 1997

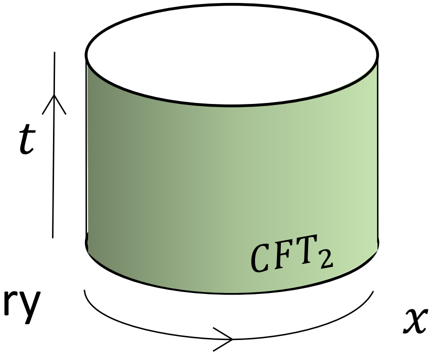
('t Hooft, Susskind, Thorn, Bousso, ...)



$AdS_{d+1} \Leftrightarrow CFT_d$

quantum gravity

quantum field theory

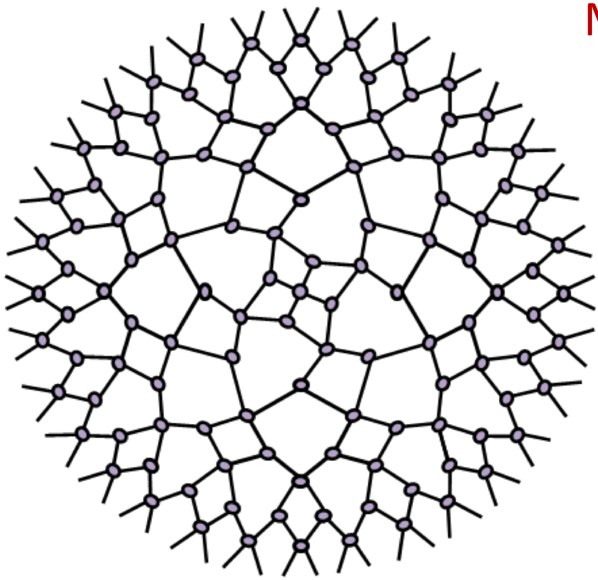


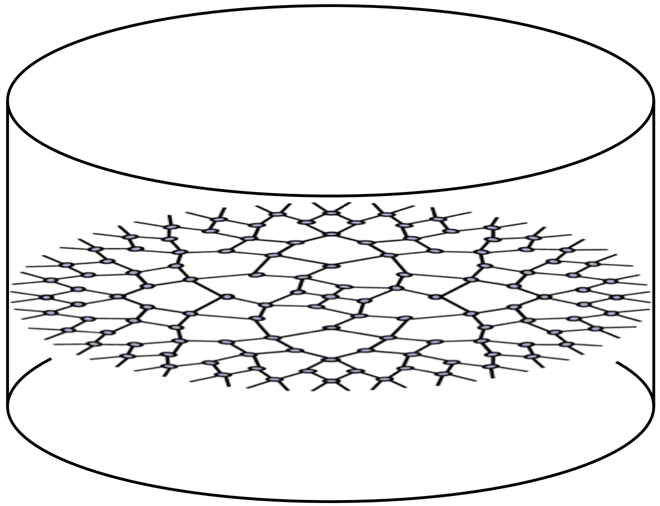
Swingle 2009, 2012

MERA = Hyperbolic space H_2

(time slice of AdS_3)

AdS_3/CFT_2

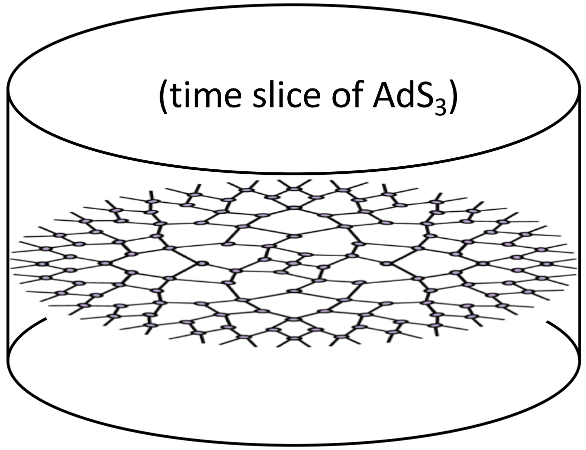




MERA as (toy model for) holography & cosmology

MERA = hyperbolic space H_2
AdS_3/CFT_2

Swingle 2009, 2012



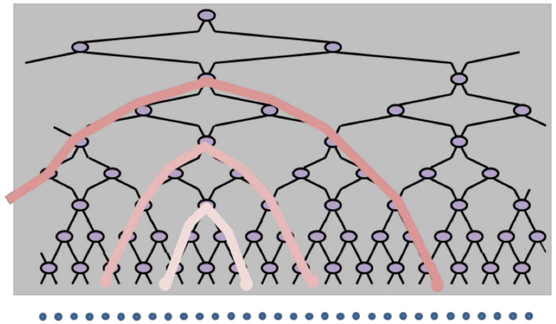
why?

1 MERA prepares CFT ground state
MERA extends in extra scale dimension

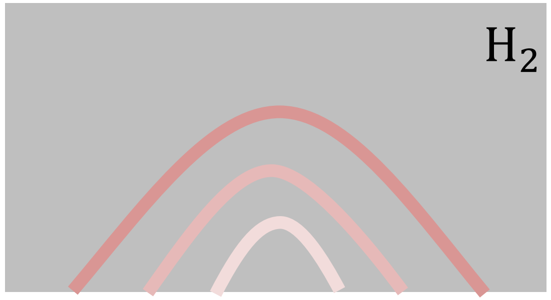
in MERA…

in hyperbolic space H_2 …

2

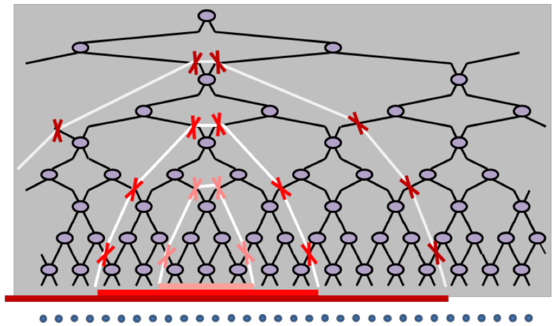


correlations $\sim e^{-D_{network}}$

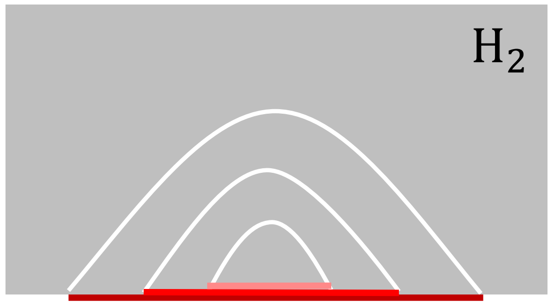


geodesics in H_2

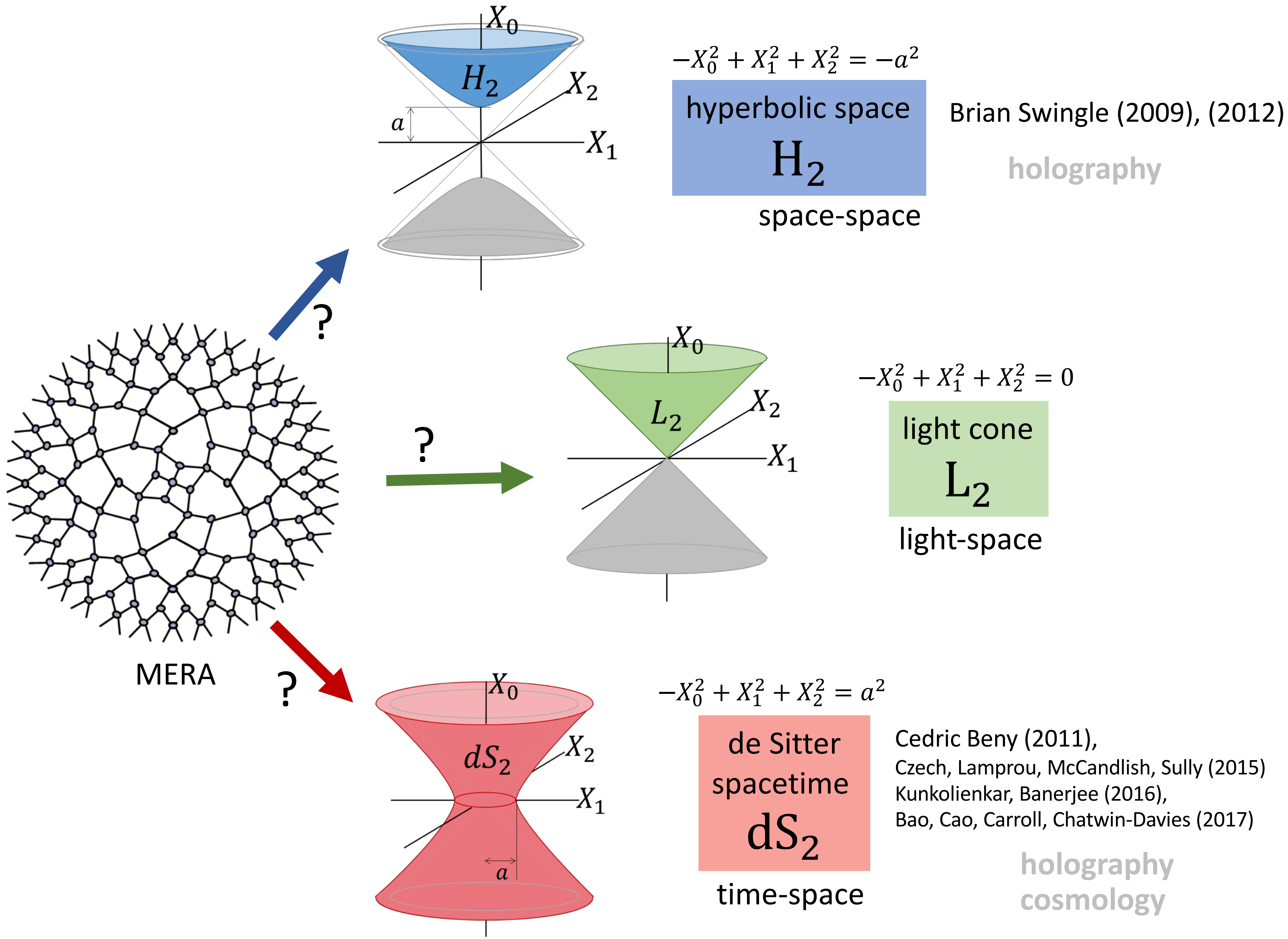
3



entanglement entropy $\sim$ minimal cut



CFT entanglement entropy $\sim$ minimal surface (Ryu-Takayanagi)



X_0
H_2
X_2
a
X_1
$-X_0^2 + X_1^2 + X_2^2 = -a^2$
hyperbolic space
H_2
space-space
Brian Swingle (2009), (2012)
holography
?
X_0
L_2
X_2
X_1
?
$-X_0^2 + X_1^2 + X_2^2 = 0$
light cone
L_2
light-space
MERA
?
X_0
dS_2
X_2
X_1
a
$-X_0^2 + X_1^2 + X_2^2 = a^2$
de Sitter
spacetime
dS_2
time-space
Cedric Beny (2011),
Czech, Lamprou, McCandlish, Sully (2015)
Kunkolienkar, Banerjee (2016),
Bao, Cao, Carroll, Chatwin-Davies (2017)
holography
cosmology

MERA as (toy model for) AdS/CFT correspondence & cosmology

10 years of debate!

MERA $=$ H_2 or dS_2 ?

Spoiler:

MERA $=$ L_2 light cone !

euclidean MERA $=$ H_2 hyperbolic space !!

lorentzian MERA $=$ dS_2 de Sitter spacetime !!!

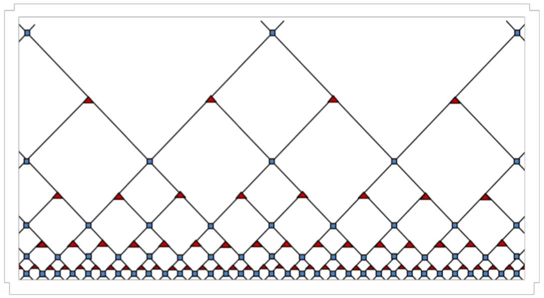
Ash Milsted
Perimeter postdoc

A. Milsted, G. Vidal arXiv:1805.12524
A. Milsted, G. Vidal arXiv:1807.02501
A. Milsted, G. Vidal arXiv:1812.00529

tensor network $=$ QFT path integral on curved spacetime

Outline

MERA tensor network



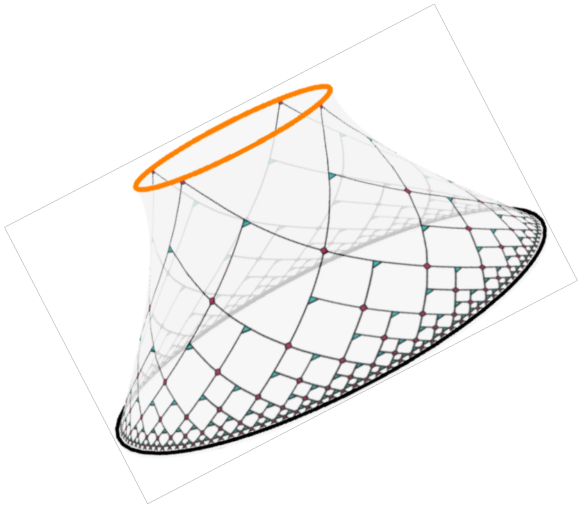
1) quantum circuit

2) renormalization group

quantum information

condensed matter

Tensor network as geometry



3) path integral

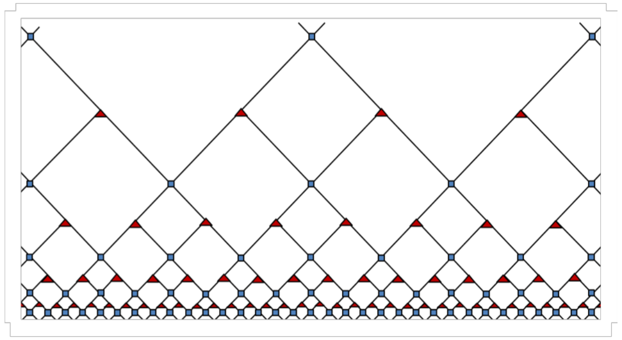
4) light cone

5) euclidean / lorentzian MERA

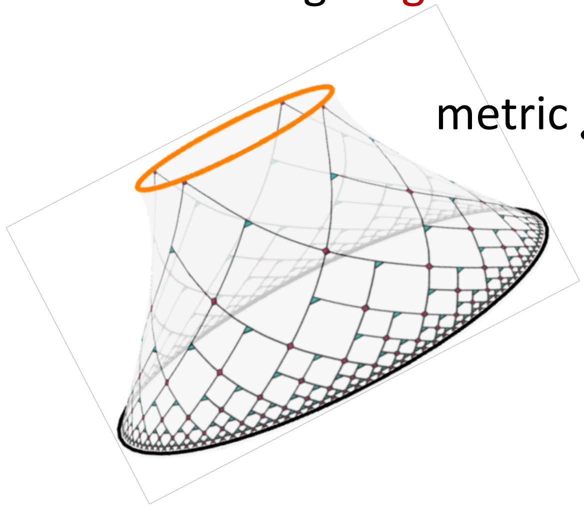
holography

cosmology

given a tensor network…



…how do we assign a geometry to it?

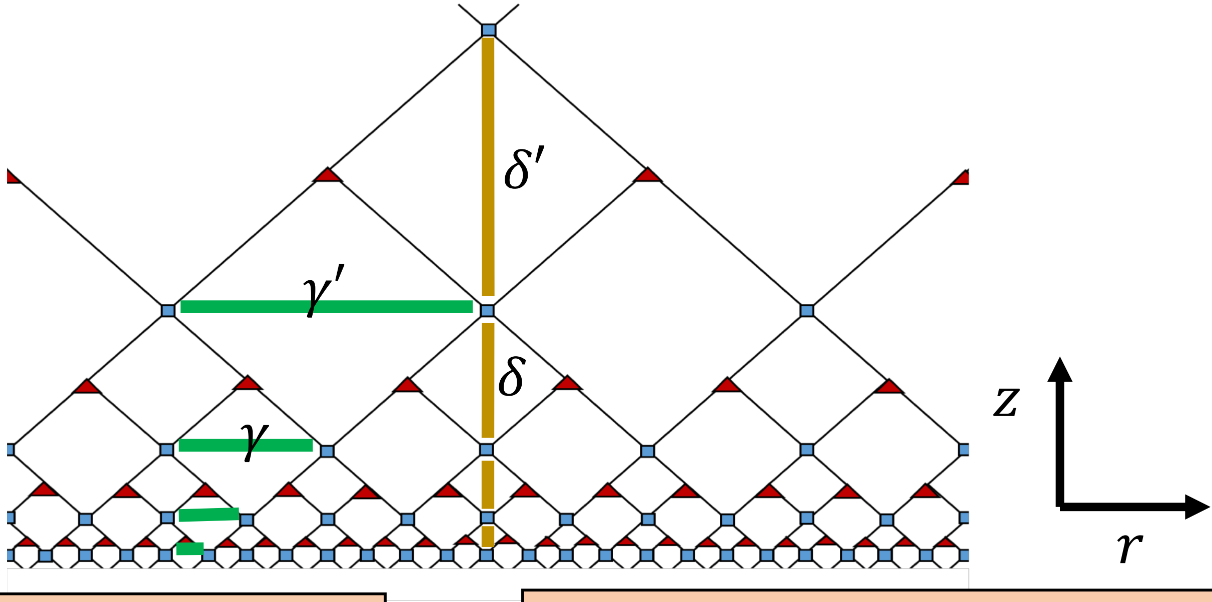


- network symmetries
- path integral

metric ?

$$ds^2 = g_{zz}(z,r)\,dz^2 + 2g_{zr}(z,r)dzdr + g_{rr}(z,r)dr^2$$

$$g_{\mu\nu}(z,r)?$$



z scale
r space

(1) translation invariance $r \to r + r_0$

$$g_{\mu\nu}(z,r) = g_{\mu\nu}(z)$$

(2) reflection symmetry* $r \to -r$

$$g_{zr}(z) = 0$$

(3) scale invariance $(z,r) \to \lambda(z,r)$

$d_\gamma = d_{\gamma'}$ Δr_{nn} between nearest neighbor tensors at fixed z is proportional to z, $\Delta r_{nn}(z) \sim z$

$d_\delta = d_{\delta'}$ Δz_{nn} between nearest neighbor tensors at fixed r is proportional to z, $\Delta z_{nn}(z) \sim z$

$$g_{rr}(z)(\Delta r_{nn})^2 \sim g_{rr}(z)z^2 = \pm\mu^2$$

$$\Rightarrow \boxed{g_{rr}(z) = \pm\mu^2/z^2}$$

$$g_{zz}(z)(\Delta z_{nn})^2 \sim g_{rr}(z)z^2 = \pm\nu^2$$

$$\Rightarrow \boxed{g_{zz}(z) = \pm\nu^2/z^2}$$

metric:

$$ds^2 = \frac{\pm R^2 dz^2 + dr^2}{z^2}$$

$+R^2$ hyperbolic space H_2
$-R^2$ de Sitter spacetime dS_2
$R \to 0$ light cone L_2

$$ds^2 = \frac{\pm d\eta^2 + dr^2}{(\eta/R)^2}$$

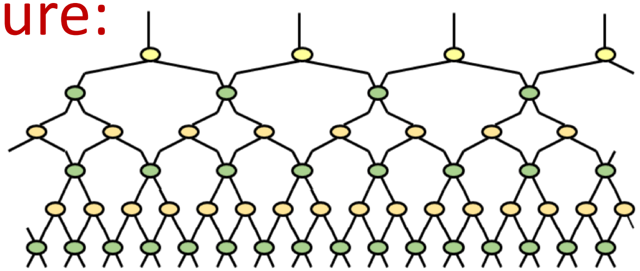
time $\eta \equiv Rz$

candidate metrics (by network symmetry):

$$\left(ds_{H_2}\right)^2 = \frac{+R^2dz^2 + dr^2}{z^2} \qquad \left(ds_{L_2}\right)^2 = \frac{dr^2}{z^2} \qquad \left(ds_{dS_2}\right)^2 = \frac{-R^2dz^2 + dr^2}{z^2}$$

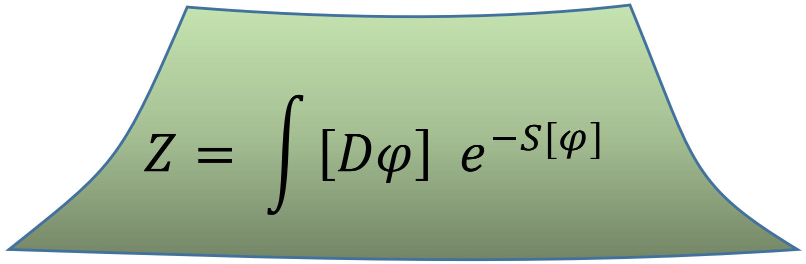
The network symmetry cannot help us choose one.
How about the content of the tensors?

conjecture:



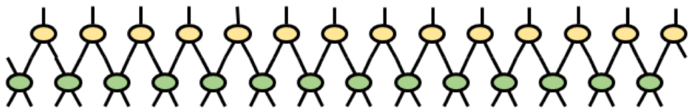
MERA

$\overset{?}{\approx}$



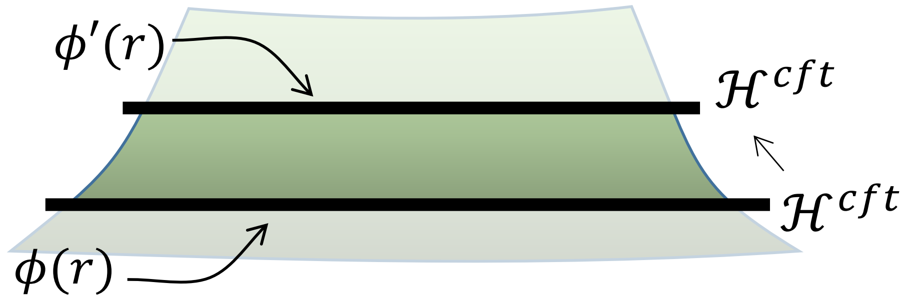
Path integral on some geometry

test:



layer of MERA

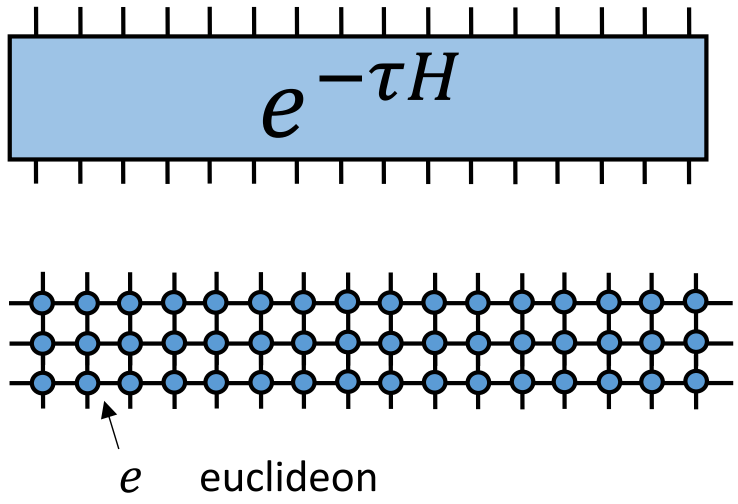
$\overset{?}{\approx}$



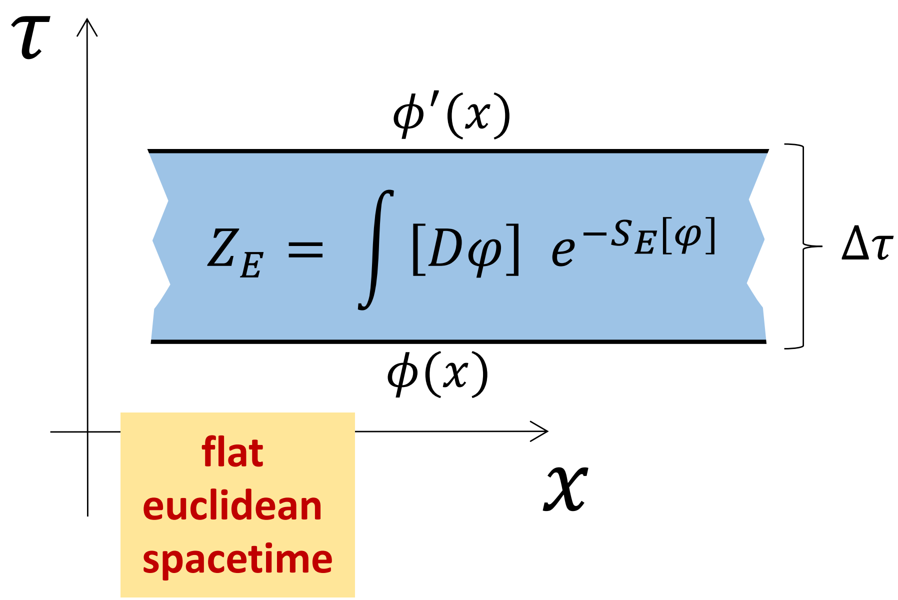
Linear map $V\colon \mathcal{H}^{cft} \to \mathcal{H}^{cft}$

$$\langle \phi(r)|V|\phi'(r)\rangle = \int_{strip} [D\varphi]\ e^{-S[\varphi]}$$

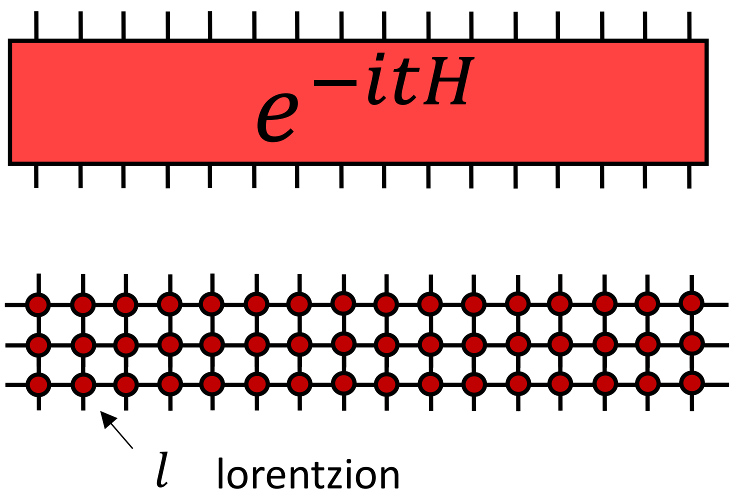
Euclidean time evolution



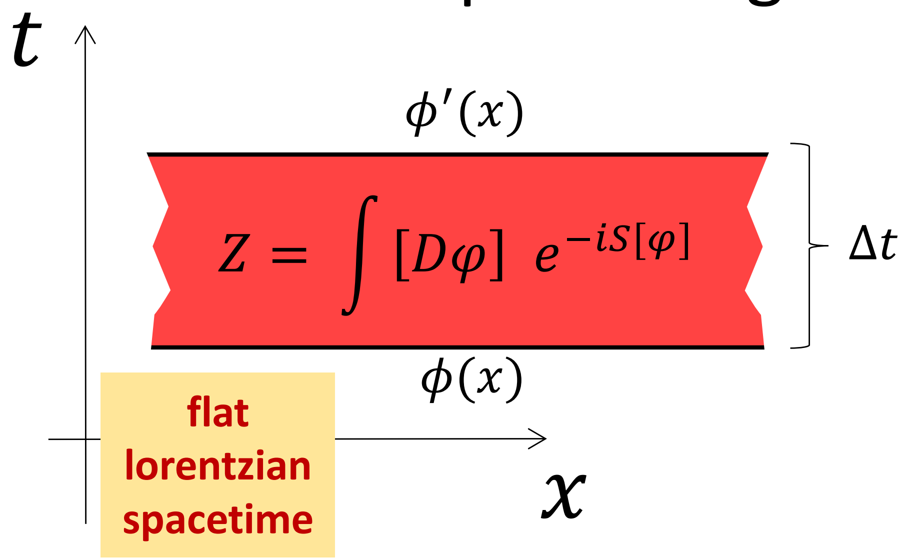
Euclidean path integral



Lorentzian time evolution



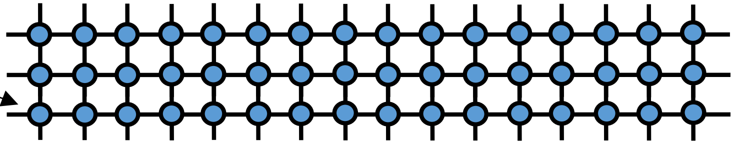
Lorentzian path integral



Euclidean evolution versus Lorentzian evolution

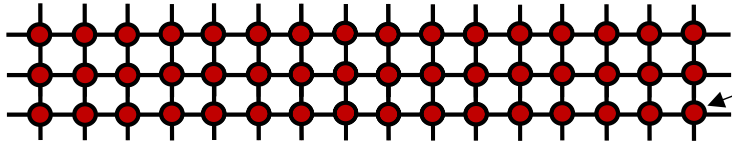
$$e^{-\tau H}$$

$$e^{-itH}$$



e

euclideon



l

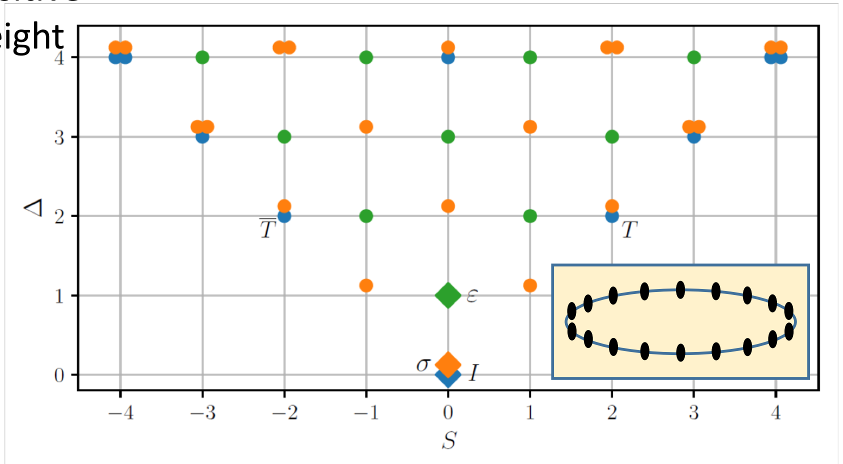
lorentzion

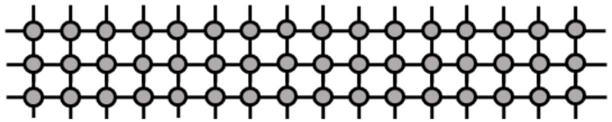
$$|\Psi_\alpha\rangle \to e^{-\tau E_\alpha}|\Psi_\alpha\rangle$$

positive weight

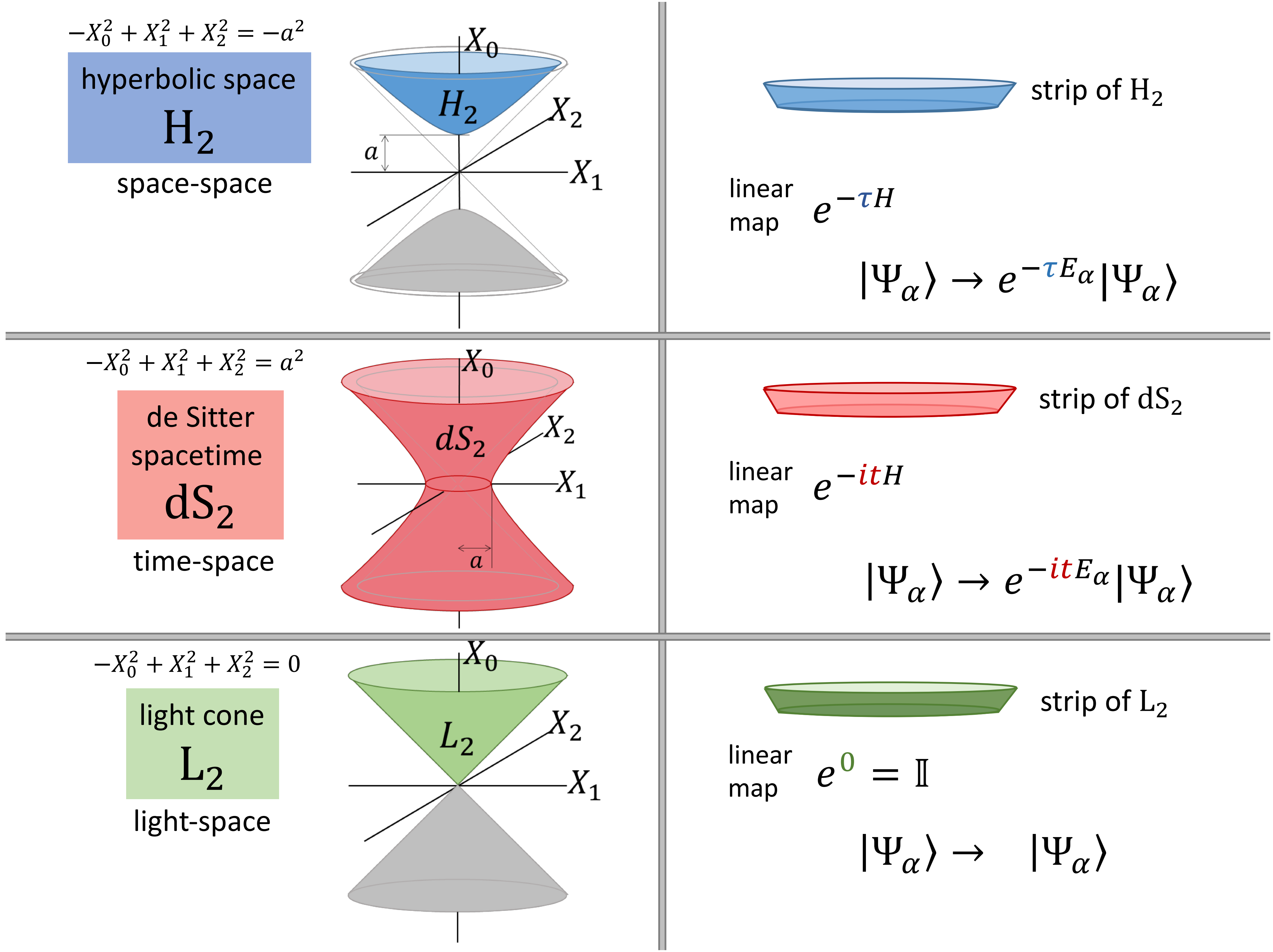
$$|\Psi_\alpha\rangle \to e^{-itE_\alpha}|\Psi_\alpha\rangle$$

complex phase



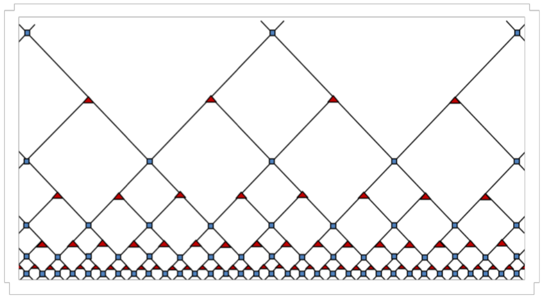


$e^{-\tau E_\alpha}$? e^{-itE_α}


$-X_0^2 + X_1^2 + X_2^2 = -a^2$
hyperbolic space
H_2
space-space
X_0
H_2
X_2
a
X_1
strip of H_2
linear map
$e^{-\tau H}$
$|\Psi_\alpha\rangle \rightarrow e^{-\tau E_\alpha}|\Psi_\alpha\rangle$
$-X_0^2 + X_1^2 + X_2^2 = a^2$
de Sitter spacetime
dS_2
time-space
X_0
dS_2
X_2
X_1
a
strip of dS_2
linear map
e^{-itH}
$|\Psi_\alpha\rangle \rightarrow e^{-itE_\alpha}|\Psi_\alpha\rangle$
$-X_0^2 + X_1^2 + X_2^2 = 0$
light cone
L_2
light-space
X_0
L_2
X_2
X_1
strip of L_2
linear map
$e^{0} = \mathbb{I}$
$|\Psi_\alpha\rangle \rightarrow \quad |\Psi_\alpha\rangle$

Outline

MERA tensor network



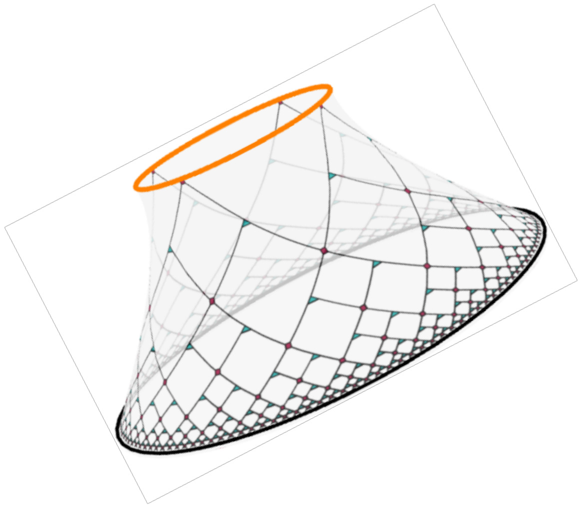
1) quantum circuit

2) renormalization group

quantum information

condensed matter

Tensor network as geometry



3) path integral

4) light cone

5) euclidean / lorentzian MERA

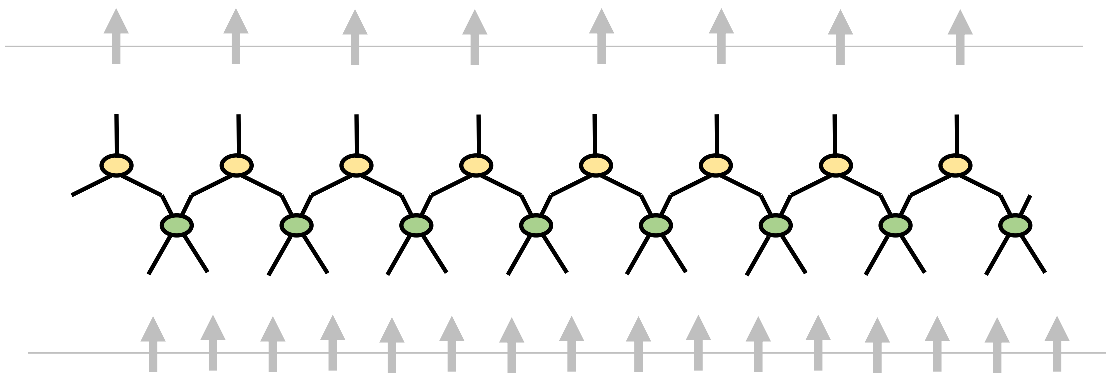
holography

cosmology

So… what linear map is implemented by a layer of MERA?

There is a problem…

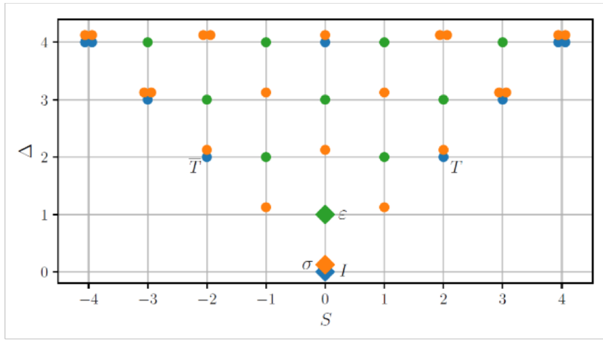
quantum
spin chain
$N = 8$



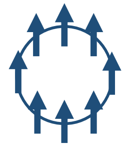
quantum
spin chain
$N = 16$

So… what linear map is implemented by a layer of MERA?

quantum
spin chain
$N = 8$

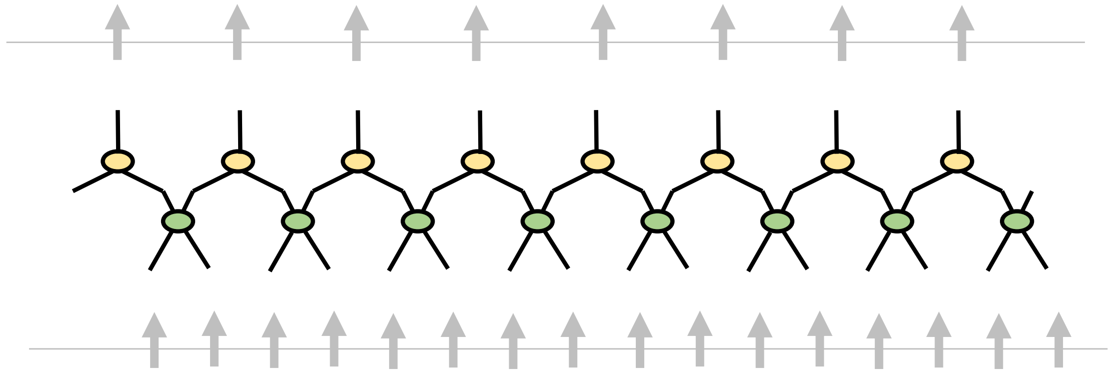


$|\Psi_\alpha^{N=8}\rangle$

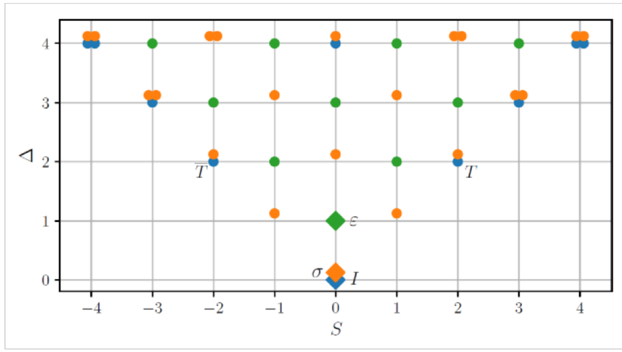


low energy basis

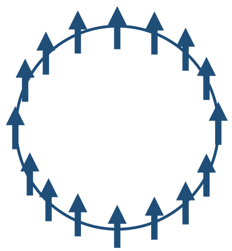
A. Milsted, G. Vidal, PRB 2017, arXiv:1706.01436
Y. Zou, A. Milsted, G. Vidal, PRL 2018, arXiv:1710.05397



quantum
spin chain
$N = 16$



$|\Psi_\alpha^{N=16}\rangle$

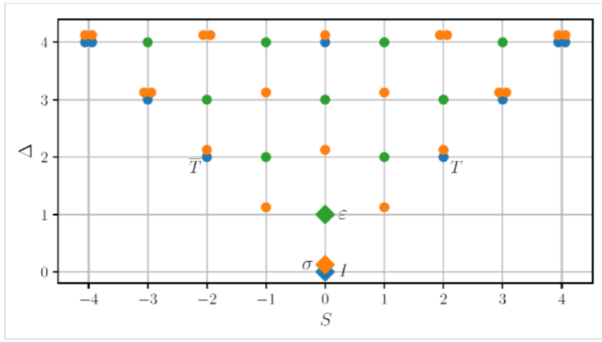


low energy basis

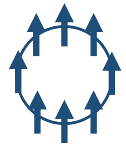
$$|\Psi_\alpha^{N=16}\rangle \to W|\Psi_\alpha^{N=16}\rangle \overset{?}{=} \sum_\beta W_{\alpha\beta}|\Psi_\beta^{N=8}\rangle$$

So… what linear map is implemented by a layer of MERA?

quantum
spin chain
$N = 8$

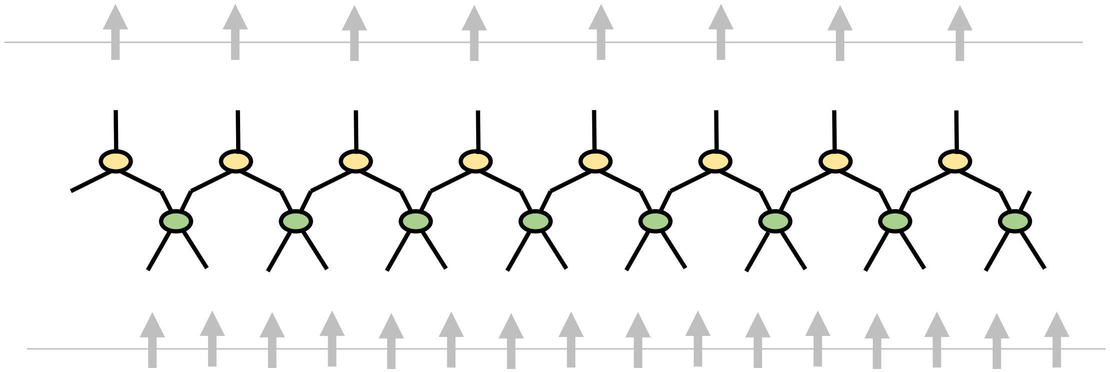


$|\Psi_\alpha^{N=8}\rangle$

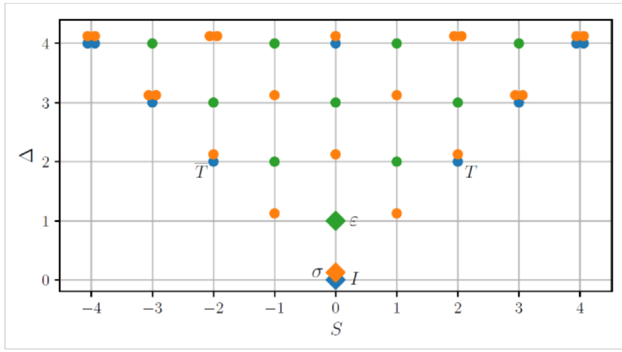


low energy basis

A. Milsted, G. Vidal, PRB 2017, arXiv:1706.01436
Y. Zou, A. Milsted, G. Vidal, PRL 2018, arXiv:1710.05397



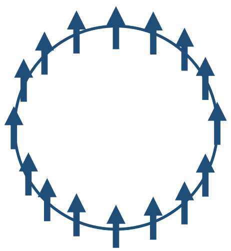
quantum
spin chain
$N = 16$



- hyperbolic space? $|\Psi_\alpha^{N=16}\rangle \rightarrow e^{-\tau H}|\Psi_\alpha^{N=8}\rangle$
- Light cone! $|\Psi_\alpha^{N=16}\rangle \rightarrow e^{0}|\Psi_\alpha^{N=8}\rangle$
- de Sitter spacetime? $|\Psi_\alpha^{N=16}\rangle \rightarrow e^{-itH}|\Psi_\alpha^{N=8}\rangle$

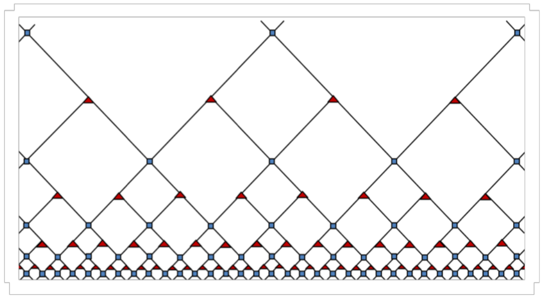
$|\Psi_\alpha^{N=16}\rangle$

low energy basis



Outline

MERA tensor network



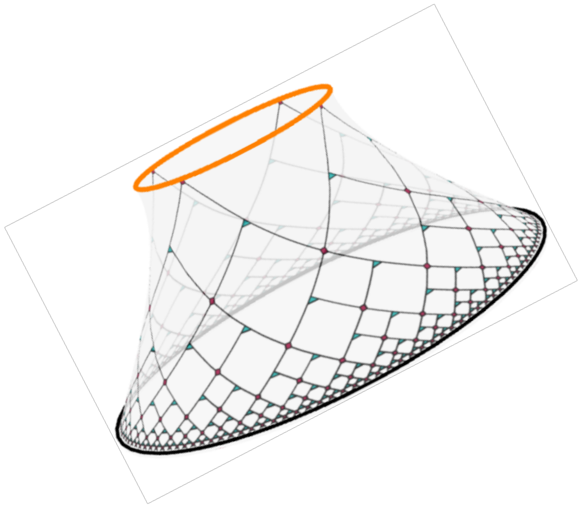
1) quantum circuit

2) renormalization group

quantum information

condensed matter

Tensor network as geometry



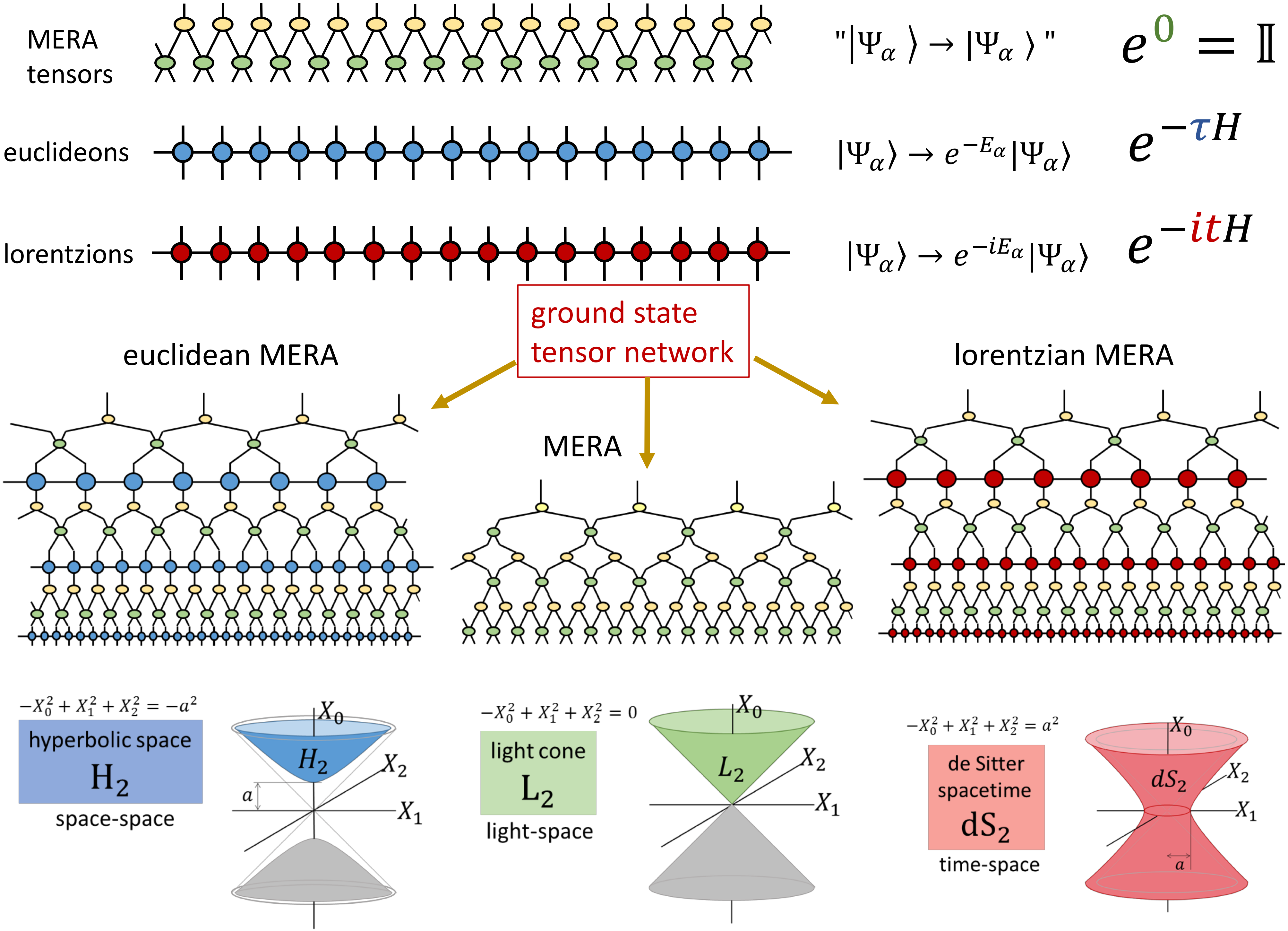
3) path integral

4) light cone

5) euclidean / lorentzian MERA

holography

cosmology



MERA tensors
"$|\Psi_\alpha\rangle \to |\Psi_\alpha\rangle$"
$e^{0} = \mathbb{I}$
euclideons
$|\Psi_\alpha\rangle \to e^{-E_\alpha}|\Psi_\alpha\rangle$
$e^{-\tau H}$
lorentzions
$|\Psi_\alpha\rangle \to e^{-iE_\alpha}|\Psi_\alpha\rangle$
e^{-itH}
ground state tensor network
euclidean MERA
MERA
lorentzian MERA
$-X_0^2 + X_1^2 + X_2^2 = -a^2$
hyperbolic space
H_2
space-space
X_0
H_2
X_2
a
X_1
$-X_0^2 + X_1^2 + X_2^2 = 0$
light cone
L_2
light-space
X_0
L_2
X_2
X_1
$-X_0^2 + X_1^2 + X_2^2 = a^2$
de Sitter spacetime
dS_2
time-space
X_0
dS_2
X_2
X_1
a

CONCLUSION

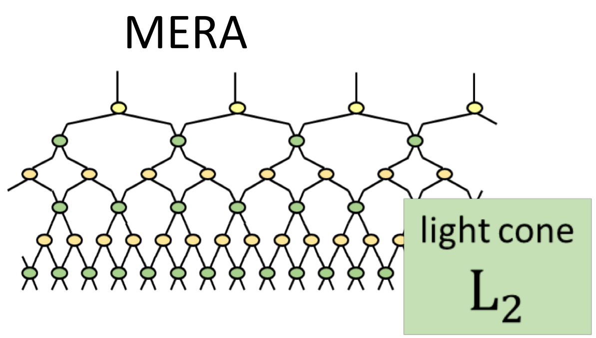
In the quest of assigning a geometry to tensor networks…

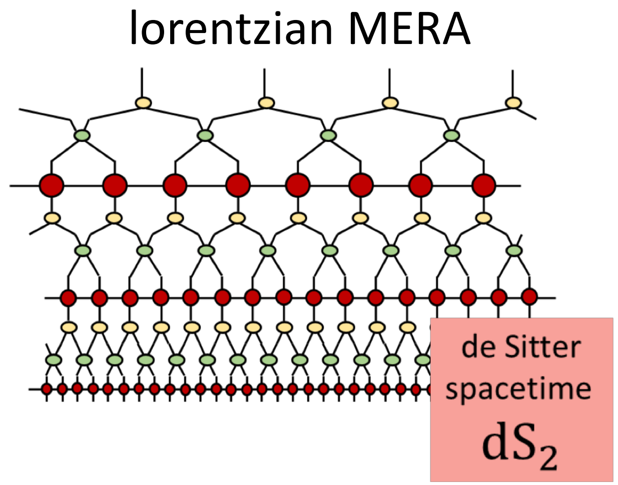
…we have understood that MERA
(and euclidean/lorenzian generalizations)
represents a CFT path integral on some curved spacetime

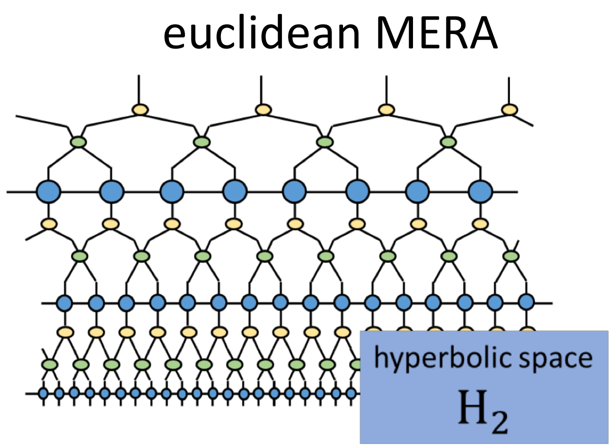
(simplest examples of a much more general construction)

MERA = Holography?

Maybe… ("MERA" is as holographic as a "2d path integral on a light cone geometry")







THANKS!