Workshop Tutorials for Introductory Physics
Solutions to EI1: **Charge and Coulomb's Law**

A. Review of Basic Ideas:

Charge and Electrostatic Force

Electrical interactions play a key role in the chemical bonding of matter and in most biological processes such as seeing, feeling, moving and thinking. Electrical interactions occur between bodies and particles which have **electric** charge which, like mass, is a fundamental property of matter.

Charge comes in discrete amounts and, like energy, it is always **conserved**. The SI unit of charge is the coulomb (symbol C). The smallest possible discrete amount of charge, the elementary charge e, has a magnitude of 1.6×10^{-19} **C**. There are two types of electric charge, which have been arbitrarily labelled **positive** and negative. Every electron has a **negative** charge, of $-1e$ (-1.6×10^{-19} C), and every proton has a charge of $+1e$. An atom or molecule which has acquired a net electric charge by the addition or removal of a whole number of electrons always has a charge which is some positive or negative integer multiple of e.

Coulomb's law describes the interaction between two charged particles which are not chemically bound to each other and which are at rest. The interaction involves a pair of forces as described by Newton's third law of motion. Particle 1 exerts a force on particle 2 at the same time as particle 2 exerts a force on particle 1. These two forces have exactly the same magnitude but they act in opposite directions along the line joining the particles. If both charges have the same sign the two forces are repulsive but opposites attract each other. The common magnitude of the two forces is given by the formula: $F = k\dfrac{q_1 q_2}{r^2}$

where q_1 and q_2 are the magnitudes of the two charges and r is the distance between them; k is a universal constant; $k = 9.0 \times 10^9$ N.m^2.C^{-2}. Even if the charges are different the two forces still have the same magnitude.

The total amounts of negative and positive charge in the universe seem to be exactly balanced. Most objects normally have the same amount of positive charge as they do negative charge, and so are overall electrically **neutral**. However it is possible to "charge up" an object by adding or **removing** some charged particles such as electrons or ions. When you make an object negative by adding extra electrons, you must be getting those electrons from something which is becoming positive. The total amount of charge is conserved, it is just moved around.

Discussion Question

Protons are in the nucleus of an atom, surrounded by electrons. It takes much less energy to move electrons from one atom to another than to break up the nucleus (nuclear fission) to allow protons to be moved. (An exception is the hydrogen nucleus which is a single proton.) Charging can also occur by moving whole atoms or molecules.

B. Activity Questions:

1. Tape Charge

Large organic molecules, such as are found in sticky tape or combs and hair or glass/plastic and cloth/fur, break easily and leave these items charged. The tape pieces repel each other because they have picked up a net charge, so there is an electric field between them due to the charges. Hence they can interact without touching.

stick tape on desk

pull off, tape repels
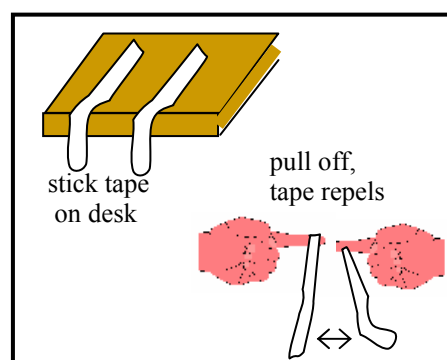
2. van de Graaff generator

You may feel tingly when you touch the generator, or you may feel nothing at all. The generator charges you up to a very high voltage, which means a lot of extra charges. Usually the dome becomes positive, so negative charges move from you to the dome, leaving you positively charged. People's hair tends to stand up because the charges exert a repulsive force on each other, the hairs try to get as far away from each other as possible and are light enough to stand up and move apart.

flow of charges

van de Graaff generator
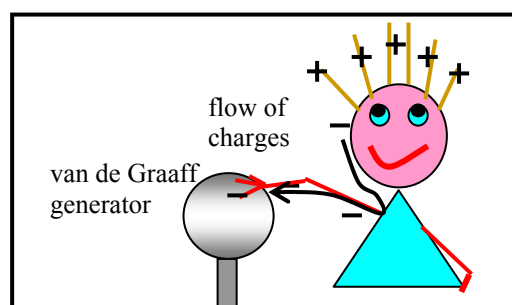
3. Charged rods
The glass rods are charged by electrons moving between the rods and the fur or silk. The plastic rods are charged by organic molecules being broken and positively charged segments stripped from the rod.
You can accelerate the rod without touching or blowing on it by holding another charged rod close by. The charges on the rods interact via a field. They attract or repel each other, accelerating the rod balanced on the watch-glass.

C. Qualitative Questions:

1. Comparing gravitational and electrostatic forces.
Newton's law of gravitation says that the gravitational force between any two objects is proportional to the mass of the objects and decreases with the square of the distance between them: $F_G = \dfrac{Gm_1m_2}{r^2}$.

Coulomb's law for electrostatics says that the force between any two charged objects is proportional to the size of the charges and decreases with the square of the distance between them: $F_E = \dfrac{kq_1q_2}{r^2}$.

Both have the same basic form in that the force varies inversely with r^2 and directly with either the mass or charge of the objects. However the Coulomb force is repulsive for like charges and attractive for unlike charges. The force of gravity is always attractive. Note also that there is only one sort of mass, positive mass, while in the case of electric charge there is both positive and negative charge.

2. Coulomb's law for electrostatics: $F_E = \dfrac{kq_1q_2}{r^2}$.

The force on one electron in the helium atom due to the nucleus is $F = \dfrac{kq_1q_2}{r^2} = \dfrac{k(-e)(2e)}{r^2}$,

where r is the distance from the nucleus to the electron, $-e$ is the charge on the electron and $+2e$ is the charge of the nucleus due to the two protons it contains.

The force on that one electron due to the nucleus is $F = \dfrac{k(-e)(2e)}{r^2}$, which has *exactly the same magnitude* as the force on the nucleus due to that electron, not less. Note that this is also the case for the gravitational force, the force on the Earth due to the gravitational attraction of a thrown tennis ball is the same as the force on the ball due to the earth. These are action reaction pairs, and Newton's third law tells us that they must experience equal and opposite forces.

D. Quantitative Question:

The levitating cat trick.
a. You require the electrostatic force up to equal the gravitational force down.

$$F_{elect} = \frac{kq_{cat}^2}{r^2} = F_{grav} = mg$$

$$\frac{9 \times 10^9 \text{ N.m}^2.\text{C}^{-2} \times q_{cat}^2}{(1.0\text{m})^2} = 2 \text{ kg} \times 9.8 \text{ m.s}^{-2}.$$

$$q_{cat}^2 = \frac{2\text{kg} \times 9.8 \text{ m.s}^{-2} \times (1.0 \text{ m})^2}{9 \times 10^9 \text{ N.m}^2.\text{C}^{-2}} = 2.0 \times 10^{-9} \text{ C}^2$$

so $q_{cat} = 5.0 \times 10^{-5}$ C.
b. One electron has 1.6×10^{-19} C. The total number of electrons is the total charge / charge per electron:
number of electrons = 5×10^{-5} C / 1.6×10^{-19} C per $e^- = 3 \times 10^{14}$ electrons.