Workshop Tutorials for Technological and Applied Physics

Solutions to ER5T: **Capacitance**

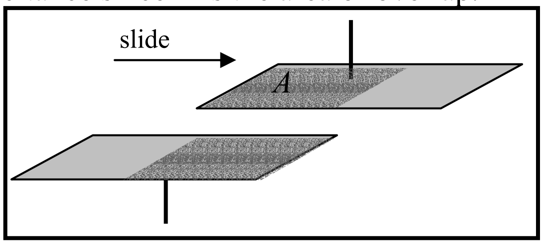
A. Qualitative Questions:

1. Capacitance of a parallel plate capacitor.

a. Since $C = \dfrac{\varepsilon_0 A}{d}$,reducing d to half its value will double the capacitance.

b. Doubling the area of both plates will again double the capacitance (assuming an ideal capacitor).

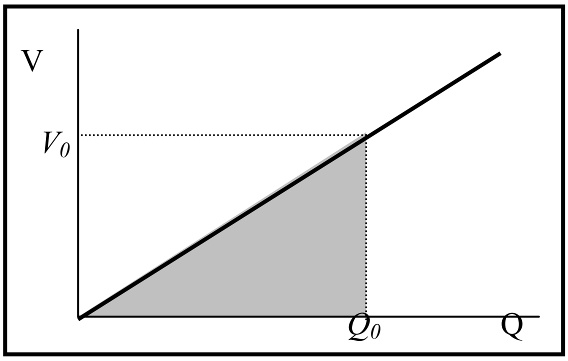
c. Doubling the area of one plate only will not change the capacitance since A is the area of overlap.



d. If the area of overlap is 50% of its original value, then the capacitance also halves.

e. Doubling the potential difference between the plates results in no change in the capacitance. The capacitance is determined by geometrical quantities.

2. Charge, capacitance and potential difference.



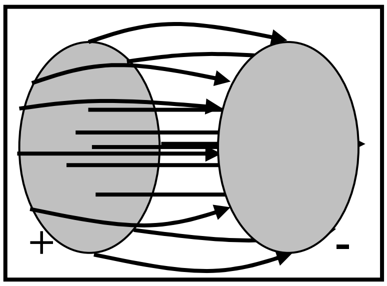
a. See diagram opposite.

b. The slope of the graph is the capacitance, $C = Q/V$. The unit CV^{-1} is also called the farad, in honour of Michael Faraday who did a lot of the early work in electromagnetism.

c. See diagram opposite. The area under the curve, shaded grey, is a triangle and has area ½ base × height $= ½\, V_0 \times Q_{oi} = ½\, V_0\, (V_0 \times C) = ½\, V_0^{\,2}\, C$ which is the energy stored in the capacitor.

B. Activity Questions:

1. Variable capacitor I – giant capacitor



The capacitance is inversely proportional to the separation of the plates, moving the plates closer together increases the capacitance. The paper strips lift and align with the field lines when the field is strong enough. The strips become charged by the plate to which they are attached, and are both repelled by this plate and attracted towards the opposite plate.

2. Variable capacitor II – tuning capacitor

Notice that the capacitor is a series of leaves. Rotating the "stem" rotates one set of leaves so that the area of overlap changes. This changes the value of the capacitor.

Variable capacitors can be used in tuning devices such as radios where dialing up the radio station is just twisting the "stem". The variable capacitor is part of the resonant circuit where maximum response to the transmitted signal depends on matching the resonant frequency of the circuit with the frequency of the signals carrier waves.

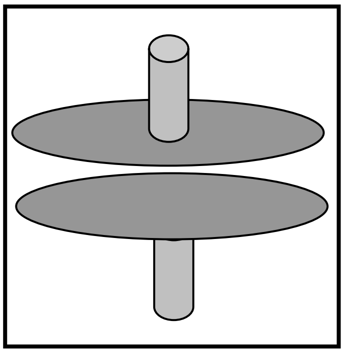
3. Energy stored by a capacitor

We can use the battery to charge up the capacitor and store energy $U = \frac{1}{2} CV^2$ (in the form of stored charge or an electric field). If we then disconnect the capacitor from the battery and connect the leads across the small electric motor fitted with a 'propeller' – the stored electrical energy is converted into mechanical energy – in the form of rotational motion.

Changing the supply voltage does not change the capacitance, but it does change the amount of energy stored, in the same way that pouring water into a bucket does not change the capacity of the bucket, but it does change the amount of water actually in it.

C. Quantitative Questions:

1. A parallel plate capacitor has circular plates of 8.2 cm radius separated by 1.3mm of air. They are connected to a 240V power supply and allowed to charge up before being disconnected.



a. The capacitance is given by;

$$C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 \pi r^2}{d} \text{ and}$$

$$C = \frac{(8.85\times10^{-12}\,\mathrm{C^2N^{-1}m^{-2}})\pi(8.2\times10^{-2}\,\mathrm{m})^2}{1.3\times10^{-3}\,\mathrm{m}} = 1.4 \times 10^{-10}\mathrm{F} = 140 \text{ pF}.$$

b. The charge on the plates is given by $q = CV$,

thus $q = (1.4 \times 10^{-10}\text{ F})(240\text{ V}) = 3.4 \times 10^{-8}\text{ C} = 34\text{ nC}$.

c. The electrical energy stored between the plates is;

$$U = \tfrac{1}{2} CV^2 = 0.5\,(1.4 \times 10^{-10}\mathrm{F})(240\text{ V})^2 = 4.0 \times 10^{-6}\text{ J}$$

d. If the plates are pulled apart without affecting the charge distribution, the electric field between the plates remains unchanged. The field line pattern and density does not change thus the electric field does not change.

e. The potential difference doubles to 480 V, using $E = V/d$.

f. The electrical energy stored between the plates is 8.0×10^{-6} J. It has also doubled.

g. The electrical energy stored in part **f** is greater than that in part **c**. In going from c to f we have added energy by doing work on the system when pulling the plates further apart. This has increased the energy of the system.

2. The coaxial cable acts as a cylindrical capacitor. The capacitance of a cylindrical capacitor is given by $C = 2L\pi\varepsilon_0/\ln(b/a)$ where b and a are the inner radius of the outer conductor and the outer radius of the inner conductor respectively, and L is the length of the capacitor. In this case we have a dielectric separating the conductors, so we must replace ε_0 with $\kappa\varepsilon_0$ which gives

$C = 2L\pi\kappa\varepsilon_0/\ln(b/a)$

$= 2\times3\text{ m} \times 2.6 \times 8.85\times10^{-12}\text{ F.m}^{-1}/\ln(3\text{mm}/0.5\text{mm})$

$C = 8 \times 10^{-11}\text{ F} = 80\text{ pF}$.