Workshop Tutorials for Physics

Solutions to ER11: **AC Circuits**

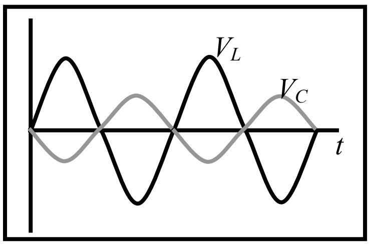
A. Qualitative Questions:

1. An inductor is connected to an AC power supply. If the frequency of the power supply is doubled:
a. The inductance, L, will not change. The inductance depends on the physical characteristics of the inductor, characteristics such as the number of turns of wire and the cross sectional area.
b. The inductive reactance, X_L, does change. The voltage drop developed across the inductor depends on the rate of change of flux and that depends on the frequency of the input voltage. Since $X_L = 2\pi fL$, when the frequency doubles so does the inductive reactance.
c. The capacitive reactance also depends on frequency and hence changes. The capacitive reactance is: $X_C = \frac{1}{2\pi fC}$. When the frequency is doubled the capacitive reactance is halved.

2. Current and voltage in capacitors and inductors.
a. The current through and voltage across a capacitor are out of phase. The voltage across the capacitor depends on the charge stored on the capacitor. As the voltage (and charge stored) increases through the first part of any cycle, the current will decrease from its initial maximum value. Then as the current reverses the capacitor begins to discharge and the voltage across it starts to decrease. The instantaneous current in the circuit leads the instantaneous voltage across the capacitor by one-quarter of a cycle.
b. The current through and voltage across an inductor are also out of phase.. The voltage across the inductor is the back *emf* developed because of the changing magnetic flux linking the coils of the inductor. The magnetic flux is changing because the current is an alternating current. Once again as the current decreases from its initial maximum, a voltage develops across the inductor, but in this case the direction is opposite. Once again the phase difference is one quarter of a cycle, but now as the voltage developed is in the opposite direction, i.e. the voltage is maximum negative as the current goes to zero, the instantaneous current in the circuit lags the instantaneous voltage across the inductor by one-quarter of a cycle.

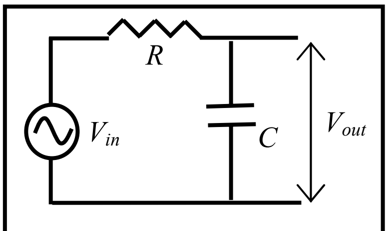
B. Activity Questions:

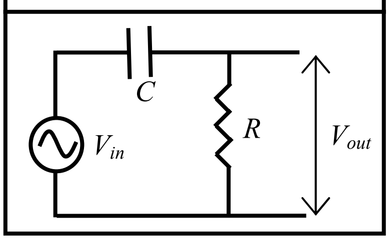
1. Series RLC circuit



See diagram opposite. The voltages across the inductor and the capacitor depend on the reactance of the two components. The reactance, X, depends on the frequency, f: $X_C = 1/\,2\pi fC$ and $X_L = 2\pi fL$. When $X_c = X_L$, the voltage drop across L and C is zero, since V_C and V_L are 180^o out of phase and equal in magnitude, cancelling each other out.
So the impedance of the circuit = R, the resistance of the globe. The current will be a maximum and the bulb will glow most brightly. Varying the inductance, L, of the inductor until $X_c = X_L$ will lead to a similar effect.

2. High pass and low pass filters





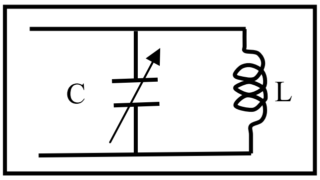
The top circuit is a low pass filter. As the frequency changes, X_C changes, hence V_C changes. When the frequency is low the reactance is high and most of the voltage is dropped across the capacitance rather than the resistance. Thus V_{out} will be high. We call this a low pass filter as low frequencies provide a significant output, but high frequencies do not.
The bottom circuit is a high pass filter. The output is taken across the resistor now. At high frequencies the reactance of the capacitor is low, so the voltage dropped across the capacitor is low while that across the resistor is high. Hence for a high pass filter we take V_{out} across the resistor.

3. Tuning circuit

A tuning circuit consists of a inductor and a capacitor, usually in parallel.

Either the inductance of the inductor or the capacitance of the capacitor must be able to be varied. By varying either the capacitance or inductance the resonant frequency of the circuit is varied. When the resonant frequency matches an incoming signal, for example the carrier wave of your favourite radio station, then the circuit is tuned to that frequency and the radio program can be heard.

C. Quantitative Question:

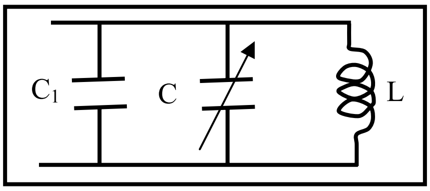


1. A variable capacitor (10 pF to 365 pF) is used in a tuning circuit.

a. The frequency decreases with increasing C or L.

$f_{max} = \frac{\omega_{max}}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC_{min}}}$, and $f_{min} = \frac{\omega_{min}}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC_{max}}}$.

The ratio of maximum to minimum is therefore $f_{max}/f_{min} = \sqrt{\frac{C_{max}}{C_{min}}} = \sqrt{\frac{365\,\text{pF}}{10\,\text{pF}}} = 6.0$



A second capacitor, C_1 is in parallel with the variable capacitor. So the total capacitance is now $C_{total} = C + C_1$.

b. We want to reduce the frequency ratio by a factor a 2, i.e. f_{max}/f_{min} = 3.

We can write this ratio for the new circuit as $f_{max}/f_{min} = \sqrt{\frac{C_{max}+C_1}{C_{min}+C_1}}$ =3.

Rearranging for C_1 gives: $C_1 = (C_{max} - 9C_{min})./8$ = (365 pF – 9 × 10 pF)/8 = 34 pF

c. We want to tune in to the AM radio band, 540 kHz to 1600 kHz. To find the appropriate value of L we can use our expression for f_{max} from part **a** and remembering to include the extra capacitor, C_1**:**

$f_{max} = \frac{1}{2\pi}\sqrt{\frac{1}{L(C_{min}+C_1)}}$ and rearranging this for L: $(2\pi f_{max})^2 = \frac{1}{L(C_{min}+C_1)}$

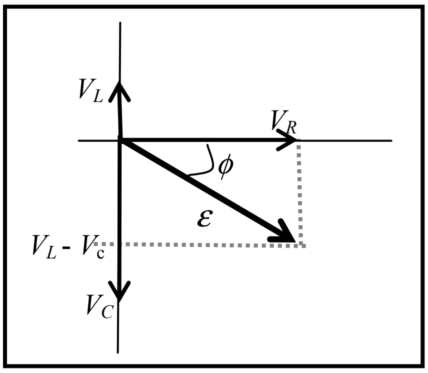
and finally $L = \frac{1}{(2\pi f_{max})^2(C_{min}+C_1)} = \frac{1}{(2\pi\times 1.6\times 10^6\,\text{Hz})^2(10\times 10^{-12}\,\text{F}+34\times 10^{-12}\,\text{F})} = 2.2\times 10^{-4}$ H = 0.22 mH.

Note that we get the same answer of we use the expression for minimum frequency instead.

2. Series LCR circuit with ε = 240 V (RMS) f = 50 Hz. R = 50 Ω, C = 50 μF and I = 0.05H.

a. The total impedance, Z, of the circuit depends upon the angular frequency, ω, of the power supply, which is $\omega = 2\pi f = 2\pi \times 50$ Hz = 314 rad.s^{-1}. The total impedence, Z, is given by

$$Z = \sqrt{R^2 + (\omega L - \tfrac{1}{\omega C})^2} = \sqrt{50^2 + (314\times 0.05 - \tfrac{1}{314\times 50\times 10^{-6}})^2} = 69\ \Omega$$



a. See diagram opposite.

Using Ohm's law; $V_L = iX_L$, $V_C = iX_C$, and $V_R = iR$.

The impedances of the components are:

$X_L = \omega L$ = 16 Ω, $X_C = \frac{1}{\omega C}$ = 64 Ω, R = 50Ω.

We wish to find the angle, ϕ, between V_R and ε:

$\tan\phi = (V_L - V_C)/V_R = (iX_L - iX_C)/iR = (X_L - X_C)/R$

=(16 Ω – 64 Ω)/50 Ω = -0.95.

and finally, ϕ = -44^o. (So current leads ε.)

b. The resonant frequency, ω_o, is : $\omega_o = \frac{1}{\sqrt{LC}}$ = 630 rad.s^{-1}. This is twice the source frequency, so the circuit is a long way from resonance.

c. $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$ = 2. The quality factor is an indication of the bandwidth, the distance between the half power points, $Q = \frac{\omega_o}{\Delta\omega}$, where Δω is the bandwidth. So $\Delta\omega = \omega_o/Q$ = 630 rad.s^{-1}/2 = 315 rad.s^{-1}. This means that the *emf* frequency, 314 rad.s^{-1}, is just at the lower end of the bandwidth.