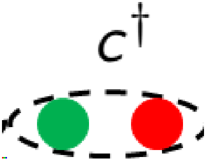
Combating quasiparticle poisoning with multiple Floquet Majorana modes

Raditya Weda Bomantara

Centre for Engineered Quantum Systems
University of Sydney

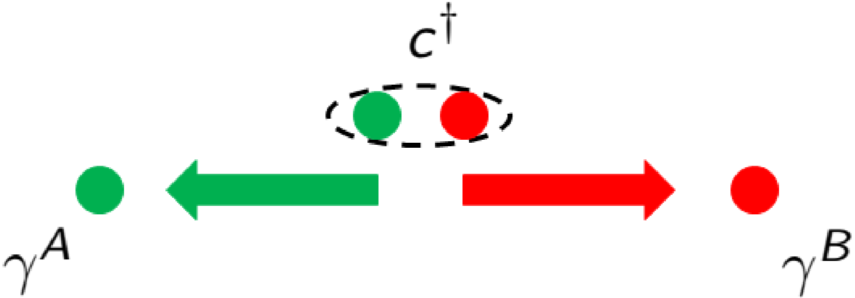
Majorana fermions for quantum computing

- Fermions can be occupied or unoccupied, i.e., $c^\dagger c = 0, 1$
- Write $c^\dagger = \gamma^A + \mathrm{i}\gamma^B$, with $\gamma^S = \gamma^{S\dagger}$ and $\left\{\gamma^S, \gamma^{S'}\right\} = 2\delta_{S,S'}$.
- Occupancy of the actual fermion is encoded by $\mathrm{i}\gamma^A\gamma^B = \pm 1$.
- When γ^A and γ^B are spatially separated, can encode a qubit nonlocally.



Majorana fermions for quantum computing

- Fermions can be occupied or unoccupied, i.e., $c^\dagger c = 0, 1$
- Write $c^\dagger = \gamma^A + \mathrm{i}\gamma^B$, with $\gamma^S = \gamma^{S\dagger}$ and $\left\{\gamma^S, \gamma^{S'}\right\} = 2\delta_{S,S'}$.
- Occupancy of the actual fermion is encoded by $\mathrm{i}\gamma^A\gamma^B = \pm 1$.
- When γ^A and γ^B are spatially separated, can encode a qubit nonlocally.



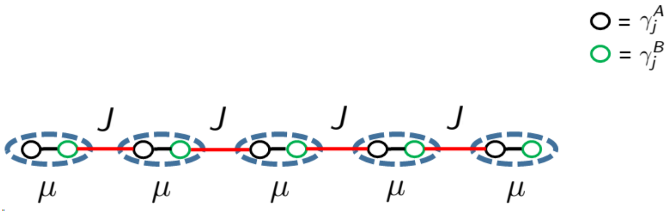
Majorana fermions as Majorana zero modes (MZMs)

- spinless p-wave superconducting system:

$$H = \sum_{j=1}^{N-1} \left(-J c_{j+1}^{\dagger} c_j + \Delta c_{j+1}^{\dagger} c_j^{\dagger} + h.c. \right) + \sum_{j=1}^{N} \mu c_j^{\dagger} c_j \ . \tag{1}$$

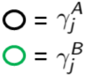
- Let $c_j^{\dagger} = \frac{1}{2}\left(\gamma_j^A + \mathrm{i}\gamma_j^B\right)$, with $\gamma_j^S = \gamma_j^{S\dagger}$ and $\left\{\gamma_j^S, \gamma_{j'}^{S'}\right\} = 2\delta_{S,S'}\delta_{j,k}$.
- Take $\Delta = J$ for simplicity,

$$H = \sum_{j=1}^{N} \frac{\mu}{2}\, \mathrm{i}\gamma_j^A \gamma_j^B + \sum_{j=1}^{N-1} J\, \mathrm{i}\gamma_{j+1}^B \gamma_j^A \ . \tag{2}$$



Majorana fermions as MZMs

- Two extreme cases:



$\mu = 0$

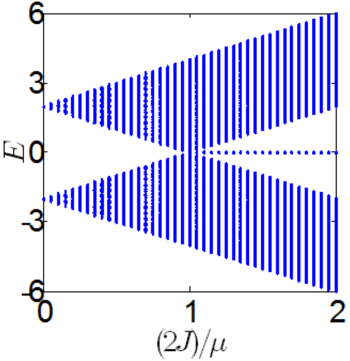
Topologically nontrivial, MZMs exist

$J = 0$

Topologically trivial, No MZMs

Majorana fermions as MZMs

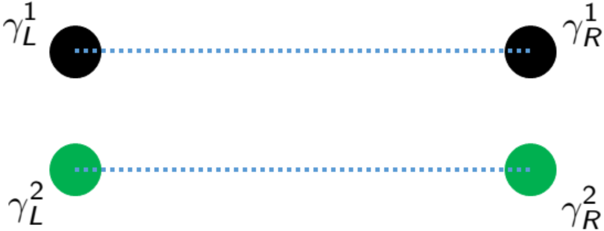
- Energy excitation at finite μ and J.



- Gap closing at $\mu = 2J$, separating two regions with and without MZMs respectively.
- No adiabatic deformation is possible from one region to the other $\implies$ they are topologically distinct!

Quantum computing with MZMs

- **Qubit definition:**
  - Prepare two *p*-wave superconductors with four MZMs in total



  - Define two basis qubit states $|0\rangle$ and $|1\rangle$ such that

$$\mathrm{i}\gamma_L^1\gamma_R^1|0\rangle = \mathrm{i}\gamma_L^2\gamma_R^2|0\rangle = |0\rangle \,, \tag{3}$$

$$\mathrm{i}\gamma_L^1\gamma_R^1|1\rangle = \mathrm{i}\gamma_L^2\gamma_R^2|1\rangle = -|1\rangle \,. \tag{4}$$

  - Note that $\gamma_L^1\gamma_R^1\gamma_L^2\gamma_R^2|S\rangle = |S\rangle$ for $S = 0, 1$.
  - The two basis states are related by $|1\rangle = \gamma_L^1\gamma_L^2|0\rangle$.

Quantum computing with MZMs

- **Quantum gate operations:**
  - Implementing *HZ* gate by braiding:

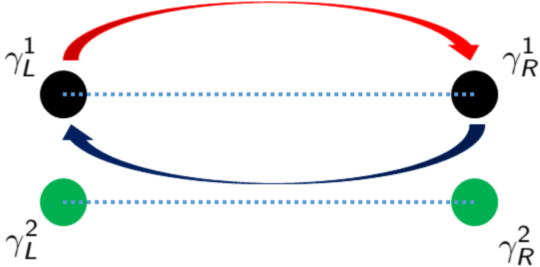
  - Braiding matrix $U = \exp\left(\frac{\pi}{4}\gamma_L^1\gamma_L^2\right) = \frac{1}{\sqrt{2}}\left(1 + \gamma_L^1\gamma_L^2\right)$.
  - Note that $U^\dagger\gamma_L^1 U = \gamma_L^2$, $U^\dagger\gamma_L^2 U = -\gamma_L^1$, $U^\dagger\gamma_R^1 U = \gamma_R^1$, and $U^\dagger\gamma_R^2 U = \gamma_R^2$.
  - Indeed,

$$U|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) , \tag{5}$$

$$U|1\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle - |0\rangle\right) . \tag{6}$$

Quantum computing with MZMs

- **Quantum gate operations:**
  - Implementing $P = \exp\left(-\mathrm{i}\frac{\pi}{4}Z\right)$ gate by braiding:



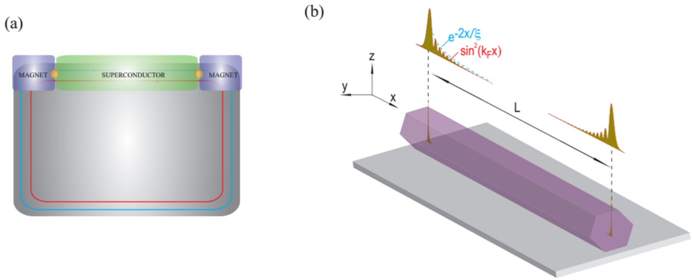
  - Braiding matrix $U = \exp\left(\frac{\pi}{4}\gamma_L^1\gamma_R^1\right) = \frac{1}{\sqrt{2}}\left(1 + \gamma_L^1\gamma_R^1\right)$.
  - Note that $U^\dagger\gamma_L^1 U = \gamma_R^1$, $U^\dagger\gamma_R^1 U = -\gamma_L^1$, $U^\dagger\gamma_L^2 U = \gamma_L^2$, and $U^\dagger\gamma_R^2 U = \gamma_R^2$.
  - Indeed,

$$U|0\rangle = \frac{1}{\sqrt{2}}\left(1 - \mathrm{i}\right)|0\rangle \,, \tag{7}$$

$$U|1\rangle = \frac{1}{\sqrt{2}}\left(1 + \mathrm{i}\right)|1\rangle \,. \tag{8}$$

Proposals to realize *p*-wave superconductors:

- Chiral edge states of topological insulators, proximitized by *s*-wave superconductivity. PRL **100**, 096407 (2008)
- Semiconducting nanowire proximitized by *s*-wave superconductivity and subject to perpendicular magnetic field. PRL **105**, 077001 (2010); PRL **105**, 177002 (2010)



Picture taken from RIV NUOVO CIMENTO **11**, 523-593 (2017)

Quasiparticle poisoning problem

- Majorana-based quantum computing relies on the conservation of total Majorana parity $\gamma_L^1\gamma_R^1\gamma_L^2\gamma_R^2$.
- Current proposals to realize *p*-wave superconductors necessarily involve coupling with environments.
- **Quasiparticle poisoning:** The unwanted flow of Majorana fermions in and out the system $\implies$ total Majorana parity is no longer conserved.
- Results in the low coherence time of MZMs, i.e., between 10 ns to 0.1 ms. PRB **85**, 174533 (2012)
- Either fast quantum computation or supplement with active quantum error corrections.

Time-periodic p-wave superconductors

$$
\begin{aligned}
H(t) &= \begin{cases} H_1 & \text{for } MT < t \leq (M+\frac{1}{2})T \\ H_2 & \text{for } (M+\frac{1}{2})T < t < (M+1)T \end{cases} , \\
H_S &= \sum_{j}^{N-1} \left(-J_S c_{j+1}^\dagger c_j + \Delta_S c_{j+1}^\dagger c_j^\dagger + h.c. \right) + \mu_S \sum_j^N c_j^\dagger c_j \; , \qquad (9)
\end{aligned}
$$

- Under periodic boundary conditions,

$$
\begin{aligned}
H(t) &= \frac{1}{2} \sum_k \psi_k^\dagger h(k,t) \psi_k \; , \qquad (10) \\
h(k,t) &= \left[\mu(t) - 2J(t)\cos(k) \right] \sigma_z + 2\Delta(t) \sin(k) \sigma_y \; , \qquad (11)
\end{aligned}
$$

where $\psi_k = \begin{pmatrix} c_k \\ c_{-k}^\dagger \end{pmatrix}$.

Time-periodic *p*-wave superconductors

- symmetric time-frame Floquet operator,

$$\begin{aligned} U &\equiv U(t-\frac{T}{2};t+\frac{T}{2}) \\ &= \exp\left(-\mathrm{i}\frac{H_1 T}{4\hbar}\right)\times\exp\left(-\mathrm{i}\frac{H_2 T}{2\hbar}\right)\times\exp\left(-\mathrm{i}\frac{H_1 T}{4\hbar}\right) \end{aligned} \quad (12)$$

- Its eigenvalues are of the form $\exp\left(-\mathrm{i}\varepsilon T/\hbar\right)$.
- $\varepsilon \in (-\frac{\hbar\pi}{T}, \frac{\hbar\pi}{T}]$ is called quasienergy.
- In addition to MZMs, Majorana fermions can also exist as $\frac{\hbar\pi}{T}$ quasienergy excitations, i.e., Majorana π modes (MPMs).

Time-periodic p-wave superconductors

- **Bulk-edge correspondence:** The presence of MZMs and MPMs can be determined from bulk properties.
- Under PBC, the momentum space Floquet operator is

$$\begin{aligned} u(k) &= F(k)G(k) \,, \\ F(k) &= \exp\left(-\mathrm{i}\frac{h_1(k)T}{4\hbar}\right) \times \exp\left(-\mathrm{i}\frac{h_2(k)T}{4\hbar}\right) \,, \\ G(k) &= \exp\left(-\mathrm{i}\frac{h_2(k)T}{4\hbar}\right) \times \exp\left(-\mathrm{i}\frac{h_1(k)T}{4\hbar}\right) \,, \end{aligned} \tag{13}$$

where

$$h_S(k) = \left[\mu_S - 2J_S \cos(k)\right] \sigma_z + 2\Delta_S \sin(k)\sigma_y \,. \tag{14}$$

Time-periodic p-wave superconductors

- Transform $F(k)$ and $G(k)$ to canonical basis, such that

$$\sigma_z F(k) \sigma_z = G(k)^\dagger \ . \tag{15}$$

- Write

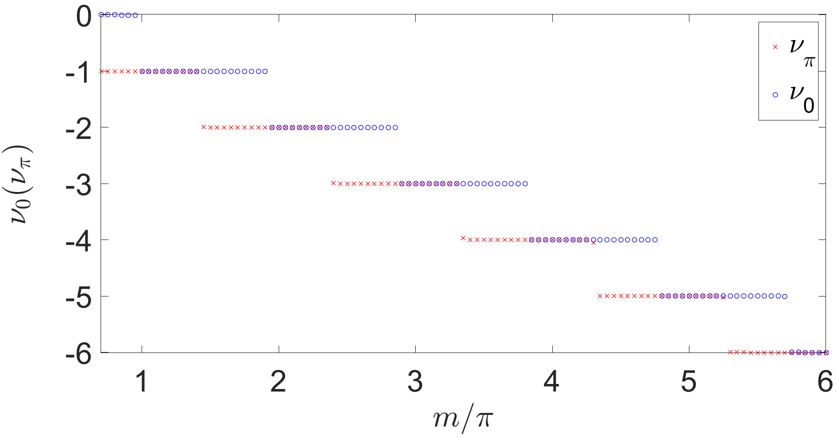
$$F(k) = \begin{pmatrix} A(k) & B(k) \\ C(k) & D(k) \end{pmatrix} \ . \tag{16}$$

- Number of MZMs and MPMs are given by

$$\begin{aligned} \nu_0 &= \frac{1}{2\pi \mathrm{i}} \int dk B^{-1} \frac{dB}{dk} \ , \\ \nu_\pi &= \frac{1}{2\pi \mathrm{i}} \int dk D^{-1} \frac{dD}{dk} \ . \end{aligned} \tag{17}$$

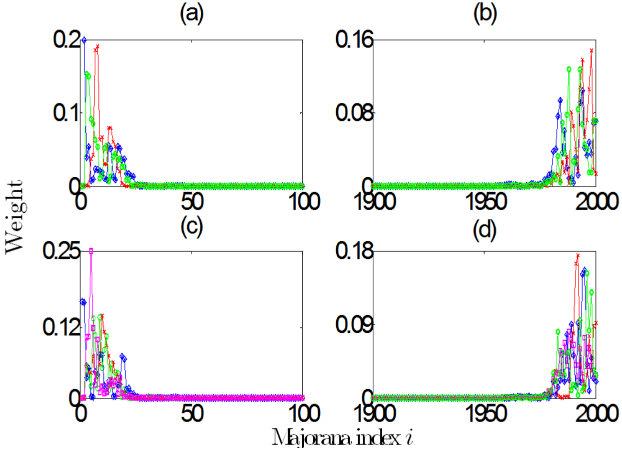
Time-periodic p-wave superconductors

- **Main trick:** $\mu_2 = m\mu_1$, $J_2 = -mJ_1$, and $\Delta_2 = -m\Delta_1$, where $m \in \mathbb{R}$.



Time-periodic p-wave superconductors

- At $m = 3.6\pi$, there are 3 pairs of MZMs and 4 pairs of MPMs (14 Majoranas in total)
- By writing each Majorana as $\gamma = \sum_i w_i \gamma_i$, where $c_j = \gamma_{2j} - \mathrm{i}\gamma_{2j+1}$,



Majorana stabilizer codes to combat quasiparticle poisoning

- Using our proposed model at $m = 3.6\pi$,
  - 3 pairs of MZMs: $\gamma_{0,L,1}$, $\gamma_{0,L,2}$, $\gamma_{0,L,3}$, $\gamma_{0,R,1}$, $\gamma_{0,R,2}$, $\gamma_{0,R,3}$.
  - 4 pairs of MPMs: $\gamma_{0,L,1}$, $\gamma_{0,L,2}$, $\gamma_{0,L,3}$, $\gamma_{0,L,4}$ $\gamma_{0,R,1}$, $\gamma_{0,R,2}$, $\gamma_{0,R,3}$, $\gamma_{0,R,4}$.
- Six weight-four Majorana stabilizers

$$\begin{aligned}
\mathcal{S}_1 &= \gamma_{0,L,1}\gamma_{0,R,1}\gamma_{\pi,L,1}\gamma_{\pi,R,1} \ , \\
\mathcal{S}_2 &= \gamma_{0,L,1}\gamma_{0,R,2}\gamma_{\pi,L,1}\gamma_{\pi,R,2} \ , \\
\mathcal{S}_3 &= \gamma_{0,L,1}\gamma_{0,R,1}\gamma_{\pi,L,2}\gamma_{\pi,R,2} \ , \\
\mathcal{S}_4 &= \gamma_{0,L,2}\gamma_{0,R,3}\gamma_{\pi,L,3}\gamma_{\pi,R,3} \ , \\
\mathcal{S}_5 &= \gamma_{0,L,3}\gamma_{0,R,3}\gamma_{\pi,L,3}\gamma_{\pi,R,4} \ , \\
\mathcal{S}_6 &= \gamma_{0,L,2}\gamma_{0,R,3}\gamma_{\pi,L,4}\gamma_{\pi,R,4} \ . \qquad (18)
\end{aligned}$$

- Logical operators,

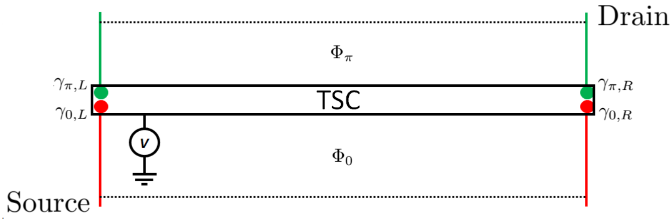
$$\begin{aligned}
Z_L &= \gamma_{0,L,1}\gamma_{0,L,2}\gamma_{0,L,3}\gamma_{0,R,1}\gamma_{0,R,2}\gamma_{0,R,3} \ , \\
X_L &= \gamma_{0,L,1}\gamma_{0,R,1}\gamma_{0,R,2} \ . \qquad (19)
\end{aligned}$$

Majorana stabilizer codes to combat quasiparticle poisoning

- **Error model:** Application of any single Majorana operator.
- Any of such errors anticommutes with a *unique* set of stabilizers,

Error	Anticommutes with
$\gamma_{0,L,1}$	$\mathcal{S}_1$, $\mathcal{S}_2$, and $\mathcal{S}_3$
$\gamma_{0,L,2}$	$\mathcal{S}_4$ and $\mathcal{S}_6$
$\gamma_{0,L,3}$	$\mathcal{S}_5$
$\gamma_{0,R,1}$	$\mathcal{S}_1$ and $\mathcal{S}_3$
$\gamma_{0,R,2}$	$\mathcal{S}_2$
$\gamma_{0,R,3}$	$\mathcal{S}_4$, $\mathcal{S}_5$, and $\mathcal{S}_6$
$\gamma_{\pi,L,1}$	$\mathcal{S}_1$, and $\mathcal{S}_2$
$\gamma_{\pi,L,2}$	$\mathcal{S}_3$
$\gamma_{\pi,L,3}$	$\mathcal{S}_4$ and $\mathcal{S}_5$
$\gamma_{\pi,L,4}$	$\mathcal{S}_6$
$\gamma_{\pi,R,1}$	$\mathcal{S}_1$
$\gamma_{\pi,R,2}$	$\mathcal{S}_2$ and $\mathcal{S}_3$
$\gamma_{\pi,R,3}$	$\mathcal{S}_4$
$\gamma_{\pi,R,4}$	$\mathcal{S}_5$ and $\mathcal{S}_6$

Stabilizer measurements via four-terminal conductance

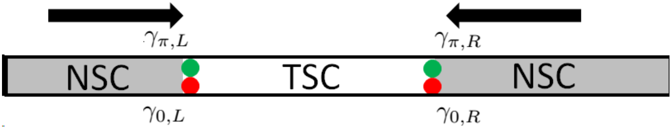


- Using third-order Floquet perturbation theory, PRB **101**, 085401 (2020)

$$\bar{G} = a_0 + a_1\langle \mathrm{i}\gamma_{0,L}\gamma_{0,R}\rangle \sin\left[\frac{e}{\hbar}(\Phi_0 - \phi_0)\right] + a_2\langle \mathrm{i}\gamma_{\pi,L}\gamma_{\pi,R}\rangle \sin\left[\frac{e}{\hbar}(\Phi_\pi - \phi_\pi)\right] + a_3\langle \gamma_{0,L}\gamma_{0,R}\gamma_{\pi,L}\gamma_{\pi,R}\rangle \cos\left[\frac{e}{\hbar}(\Phi_\pi - \phi_\pi - \Phi_0 + \phi_0)\right], \quad (20)$$

State initialization

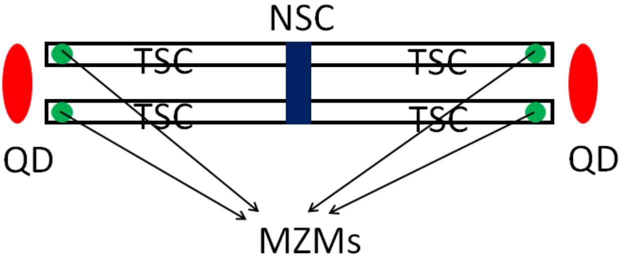
- Tune some system parameters to move the topologically nontrivial edges closer to each other.



- Hybridization of Majoranas lead to splitting in quasienergy degeneracy.

Integration into scalable Majorana-qubit architectures

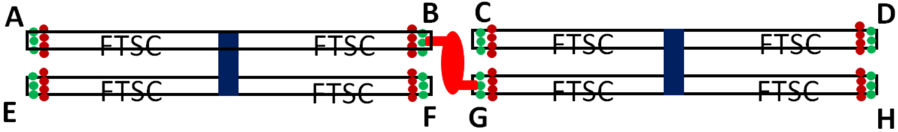
- Recall the two-sided tetron design PRB **95**, 235305 (2017)



- Apply the proposed time-periodic drive

Integration into scalable Majorana-qubit architectures

- Apply quantum gate operations via a series of measurements like its static counterparts.
- Inherent active quantum error corrections for each tetron to mitigate quasiparticle poisoning effect.



Summary and potential future direction

Summary:

- Majorana-based qubits are themselves topologically protected and allow topologically protected Clifford gate operations.
- In their current experimental realizations, quasiparticle poisoning is unavoidable.
- Time-periodic drive allows arbitrarily many Majorana modes to emerge at each end of the system.
- With 14 Majorana modes in total, a stabilizer code capable of correcting a single quasiparticle poisoning event can be implemented.
- Compatibility with scalable Majorana-qubit architectures.

Possible future direction:

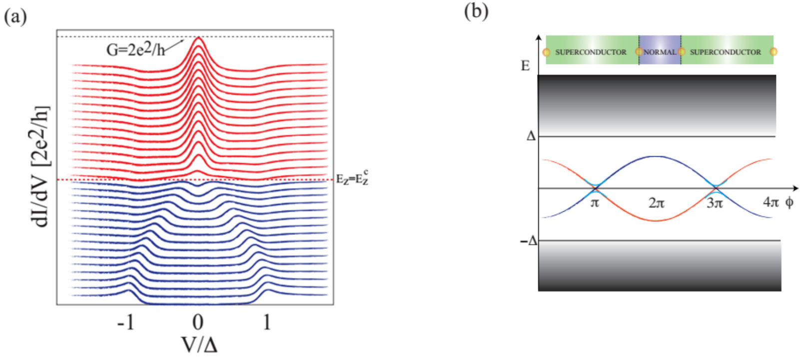
- Application in designing topological codes with lower space-overhead.
- Work towards experimental realizations of Floquet Majorana fermions:
  - Proposals for detecting MPMs.
  - Replace periodic quench in the current model with more experimentally friendly time periodic functions.

More info, see arXiv:1912.03827

Appendix

Detection of MZMs

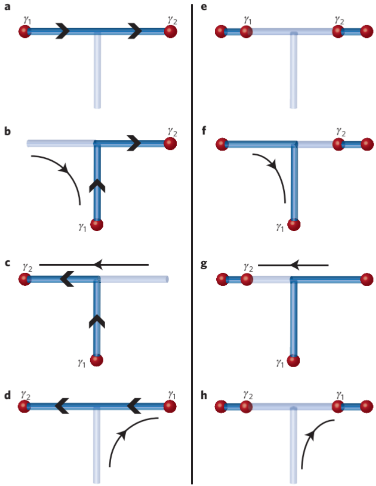
- Zero bias peak in differential conductance.
  Science **336**, 1003-1007 (2012); PRL **119**, 136803 (2017); Nature **556**, 74 (2018)
- 4π Josephson effect. Nat. Phys. **8**, 795 (2012)



Picture taken from RIV NUOVO CIMENTO **11**, 523-593 (2017)

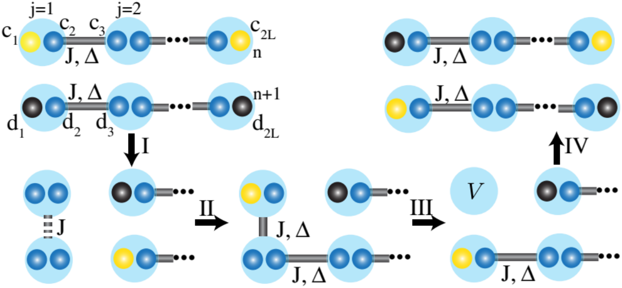
Some braiding proposals

- T-junction Nat. Phys. **7**, 412 (2011)



Some braiding proposals

- Array of superconducting wires PRL **111**, 203001 (2013)



Analytical calculations of ν_0 and ν_π

- Focus on $\mu_1 = J_1 = \Delta_1 = \delta$.

$$
\begin{aligned}
F(k) &= \begin{pmatrix} c(\theta_-)c(m\theta_+) - \mathrm{i}s(\theta_-)s(m\theta_+) & e^{-\mathrm{i}k/2}\left[c(\theta_-)s(m\theta_+) + \mathrm{i}c(m\theta_+)s(\theta_-)\right] \\ -e^{\mathrm{i}k/2}\left[c(\theta_-)s(m\theta_+) - \mathrm{i}c(m\theta_+)s(\theta_-)\right] & c(\theta_-)c(m\theta_+) + \mathrm{i}s(\theta_-)s(m\theta_+) \end{pmatrix} , \\
\theta_\pm &= \frac{\delta T}{4\hbar}\sqrt{2(1 \pm \cos(k))}
\end{aligned}
\quad (21)
$$

- Define $z = \theta_- + \mathrm{i}m\theta_+$,

$$
\begin{aligned}
\nu_0 &= -\frac{1}{2} - \frac{1}{4\pi\mathrm{i}} \oint \frac{s(\mathrm{Re}(z))s(\mathrm{Im}(z))dz + \mathrm{i}c(\mathrm{Re}(z))c(\mathrm{Im}(z))dz^*}{c(\mathrm{Re}(z))s(\mathrm{Im}(z)) + \mathrm{i}c(\mathrm{Im}(z))s(\mathrm{Re}(z))} , \\
\nu_\pi &= -\frac{1}{4\pi\mathrm{i}} \oint \frac{\sin(\mathrm{Re}(z))c(\mathrm{Im}(z))dz^* - \mathrm{i}c(\mathrm{Re}(z))s(\mathrm{Im}(z))dz}{c(\mathrm{Re}(z))c(\mathrm{Im}(z)) + \mathrm{i}s(\mathrm{Im}(z))s(\mathrm{Re}(z))} ,
\end{aligned}
\quad (22)
$$

- Let $n\pi/2 < m < n\pi/2 + \pi$, where $n \in \mathbb{Z}$.
- By residue theorem,

$$
\begin{aligned}
\nu_0 &= n , \\
\nu_\pi &= \frac{n+1}{2} .
\end{aligned}
\quad (23)
$$