Workshop Tutorials for Introductory Physics

Solutions to MI11: **Rotational Dynamics**

A. Review of Basic Ideas:

Spinning around

When we want to describe the movement of an object we can talk about its velocity and its acceleration. But what about something like a CD which stays in the same place but spins around? Different points on the CD are moving at **different** velocities, but they all trace out the same **angle**, θ, in a given time. For spinning objects we can define an angular velocity and an angular acceleration.

The angular velocity, ω, is the change in angle divided by the time taken, which for a given point is also equal to the velocity, v, of that point divided by its distance, r, from the centre.

$\omega = \Delta\theta / \Delta t = \boldsymbol{v} / \boldsymbol{r}$

The angular acceleration, α, is the rate of change of the angular **velocity**, just like linear acceleration is the rate of change of linear velocity. To make something accelerate you need to apply a **force**, and to give something an angular acceleration you need to apply a **torque**. The torque is equal to the force times the **distance** from the pivot point. We also need to allow for the angle at which the force is applied. If the force is applied pointing directly towards the pivot point then it won't make the body rotate. A force applied at right angle to this direction will have the **maximum** effect. The torque is given by

$\tau = r \times F = rF \sin\theta.$

When we want to calculate the acceleration of a body subject to a force we use Newton's **second** law, $F_{net} = ma$. To find the angular acceleration of a body subject to a torque we use the rotational equivalent to Newton's second law which is $\tau_{net} = I\alpha$. The quantity $\boldsymbol{I}$ is called the moment of inertia of a body, and is a measure of how hard it is to make the body rotate, or to stop it from rotating.

We can define an energy associated with rotation and an angular momentum. If there is no net torque these are **conserved**, just like energy and momentum are conserved in linear motion.

Discussion question

Linear variable	**Rotational variable**	**relationship**
velocity	angular velocity	$\omega = v/r$
acceleration	angular acceleration	$\alpha = a/r$
force	torque	$\tau = r \times F = rF \sin\theta$
mass	moment of inertia	$I = \Sigma_i m_i r_i^2$
momentum	angular momentum	$L = r \times p = rp\sin\theta$
kinetic energy	rotational kinetic energy	$K = ½ I\omega^2 = ½ \Sigma_i m_i v_i^2$

B. Activity Questions:

1. Clocks

The second hand goes around the clock face, that is through 2π radians, in 1 min.

So its angular speed is 2π radian/60 seconds, that is 0.105 rad.s^{-1}.

The minute hand goes around the clock face in one hour, so $\omega = 2\pi / 3600$ rad.s^{-1} = 1.75×10^{-3} rad.s^{-1}.

The hour hand goes around in 12 hours, 12 hours is 12 hours $\times$ 60 min/hour $\times$ 60 s/min = 43200 s.

So its angular speed is 2π rad / 43200s = 1.45×10^{-4} rad.s^{-1}.

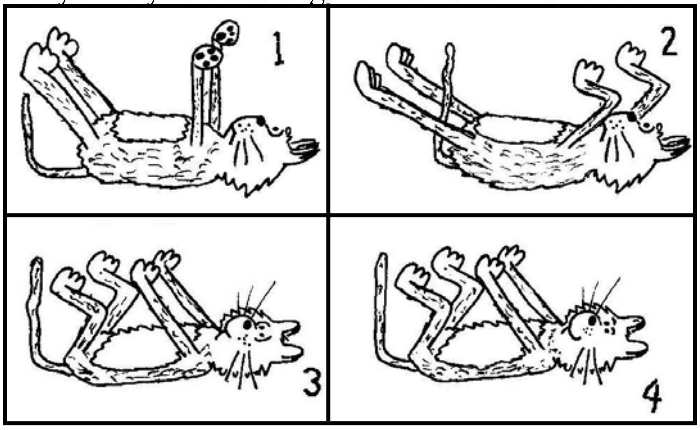
The larger the clock the bigger the linear speed of the hands, but the angular speed is the same.

2. Rotating Stool

The angular momentum of the system (person and weights) is conserved. When you stretch your hands out the system has a larger rotational inertia and a smaller angular velocity. When the hands are pulled inward towards the body the rotational inertia decreases and hence the angular velocity increases.

3. Falling cats

Conservation of angular momentum is not violated, at any time your *total* angular momentum is zero.

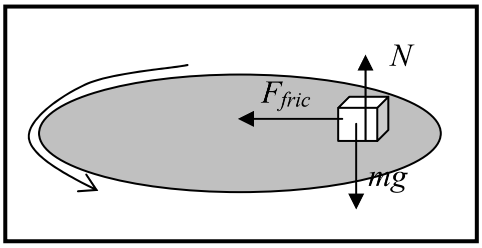


The procedure is as follows:

1. Falling with all four limbs sticking straight out.
2. Pull in front legs (arms) and rotate them 60° clockwise. Outstretched rear legs have to rotate 30° anti-clockwise.
3. Extend front legs (arms) and rotate them 30°anti-clockwise, and pull in back legs which have to rotate 60° clockwise.

You should now be rotated 30° clockwise. Repeat this 5 times and you'll be facing the right way and ready to land!

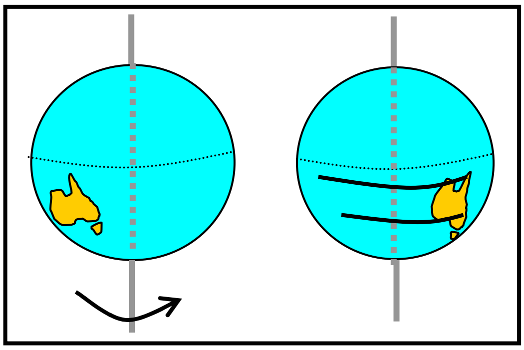
4. Objects on a rotation platform



Speed of rotation, distance from the centre and friction affect slipping; mass doesn't affect slipping

The box will slide off at a tangent to the curve, in the direction of its velocity vector. At the edges of the platform the linear acceleration is greatest, hence it is most likely to slip off when close to the edge.

C. Qualitative Questions:



1. Rebecca is in Cairns, Brent is in Sydney.

a. Both Rebecca and Brent move 2π radians (one rotation) in 24 hours as the Earth spins, so they have the same angular velocity.

b. Rebecca is further north than Brent, and closer to the equator (Southern hemisphere), hence her distance from the axis of rotation of the Earth is greater than Brent's. She travels a greater distance in the same time, 24 hours, so she must have a greater linear velocity, *v*.

2. A spoked wheel has a moment of inertia twice that of a solid wheel, so it takes twice the torque to achieve a given angular acceleration. If you accelerate the wheel while riding, while putting a given amount of energy into it, more energy goes into translational motion for the solid wheel than for the spoked wheel, for a given mass and radius, hence the solid wheel will go faster (greater *v*). This is why solid wheels are used in the Olympics. However the wheel also has to take half the weight of the bicycle plus rider, so it needs to be strong, but it also needs to be very light, so the bike doesn't get too heavy. A spoked wheel is a good compromise for strength and weight, unless you have access to expensive modern materials which are both very strong and very light.

D. Quantitative Question:

a. One complete rotation takes 100 ms, so the angular velocity is
$\omega = 2\pi f = 2\pi(1/T) = 2\pi(1/0.100\text{s}) = 62.8\ \text{rad.s}^{-1}$
(This is 6000 rpm, about the red-line on the tachometer of most 4 cylinder cars.)

b. If we treat the actin molecule as a rod 1 μm long with a mass of 2×10^{-22}kg, pivoted at one end, the moment of inertia of the actin propeller is $I = ml^2/3 = 2 \times 10^{-22}\text{kg} \times (1.0 \times 10^{-6}\text{m})^2 / 3 = 7 \times 10^{-35}\ \text{kg.m}^2$.

c. The angular momentum is $L = I\omega = 7 \times 10^{-35}\ \text{kg.m}^2 \times 62.8\ \text{rad.s}^{-1} = 4.4 \times 10^{-33}\ \text{kg.m}^2\ \text{rad.s}^{-1}$

d. Assuming constant angular acceleration, if it takes 100 ms to perform a rotation starting from rest, the angular acceleration of the actin is $\alpha = \Delta\omega/\Delta t = (62.8\ \text{rad.s}^{-1} - 0)/0.100\text{s} = 628\ \text{rad.s}^{-2}$