Workshop Tutorials for Introductory Physics
Solutions to MI12: **Gravity and Kepler's Laws**

A. Review of Basic Ideas:

Gravity and Kepler's laws

Our view of the universe and our **solar** system has evolved greatly over the past five hundred years. In 1543 when Copernicus wrote a description of the universe that put the Sun rather than the **Earth** at the centre, he was largely ignored and his theories considered implausible. Worse than this, they were in contradiction with scripture, and the church was liable to punish any who supported Copernicus' views. About a century later his supporter **Galileo** was interviewed by the Inquisition and put under house arrest for the rest of his life and forbidden to publish. He got off lightly. But despite the best efforts of the church, **astronomers** could observe the motions of the planets, and soon it became known that the planets did indeed orbit the Sun.

Kepler described the orbits of the planets and was the first to recognise that the orbits were not necessarily circular, but could be, and in fact they are, **elliptical**. Kepler came up with three laws, now known as Kepler's laws. The first is that all the planets move in an elliptical orbit with the **Sun** at one focus. The second is that a line joining any planet to the Sun sweeps out equal **areas** in equal times. The third is that the square of a planet's period, T, is proportional to the cube of its orbit's semi-major axis, r, or $T^2 = Cr^3$, where C is a constant which has the same value for all planets.

Kepler's laws were based on observation, and offered no explanation as to why the planets behaved as they did. What **force** kept the planets orbiting? Newton's law of **gravitation** provided the explanation. Newton postulated that there is a force of attraction between any two bodies, and this force is proportional to the masses of the bodies and inversely proportional to the square of the distance between the bodies; $F = Gm_1m_2 / r^2$. The constant G is the universal gravitational constant. It is possible to derive Kepler's laws from Newton's law of gravitation, and it turns out the constant C is equal to $4\pi^2/GM_s$, where M_s is the mass of the Sun.

Newton's law of gravitation applies to **all** objects in the universe, and Kepler's laws apply to any orbiting body, or satellite. Kepler's laws describe the motion of our natural satellite, the **Moon**, and all the man made satellites orbiting the Earth. They describe the motions of comets and asteroids and the moons of Jupiter.

Discussion questions

The mass of a building is negligible compared to the mass of the Earth. Hence the gravitational force of attraction which depends on both your mass and the mass of the attracting object is negligible when compared to that of the Earth (your weight).

G has a very small value (6.67×10^{-11} N.m^2.kg^{-2}) which in a laboratory situation is difficult to measure.

B. Activity Questions:

1. Drawing orbits

The planets move in elliptical orbits around the Sun. The closest point to the Sun is called the perihelion and the furthest point is called the aphelion. The Sun is at one focus of the ellipse. The string should be a length equal to the sum of the aphelion and the perihelion, so that the pencil is at most the aphelion distance from the Sun. The distance between the foci is equal to the difference between the aphelion and perihelion distances. The eccentricity of the orbit is the ratio of the distance between the foci to the length of the major axis, which is (aphelion – perihelion) / (aphelion + perihelion), which is equal to 0 for circle and is between 0 and 1 for an ellipse.

Planet	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto
Aphelion	70	109	152	249	817	1515	3004	4546	7304
Perihelion	46	108	147	206	741	1353	2741	4445	4435

Distances $\times 10^6$ km

2. Models of the solar system
Our understanding of the motion of the planets has changed greatly over the last few hundred years. You may see some of these changes by noting the differences between the models – the most obvious is that early models had the Earth at the centre of the solar system with everything else, including the Sun, orbiting around it. Modern models of the solar system place the Sun at the centre.

3. Kepler's Second Law
The area swept out per unit time by the line joining the planet and the Sun is constant. The distance between the Sun and planet varies, because the orbit is elliptical, hence the length of this line varies in time. For the area swept out per unit time to be constant the velocity must vary, decreasing as the planet moves further from the Sun (towards aphelion) and increasing as it moves closer (towards perihelion).

C. Qualitative Questions:

1. The radius of the Earth is approximately 6,400 km. The International Space Station orbits at an altitude approximately 400 km above the Earth's surface, or at a radius of around 6,800 km from the Earth's centre. Hence the force of gravity experienced by the space station and its occupants is almost as great as that experienced by people on the surface of the Earth. Astronauts and cosmonauts feel weightless when orbiting the Earth because there is no contact force between them and the space station. An astronaut standing on the floor in the space station is accelerating towards the Earth at the same rate and with the same velocity as the space station, hence he or she is not "pushed" against the floor the way someone on Earth is, and they feel "weightless" because they are in free fall, although the astronaut still has almost the same weight as on Earth.

2. The communications satellite is falling and a gravitational force is acting on it. The difference between it and the ball is that the satellite has sufficient tangential speed so that it moves sideways at such a rate as to fall "past" the Earth and so continue in an orbit around the Earth.

D. Quantitative Question:

Many science fiction stories feature men going to Mars and meeting Martians. In some stories the space travelers are able to walk around quite comfortably and even breathe the Martian atmosphere.

a. The gravitational force between two bodies is given by $F = Gm_1m_2 / r^2$. Using Newton's second law, $F = ma$, we can write for the acceleration due to gravity at the surface of Mars:

$g_{Mars} = GM_{Mars} / r_{Mars}^2 = (6.67 \times 10^{-11}$ N.m^2.kg^{-2} $\times 0.642 \times 10^{24}$ kg $) / (3397 \times 10^3$ m$)^2 = 3.7$ m.s^{-2}.

(Hint: use $F = ma = Gm_1m_2 / r^2$.)

b. The escape velocity can be calculated using conservation of energy. Consider throwing a ball into the air with velocity v, it initially has some kinetic energy and some gravitational potential energy. When it reaches its maximum height it has only gravitational potential energy, so we can write:

$E_{initial} = ½ mv^2 - GM_{Earth}m/R_{Earth} = E_{final} = - GM_{Earth}m/R_{max}$.

If we take the case of the ball actually leaving the Earth's gravitational field totally then $R_{max} = \infty$, and substituting this into the equation above gives: $½ mv^2 - GM_{Earth}m/R_{Earth} = - GM_{Earth}m/\infty = 0$.

Rearranging for v^2 gives:

$v^2 = 2GM_{Earth}/R_{Earth} = 2 \times 6.67 \times 10^{-11}$ N.m^2.kg^{-2} $\times 5.97 \times 10^{24}$ kg $/ 6378 \times 10^3$ m $= 1.25 \times 10^8$ m^2.s^{-2}

so $v = 1.12 \times 10^4$ m.s$^{-1} = 11.2$ km.s^{-1}.

c. On Mars:

$v^2 = 2GM_{Mars}/R_{Mars} = 2 \times 6.67 \times 10^{-11}$ N.m^2.kg^{-2} $\times 0.642 \times 10^{24}$ kg $/ 3397 \times 10^3$ m $= 2.52 \times 10^6$ m^2.s^{-2}.

and $v = 5.02 \times 10^3$ m.s$^{-1} = 5.02$ km.s^{-1}.

d. The escape velocity on Mars is less than half that on Earth, hence the amount of energy needed to launch a space ship from Mars is less than ¼ that needed to launch a space ship from Earth, hence Mars would make a better launching base for interstellar travel.