Workshop Tutorials for Introductory Physics

Solutions to MI2: **Using Vectors**

A. Review of Basic Ideas:

A sense of direction.

Every one has at some point asked someone else for **directions** or given someone else directions to get somewhere. When you listen to someone describing how to get to a place they *sometimes* talk about how **far** to walk or drive, but they ***always*** describe which direction to travel in. They may do this by simply **pointing**, or saying keep going straight, or go left, then right, or go west then north. If you live 20 km from university, then you need to travel at least **20 km** to get there. But if you go 20 km in the wrong direction, you will not get there. A knowledge of direction is something you use all the time.

Direction is very important in physics as well. A lot of the quantities that we talk about in physics have a direction. Common examples are **displacement**, momentum, and force. To describe these quantities we use **vectors**.

A vector has both a **magnitude** (which tells you how big it is) and a direction. There are lots of different ways of representing vectors, some of which you regularly use already. If someone asks you where Hurstville is you might say "about 20km south west of the city", or you might give them a map reference, such as "map 75, grid reference H6". Usually we need to define some reference point, called an **origin**, so that we know where we're measuring from, and some directions such as north, south, east and west, up and down. In physics we usually use **Cartesian** coordinates, which are commonly labeled *x,y* and *z.*

We can describe a vector by giving its components in each of the coordinate directions. To do this we need to define unit vectors. Unit vectors have a length or magnitude of 1, and point in the direction of the axes. The unit vector $\hat{\mathbf{i}}$ points in the direction of the *x* axis and the unit vector $\hat{\mathbf{j}}$ points in the y direction. In 2 dimensions, a vector $\vec{\mathbf{A}}$ could be written as $\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$. The magnitude of the vector $\vec{\mathbf{A}}$ is $\mathrm{A} = \sqrt{A_x^2 + A_y^2}$

Given the angle, θ, that the vector makes with the *x* axis and the magnitude of the vector, we can find the components:

$A_x = \mathbf{A}\cos\boldsymbol{\theta},$ $\qquad$ $A_y = \mathrm{A}\sin\theta$

If you know what the components are it's easy to add vectors. If you add a vector $\vec{\mathbf{A}}$ to a vector $\vec{\mathbf{B}}$ you get a resultant vector, which we'll call $\vec{\mathbf{C}}$, $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ and $C_x = (A_x + B_x)$ and $C_y = (\boldsymbol{A_y} + \boldsymbol{B_y})$ so

$$\vec{\mathbf{C}} = C_x\,\hat{\mathbf{i}} + C_y\,\hat{\mathbf{j}} = (A_x + B_x)\,\hat{\mathbf{i}} + (A_y + B_y)\,\hat{\mathbf{j}}\,.$$

If the vector $\vec{\mathbf{A}}$ describes how to get from home to the beach, and the vector $\vec{\mathbf{B}}$ describes how to get from the beach to Uni, then the sum, $\vec{\mathbf{C}}$, tells you how to get directly from **home** to Uni.

Discussion question

Start by drawing a set of axis so you can define directions on your page. Listen for directions, such as north, south or left right, and distances.

B. Activity Questions:

1. Battleship

Battleship and similar games use vectors to determine the position of a ship. The vectors are usually written in terms of letter and number axes, rather than *x, y* axes, but are otherwise identical to vectors used in physics and mathematics. One way of describing the position of a pin is to give the lengths of perpendicular components, for example horizontal (numbers) and vertical (letters). Another way is to give the length of the vector and its angle to the horizontal. For example a pin at position C4 is also 5 units from the origin on a line 49^o above the horizontal.

2. Maps

Vectors are used to define positions on the maps via a letter/number grid. Most maps will also show a vector pointing north to define compass directions on the map.

3. Vector Game

The axes are chosen in advance and marked, so you know which direction is $+x$ and which direction is $+y$. For example, forward may be $+x$ and right may be $+y$. If the caller says "$5x + 3y$" you take 5 steps forwards and three steps to the right. If the caller says "$-5x - 3y$" you take five steps back and three steps left.

4. Mirrors and reflections – Coordinate Systems

In a reflection left and right seem to be reversed, but not up and down. This is because we define left and right as relative to ourselves, not our surroundings. It is important to know how you are defining your coordinate system. For example, "towards the wall" and "away from the wall" are not reversed, just as up and down are not reversed. Up and down directions in an externally defined coordinate system, as well as in your internally defined system of coordinates. They are defined externally, usually relative to the ground, hence are not reversed.

C. Qualitative Questions:

1. Barry is running around the yard chasing birds.

a. Barry's displacement can be less than the distance he traveled. Imagine if Barry ran backwards and forwards across the yard twice and finished back in the same place. His displacement (the vector quantity representing the difference between the initial and final position) would be zero but he would have run a fair distance.

b. The displacement can never be more than the distance traveled. To travel from one point in space to another, the minimum distance Barry can travel is the straight line joining the two points. He can never travel less distance than that.

c. No component of his displacement vector can be greater than the magnitude of the vector itself. Components are defined in two directions at right angles to each other. Hence the displacement is the hypotenuse of a right angle triangle. The hypotenuse will be greater than either of the two sides.

d. If the directions of the components were taken such that one was in the same direction as the displacement then one component would have the same magnitude as the displacement vector and the other would be zero.

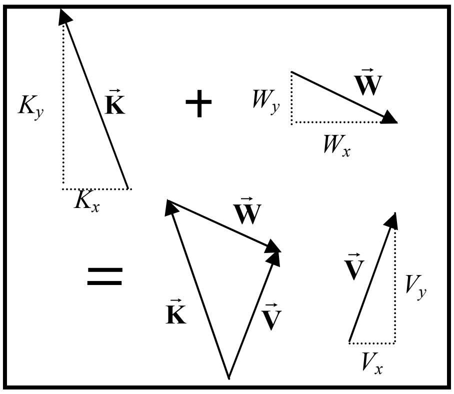
2. Kevin the duck flies north.

a. Kevin's resultant velocity is

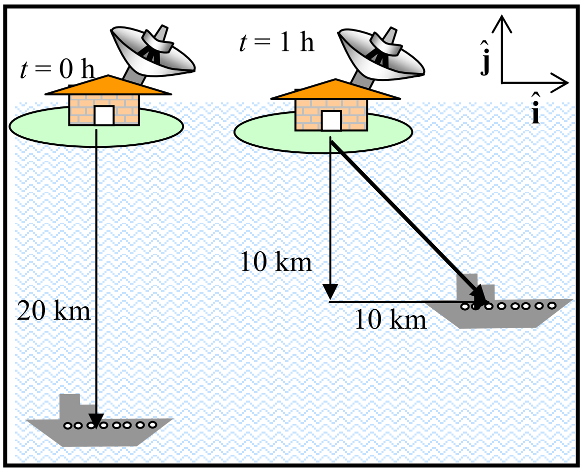
$\vec{\mathbf{V}} = \vec{\mathbf{K}} + \vec{\mathbf{W}} = K_x\hat{\mathbf{i}} + K_y\hat{\mathbf{j}} + W_x\hat{\mathbf{i}} + W_y\hat{\mathbf{j}} = (K_x + W_x)\hat{\mathbf{i}} + (K_y + W_y)\hat{\mathbf{j}}$.

b. See diagram opposite.

c. The components of $\vec{\mathbf{V}}$ are not necessarily larger than the components of $\vec{\mathbf{K}}$ and $\vec{\mathbf{W}}$. If the components of $\vec{\mathbf{K}}$ and $\vec{\mathbf{W}}$ are in different directions, i.e. one is positive and one is negative, the resultant component of $\vec{\mathbf{V}}$ will have a smaller magnitude than at least one of the components. For example, V_x opposite is smaller in magnitude than either K_x or W_x.



D. Quantitative Question:



a. See diagram opposite.

b. $\vec{\mathbf{S}}_0 = S_{x0}\hat{\mathbf{i}} + S_{y0}\hat{\mathbf{j}} = (-20 \text{ km})\hat{\mathbf{j}}$.

$\vec{\mathbf{S}}_1 = S_{x1}\hat{\mathbf{i}} + S_{y1}\hat{\mathbf{j}} = (10 \text{ km})\hat{\mathbf{i}} + (-10 \text{ km})\hat{\mathbf{j}}$.

c. The velocity is the change in position divided by the change in time. The change in position is:

$$\vec{\mathbf{S}}_1 - \vec{\mathbf{S}}_0 = (10 \text{ km})\hat{\mathbf{i}} + (-10 \text{ km})\hat{\mathbf{j}} - (-20 \text{ km})\hat{\mathbf{j}}$$

$$= (10 \text{ km})\hat{\mathbf{i}} + (+10 \text{ km})\hat{\mathbf{j}}$$

The time taken is one hour, so $\vec{\mathbf{V}} = (10 \text{ km.h}^{-1})\hat{\mathbf{i}} + (10 \text{ km.h}^{-1})\hat{\mathbf{j}}$

d. The magnitude of $\vec{\mathbf{V}}$ is $V = \sqrt{(V_x^2 + V_y^2)} = \sqrt{(10 \text{ km.h}^{-1})^2 + (10 \text{ km.h}^{-1})^2} = 14 \text{ km.h}^{-1}$.

e. Given a steady velocity, the position at any time is

$\vec{\mathbf{S}}_t = \vec{\mathbf{S}}_0 + \vec{\mathbf{V}}\,t = (10 \text{ km})\hat{\mathbf{i}} + (0 \text{ km})\hat{\mathbf{j}} + [(10 \text{ km.h}^{-1})\hat{\mathbf{i}} + (10 \text{ km.h}^{-1})\hat{\mathbf{j}}]\,t$.