Workshop Tutorials for Introductory Physics

Solutions to MI1: **Motion in a Line**

A. Review of Basic Ideas:

Describing motion

The study of motion is called **kinematics**. We can describe the motion of an object by talking about how far it has moved, how long it took to move that far, how fast it is moving and how much it is speeding up or slowing down.

Imagine watching a drag race. At the start of the race the cars are lined up at the **starting line**, not moving. Let's label this time and position as time = t_0, and position = x_0. The race starts and the cars take off. A short time later one of the cars is halfway along the track, we'll call this position x_1, and the **time** it arrives there is time $t = t_1$. The distance the car has traveled is called the **displacement**, and is given by $\Delta x = x_1 - x_0$, the time it took to travel this distance is $\boldsymbol{\Delta t = t_1 - t_0}$. The car started at rest, so its velocity at (x_0, t_0) was $v_0 = 0$. As the car accelerates away from x_0 its velocity **increases**. We can find the average velocity of the car between $t = t_0$ and $t = t_1$ by using

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_1 - x_0}{t_1 - t_0}$$

Velocity is the rate of change of displacement with time. The rate of **change** of velocity with time is called the acceleration. If the car is moving with velocity v_1 when it reaches the point x_1 at time t_1 then its **average** acceleration between $t = t_0$ and $t = t_1$ is

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_1 - v_0}{t_1 - t_0}$$

In car racing it's important not only that the car has a high velocity, but also that it has a large **acceleration** so that it can reach that velocity quickly.

If we decide to set the time $t_0 = 0$, which is usually very convenient, then we can rearrange to find our velocity at some later time, say at t_1:

$$v_1 = v_0 + a_{av}\, t_1$$

If the acceleration is **constant** then the average velocity is $v_{av} = \frac{1}{2}\,(\boldsymbol{v_0} + \boldsymbol{v_1})$.Using this and the expression for v_1 above, we can work out how far the car has gone, how fast it's moving and how quickly it's accelerating.

Discussion questions

Instantaneous velocity is the velocity an object has at some instant in time. For example, you may be driving down a hill and speeding up, so that when you reach the bottom your velocity is 70 km.h^{-1}, and a speed camera at the bottom of the hill will measure this speed. If you draw a graph of your position as a function of time then your velocity at any instant is the gradient of a tangent to the graph at that point. Your average velocity is the total distance traveled divided by the total time taken. On most days your average velocity over the entire day is zero because you end up in bed where you started, having traveled no net displacement at all.

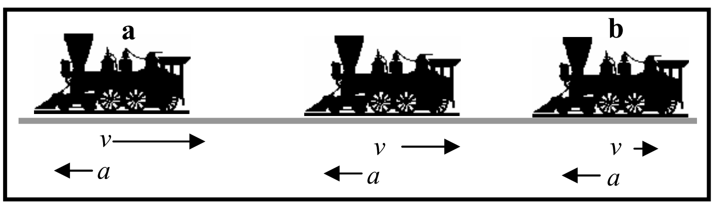
B. Activity Questions:

1. Velocities

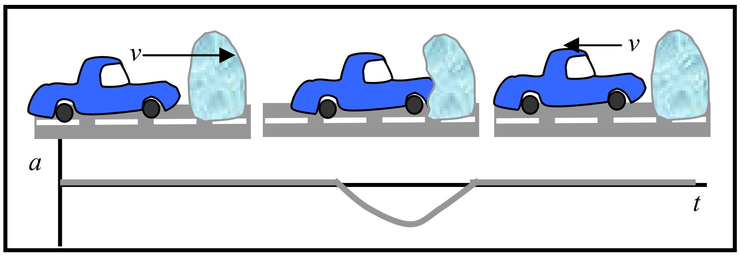
You should start by choosing a positive direction and a negative direction. Most people have a step length a little less than a metre, so if you walk at slightly more that one step per second you should be going at approximately 1 m.s^{-1}. To move at -1 m.s^{-1} you need to walk in the negative direction.

2. Train set

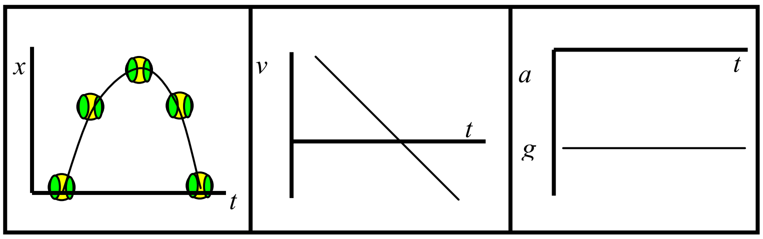
You can move the train from point **a** to point **b** such that it has a negative velocity and a positive acceleration by running the train backwards, and having it slow down as it goes.



3. Acceleration due to collision
The car slows down as it hits the sponge, and is bounced back. Hence it has a negative acceleration due to the impact, which acts during the collision with the sponge.



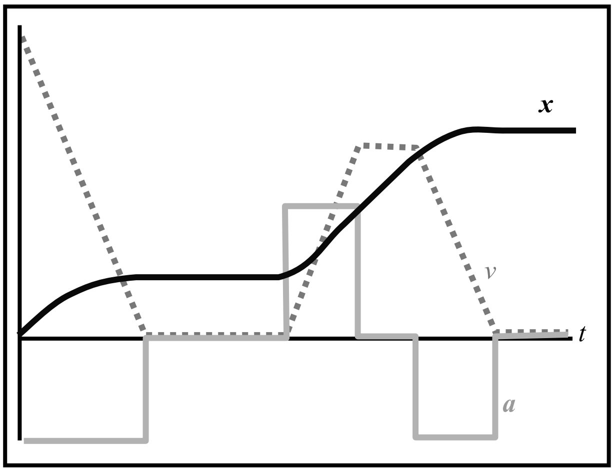
4. Acceleration due to gravity
The ball slows as it climbs until it reaches its peak, then speeds up as it falls. The acceleration of the ball is constant once it leaves your hand, and is due to gravity only. See diagrams opposite.



C. Qualitative Questions:

1. Brent is jumping on a trampoline. He leaps up into the air, and falls back again.
a. Brent always has the force of gravity acting on him. Whenever he is up in the air, that is the only force acting on him. When he is going up, the force of gravity pulls down on him and he slows down. When he is coming down again, gravity acts to speed him up. Brent is always accelerating downwards at about 9.8 m.s^{-2} except when he is in contact with the trampoline. When he lands on the trampoline it stretches and applies a force to Brent, sending him up in the air again.
b. When Brent is moving downwards, he is speeding up - his velocity is in the same direction as the acceleration. But when he is moving upwards, his velocity is opposite to the direction of his acceleration - the acceleration is downwards due to gravity, yet the velocity is upwards. An acceleration opposite in direction to your velocity simply means that you are slowing down, not speeding up.

2. You are driving west on Parramatta Road at peak hour, a graph of velocity vs time is shown.
a. In the first 5 minutes the car has a positive but decreasing velocity. Hence the car is going forwards but slowing down before coming to a halt. It is then stationary for the next 5 minutes.
b. In the next 10 minutes the car speeds up, reaching a constant velocity, and then slowing to a halt again.
c. Acceleration is the rate of change of velocity. When the velocity is a straight, increasing line, the acceleration is positive and constant. A decreasing velocity gives a negative acceleration. See plot opposite.
d. Displacement is the area under the velocity curve, or the integral of velocity with time. See plot opposite.



D. Quantitative Question:

A droplet of moisture, initially at rest in your mouth, travels 5 cm to be sneezed out at 150 km.h^{-1}.
a. Average acceleration is change in velocity divided by change in time or in this case we can use the formula: $v^2 - v_o^2 = 2a\,\Delta s$, which we rearrange to get
$a = ½ (v^2 - v_o^2)/\Delta s = ½ (42 \text{ m.s}^{-1})^2/0.05\text{m} = 17{,}640 \text{ m.s}^{-2}$. This is a huge acceleration, almost 2000 g's!
b. If your eyes are closed for 0.2 seconds and the droplet is travelling at 42 m.s^{-1} then it travels
$d = v \times t = 42\text{m.s}^{-1} \times 0.2 \text{ s} = 8.4 \text{ m}$.
c. If a droplet travels 3m before coming to a stop, its average deceleration is
$a = ½ (v^2 - v_o^2)/\Delta s = ½ (-42 \text{ m.s}^{-1})^2/5\text{m} = 176 \text{ m.s}^{-2}$.
d. The time it takes to travel this distance is $\Delta t = \Delta v/a = 42 \text{ m.s}^{1}/ 176 \text{ m.s}^{-2} = 0.24$ s.
So by the time you open your eyes again, the droplets have already traveled a few metres and stopped.