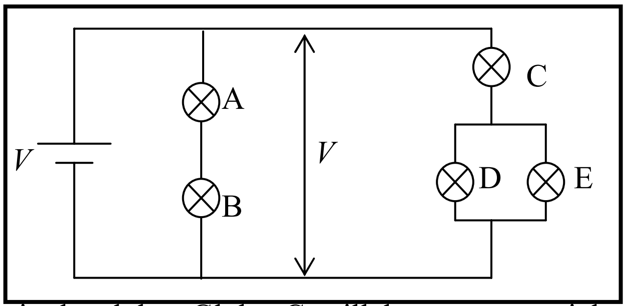
Workshop Tutorials for Physics
Solutions to ER6: **Circuits**

A. Qualitative Questions:



1. You can redraw the circuit as shown.
The voltage is the same across each arm of the circuit. The potential difference across each of A and B will be ½V. In the second arm we have globes, which we can treat as resistors in a combination of series and parallel. The total resistance of D and E will be half that of each of them individually, which is also half that of globe C. Hence the resistance of this arm is 3/2 times that of a single globe. Globe C will have a potential difference of 2/3 of the voltage V, and D and E will each have 1/3 V. Brightness increases with power which goes like the V^2, so we can rank the brightness by ranking the voltages. Hence the order of brightness will be C → A & B → D & E (brightest to dimmest).

2. Charging and discharging capacitors.

a. The capacitor discharges as $q(t) = q_0 e^{\frac{-t}{RC}}$. This can be rearranged to $t = -RC \ln \frac{q}{q_0} = RC \ln \frac{q_0}{q}$

When $q = ½\, q_o$, the time taken to discharge is simply $t = RC\ln 2$, which is independent of initial charge.

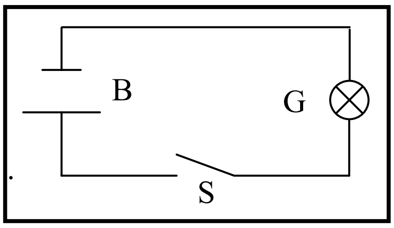
b. The voltage drop across the capacitor is given by $V(t) = V_0 e^{\frac{-t}{RC}}$ and this can be rearranged to $t = RC \ln \frac{V_0}{V}$. If the voltage drop decreases by 1V then $t = RC \ln \frac{V_0}{V_0 - 1}$, which is dependent on initial voltage and RC.

c. The decay (of both charge and voltage) is exponential. A property of exponential relations is that if the "change" can be expressed as a ratio as in **a** then the time taken for the change is independent of initial value. On the other hand if the "change" is expressed as a "measured value" as in **b** then the time taken for the change is dependent on initial value.

B. Activity Questions:

1. Torch – a simple circuit

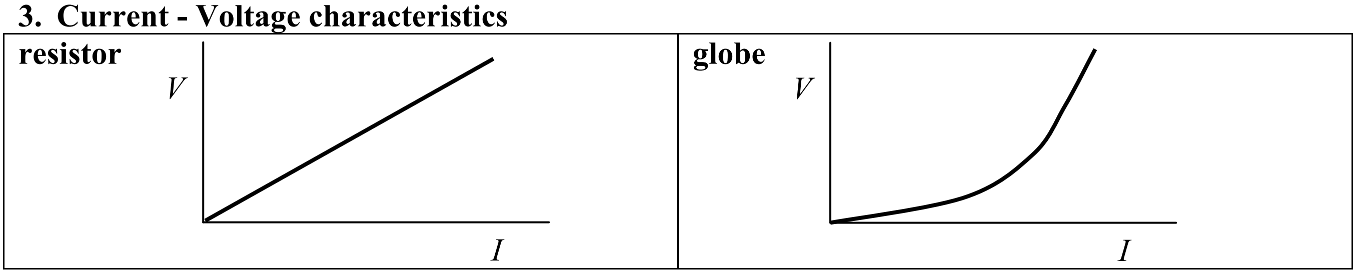
The torch has a battery, B, which supplies the voltage, a globe, G, which converts electrical energy into light, and a switch, S, which completes the circuit allowing current to flow when the torch is turned on.



2. Resistivity and resistance

Resistance increases with length, and decreases with cross section for a given material.
Resistivity is a property of the material, and does not depend on shape or size.

3. Current - Voltage characteristics

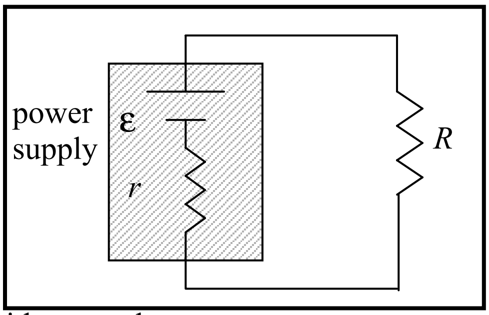


The temperature of the filament in the globe increases very quickly as the current increases. The resistance increases with temperature, hence the I-V plot for the globe is curved.

4. Toaster Man.
The current is inversely proportional to the resistance, the greater the resistance the less current can flow, and the less likely toaster man is to be electrocuted. The total resistance of resistors in series is the sum of the individual resistances.

5. Simple membrane model – resistors in parallel
When resistors are connected in parallel the total resistance is less than any individual resistance. There are more paths for the current to flow along, and so the total current is greater. Resistance is the voltage divided by the current, $R = V/I$, so a larger current means a smaller resistance for a given voltage supply.

C. Quantitative Questions:



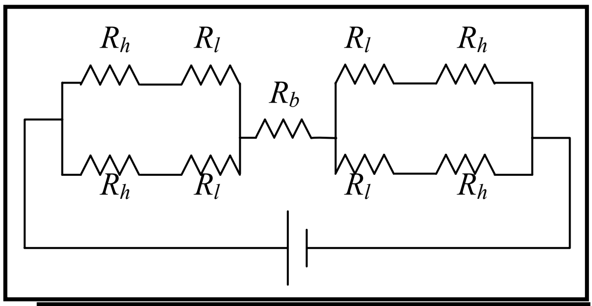
1. Internal resistance of power supplies.

a. $\varepsilon = V_R + V_r = iR + ir$.
We know from Ohm's law that V = IR, so we can write for the current $i = V_R/R$. We can rewrite the expression for ε as
$\varepsilon = iR + ir = (V_R/R)R + (V_R/R)r = V_R + (V_R/R)r$

b. Using the two values for V_R and R given, we get two equations with two unknowns:
$\varepsilon = V_R + (V_R/R)r = 1.0\text{ V} + (1.0\text{ V} / 500\ \Omega)r$
$\varepsilon = V_R + (V_R/R)r = 1.5\text{ V} + (1.5\text{ V} / 1000\ \Omega)r$
setting these equal to eachother gives
$1.0\text{ V} + (1.0\text{ V} / 500\ \Omega)r = 1.5\text{ V} + (1.5\text{ V} / 1000\ \Omega)r$ and rearranging: $(1.0/500 - 1.5/1000)\, r = 0.5\ \Omega$
$(0.5/1000)r = 0.5\ \Omega$ so $r = 1000\ \Omega$.

c. Substituting the value for r into either of the equations in **b** gives $\varepsilon = V_R + (V_R/R)r = 3.0$ V.

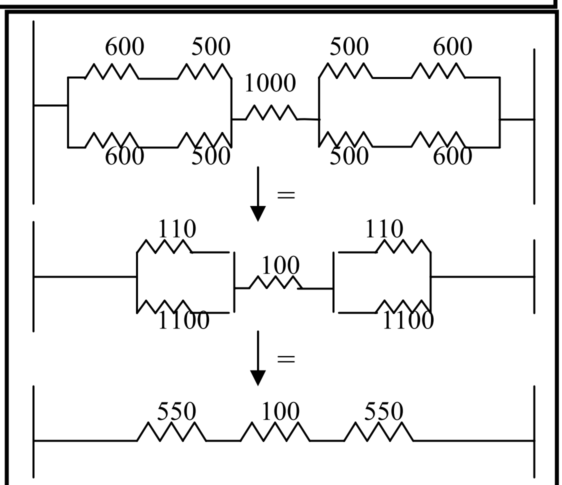
d. The efficiency is equal to P_{out} / P_{in}. The power in, $P_{in} = 20\text{ W.m}^{-2} \times 5 \times 10^4\text{ m}^2 = 0.01\text{ W} = 10\text{ mW}$.
The power out is $P_{out} = V^2/R = (1.5\text{ V})^2 / 1000\ \Omega = 2.25\times10^{-3}\text{W} = 2.3\text{ mW}$.
The efficiency is $P_{out} / P_{in} = 2.3\text{mW} / 10\text{ mW} = 0.23$ or 23%



2. Resistance of a cow.

a. See diagram opposite.
R_l – resistance of leg = 500 Ω
R_b – resistance of body = 1000 Ω
R_h – resistance of hoof = 600 Ω

b. See diagram opposite,
The total resistance of each leg+hoof
= 600 Ω + 500 Ω = 1100 Ω.
There are two of these in parallel, so the total is
$R_{legs+hoofs} = (1/1100\Omega + 1/1100\Omega)^{-1} = 550\ \Omega$.
There are two of these, one at each end of the cow, in series with the body, so the total cow resistance is 550 Ω + 550 Ω + 1000 Ω = 2100 Ω.



c. Using Ohm's law, $i = V/R = 150\text{ V} / 2100\ \Omega = 71$ mA. This is the current flowing across the entire body of the cow, the current along any given path, including that through the heart will be less than this, hence the cow is unlikely to be killed. Note that as little as 10 mA through the heart is enough to cause a human heart to fibrillate and stop beating

d. Resistors dissipate electrical energy as heat. The cow is acting as a resistor, hence she is going to produce heat and probably be burned. Electrocution victims usually suffer burns. The cow may also panic and run away and run through a fence, this is quite common and many animals including dogs injure themselves when frightened by storms.