

Workshop Tutorials for Physics

Solutions to ER10: Circuits II

A. Qualitative Questions:

1. Kirchhoff's rule for junctions states that the total currents going into a junction must be equal to the total currents coming out of a junction. This is a statement of conservation of charge, as current is just a flow of charge. Charge must be conserved, so whatever flows into a junction must flow out again. If it didn't come out again, or if more came out than went in, then charge is either being created or destroyed.

Each point in space has only one value of electrical potential at any time. Work your way mentally around *any* closed path, noting and adding up all the changes in potential as you go, making sure to count decreases as negative changes and increases as positive. When you get back to the starting point the sum must be the potential that you started with. In a circuit with just one battery, any loop which includes the battery will include a rise in potential across the battery and drops (or no change) everywhere else. The connection with energy conservation is that if you imagine that you were to take a little test charge around the loop, then the potential energy of the system at the end would be the same as it was when you started. Remember that potential is defined as potential energy per unit charge.

2. Two of the most common circuit components are resistors and capacitors.

a. When the charge moves around the circuit there is a loss in electrical potential energy due to the resistance of the resistors in the circuit. The more resistors an electron has to pass through, the more energy is lost (as thermal energy), and the less current flows. Hence the more resistors there are in series, the greater the total resistance. Resistors in parallel provide multiple paths for the current to flow along, hence more current can flow and the total resistance is less, like a multi-lane freeway.

b. Capacitors are devices to store charge. The charge, Q , stored for a given voltage, V , depends on the capacitance of the capacitors. Imagine three identical capacitors in parallel. This in effect gives us one capacitor with three times the plate area, i.e. the overall capacitance is increased and three times as much charge ($3Q$) is stored for a given voltage. Now imagine the three capacitors in series. When the same voltage as above is placed across them, each capacitor has $1/3 V$ across it, so it stores only one third the charge. The net charge is effectively $+Q/3$ on the left most plate and $-Q/3$ on the right most plate (or vice versa), hence the capacitance of the three in series is $1/3$ of a single capacitor, and much less than the capacitance of the three in parallel.

B. Activity Questions:

1. Measuring Current and Voltage

The ammeter measures the current, which is the number of charges per unit time passing through a given point on the circuit. To be able to count the charges, the ammeter must be part of the circuit and have a very low internal resistance so that it does not affect the current through the circuit.

The voltmeter is connected in parallel, because it measures the difference in potential between two points. The voltage across any arm is the same, so it is connected in parallel, forming another arm with the component we wish to measure the voltage across. It has a very high internal resistance so that very little current will flow through it, thus having little effect on the circuit.

2. Wheatstone Bridge

When there is no potential difference between points **a** and **b** there is no current flow between these points. Hence all of current I_1 flows through R_1 into R_3 , and all of current I_2 flows through R_2 into R_x . We also know that the potential difference across R_1 , must be equal to the potential difference across R_2 , i.e. that $V_1 = I_1 R_1 = I_2 R_2$. The potential differences across R_3 and into R_x must also be equal, giving $I_1 R_3 = I_2 R_x$. We can now divide the second equation by the first to give $R_3 / R_1 = R_x / R_2$, and rearrange to obtain an expression for R_x . $\therefore R_x = (R_2 R_3) / R_1$.

3. Charging capacitors

The potential difference is initially all across the resistor; as the capacitor charges the potential difference across the capacitor builds up from zero to a maximum (the supply voltage). The greater the resistance, the greater the time constant and the longer it takes for the capacitor to charge up.

C. Quantitative Questions:

1. Brent measures the potential across Rebecca's battery terminals to be 10 V, and that across his own battery to be 12.5 V. The jumper leads have a resistance of $R_j = 0.1 \Omega$, and each battery has an internal resistance of $R_b = 0.02 \Omega$.

a. Brent should connect the positive terminal of his car's battery to the positive terminal of Rebecca's battery, and connect the two negative terminals together. This will allow the good battery to push charges through the weak battery from the positive to the negative, and recharge it.

b. See diagram opposite. Note that the resistance of the jumper leads is only included once.

c. If Brent has connected the batteries together correctly the current which flows will be the total *emf* divided by the total resistance (Ohm's law) :

$$I = \frac{V_1 - V_2}{2R_b + R_j} = \frac{12.5\text{V} - 10\text{V}}{2 \times 0.02\Omega + 0.1\Omega} = 18 \text{ A.}$$

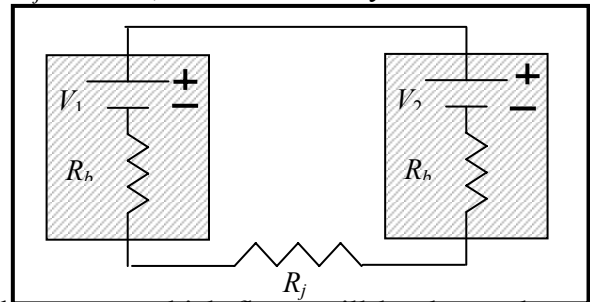
This current will decrease as the weaker battery (Rebecca's) becomes charged.

d. If Brent has connected the batteries the wrong way around, so that the *emfs* add, the current will be

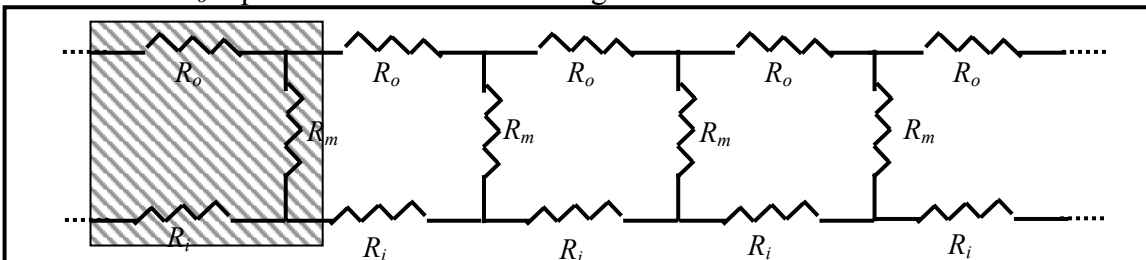
$$I = \frac{V_1 + V_2}{2R_b + R_j} = \frac{12.5\text{V} + 10\text{V}}{2 \times 0.02\Omega + 0.1\Omega} = 160 \text{ A.}$$

This is a very big current and will probably damage both the batteries and the jumper cables.

e. Brent's battery does *not* add charges (electrons) to Rebecca's battery – it adds electrical potential energy.



2. The resistances R_m represent the resistance across the membrane, R_i represents the internal resistance of the cell and R_o represents the resistance along the outside of the membrane.



If $R_o = R_i = R$, then the resistance of the single section of membrane shown shaded in the diagram is $2R + R_m$. As the circuit is of effectively infinite length, the rest of the membrane to the right has a resistance effectively the same as the total resistance, R_T . This R_T is in parallel with the membrane resistance, R_m , so

we can write the total resistance of the entire circuit shown above as $R_T = 2R + \left(\frac{1}{R_m} + \frac{1}{R_T}\right)^{-1}$.

We now wish to rearrange this to get R_T . We start by multiplying everything by $\left(\frac{1}{R_m} + \frac{1}{R_T}\right)$ which gives:

$$R_T \left(\frac{1}{R_m} + \frac{1}{R_T}\right) = 2R \left(\frac{1}{R_m} + \frac{1}{R_T}\right) + 1 \quad \text{or} \quad \left(\frac{R_T}{R_m}\right) + 1 = \left(\frac{2R}{R_m} + \frac{2R}{R_T}\right) + 1, \text{ and getting rid of the ones and}$$

multiplying by R_T and R_m gives:

$R_T^2 = 2RR_T + 2RR_m$ or $R_T^2 - 2RR_T - 2RR_m = 0$ which is a quadratic of form $ax^2 + bx + c = 0$, and can be solved by inspection or using the quadratic formula to give:

$R_T = R \pm [R^2 + 2RR_m]^{1/2}$ we take the positive solution, $R_T = R + [R^2 + 2RR_m]^{1/2}$ because $R (= R_i = R_o)$ is in series with the rest of the circuit, hence the total resistance must be greater than R .