

# Workshop Tutorials for Physics

## Solutions to MR1: Motion

### A. Qualitative Questions:

1. If you throw a ball up with vertical velocity  $v$ , it will rise until all its kinetic energy,  $\frac{1}{2}mv^2$  is converted to gravitational potential energy, it will then fall, and that gravitational potential energy will be converted back into kinetic energy. Ignoring any work done on the ball by air resistance, it will have a velocity  $v$  when it reaches the same height from which it was thrown. From here it will continue to descend and accelerate at  $g$ , the same as if it was thrown down with a velocity  $v$ . Hence both balls will have the same velocity when they reach the ground. (Although the one thrown down will reach the ground first.)

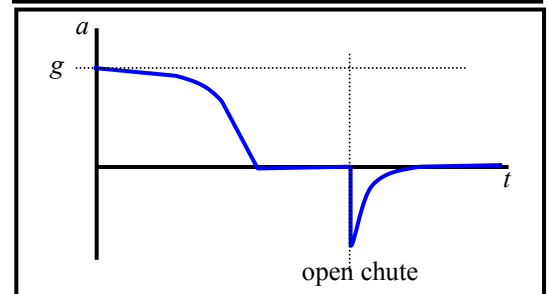
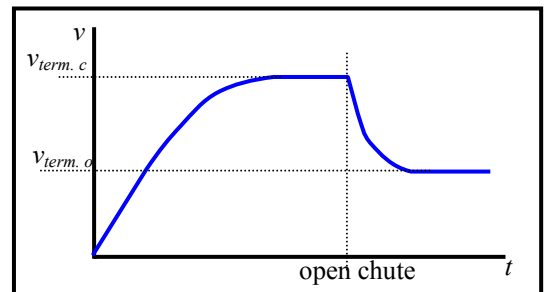
2. A skydiver's acceleration is given by  $a = g - bv^2$  where  $g$  is the acceleration due to gravity,  $v$  is his velocity and  $b$  is a constant (the co-efficient of drag). When he has reached terminal velocity the acceleration is zero,  $a = g - bv^2 = 0$ .

a. Factors which affect the value of  $b$  include the cross sectional area of the skydiver and parachute and the material on the surface pushing against the air.

b. When he opens his parachute he greatly increases the value of  $b$  by increasing the cross sectional area.

c. See opposite. The diver's velocity increases with his parachute closed until he reaches terminal velocity,  $v_{term\ c}$ , when  $g - bv^2 = 0$  and  $bv^2 = g$ . When he opens the parachute he slows down until he reaches a new lower terminal velocity,  $v_{term\ c}$ .

d. The skydiver's acceleration is the derivative of the velocity,  $dv/dt$ . It decreases to zero as the air resistance increases. When the parachute is opened the air resistance increases to greater than the gravitational force, and the acceleration is upwards, decreasing to zero again as the new terminal velocity is reached.



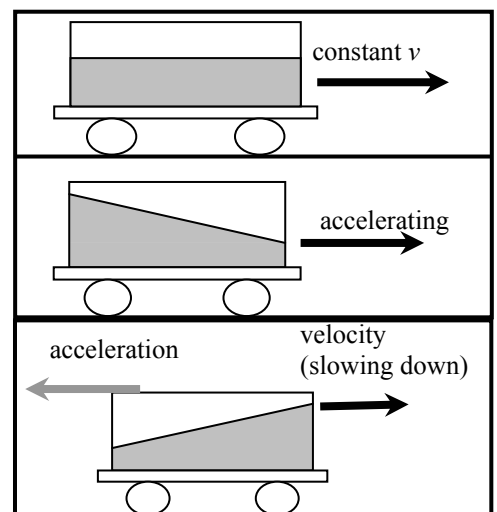
### B. Activity Questions:

#### 1. Accelerometer

The surface of the fluid in the accelerometer should be fairly flat and horizontal if you pull it smoothly at constant speed, as shown opposite. This shows that, when it is moving at constant speed, there is no **net** force on the fluid; it looks just as it would if it were standing still (if you ignore any bumps and vibrations).

When you accelerate the accelerometer forwards the fluid's surface will make an angle to the horizontal. The direction of the slope of the fluid shows you the direction of the acceleration. The fluid surface is at an angle because the net force on the fluid is no longer zero. The fluid collects at the back of the accelerometer when it accelerates.

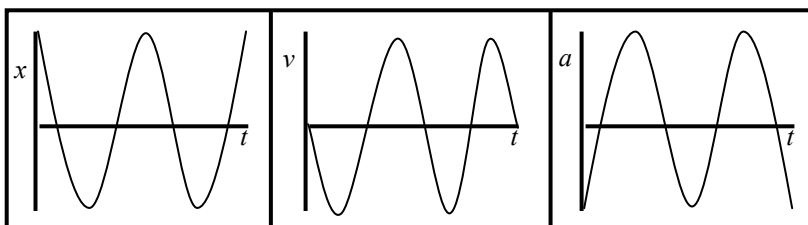
If you push the accelerometer and let go, it will slow down and eventually stop. The slope of the fluid 'points' towards the back, showing that the acceleration is in the **opposite** direction to the velocity of the accelerometer when it is slowing down.



## 2. Pendulum

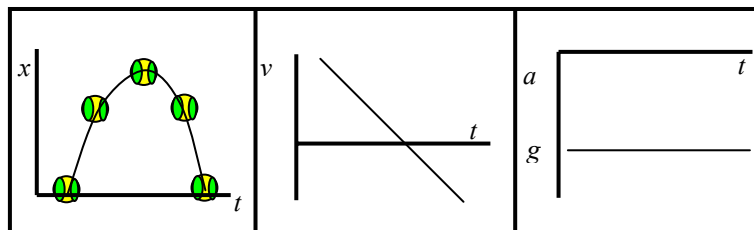
See diagrams opposite. The displacement is sinusoidal,  $x = A\sin(\omega t)$ . The velocity is zero when  $x$  is a maximum. The acceleration goes like  $-x$ .

You can have a zero velocity but a non-zero acceleration. At the top of the pendulum's swing, just as it reverses direction, the bob's acceleration is greatest (which means the velocity is changing at the greatest rate), yet at that instant the velocity itself is zero.



## 3. Acceleration due to gravity

The ball slows as it climbs until it reaches its peak, then speeds up as it falls. The acceleration of the ball is constant once it leaves your hand, and is due to gravity only. See diagrams opposite.



## C. Quantitative Questions:

1. Your car will accelerate from 0 to 100 km.h<sup>-1</sup> in 12 seconds, which is 0 m.s<sup>-1</sup> to 28 m.s<sup>-1</sup> in 12s.

a. Assuming a uniform acceleration, the acceleration is  $a = dv/dt = (28 \text{ m.s}^{-1} - 0 \text{ m.s}^{-1}) / 12 \text{ s} = 2.3 \text{ m.s}^{-2}$ .  
50 km.h<sup>-1</sup> = 14 m.s<sup>-1</sup>, to find the time taken to reach this speed we use  $v = v_0 + at$ , rearranged to give  
 $t = (v - v_0)/a = (14 \text{ m.s}^{-1} - 0 \text{ m.s}^{-1})/2.3 \text{ m.s}^{-2} = 6.1 \text{ s}$ .

b. To reach 60 km.h<sup>-1</sup> (= 17 m.s<sup>-1</sup>) takes  $t = (v - v_0)/a = (17 \text{ m.s}^{-1} - 0 \text{ m.s}^{-1})/2.3 \text{ m.s}^{-2} = 7.2 \text{ s}$

c. The distance traveled in the time taken to reach the new speed limit is

$$x = v_0 t + \frac{1}{2} a t^2 = 0 + \frac{1}{2} \times 2.3 \text{ m.s}^{-2} \times (6.1 \text{ s})^2 = 43 \text{ m}.$$

The distance traveled in the time taken to reach the old speed limit is

$$x = v_0 t + \frac{1}{2} a t^2 = 0 + \frac{1}{2} \times 2.3 \text{ m.s}^{-2} \times (7.2 \text{ s})^2 = 60 \text{ m}.$$

d. At the new speed limit it takes 6.1 s to travel the first 43 m, leaving 5000 m – 43 m = 4957 m to go.

This will take  $t = x/v = 4957 \text{ m} / 14 \text{ m.s}^{-1} = 354 \text{ s}$ , add the initial 6.1 s, giving 360 s or 6 minutes.

At the old speed limit it takes 7.2 s to travel the first 60 m, leaving 5000 m – 60 m = 4940 m to go.

This will take  $t = x/v = 4940 \text{ m} / 17 \text{ m.s}^{-1} = 291 \text{ s}$ , add the initial 7.2 s, giving 298 s or 5 minutes.

It takes only one extra minute for this short trip, at a speed with a much shorter stopping distance.

2. The ball's velocity has horizontal and vertical components which can be treated independently.

The horizontal component of the initial velocity is  $v_x = v_0 \cos\theta$ .

There is no acceleration in the horizontal direction, so this is the horizontal velocity of the ball for its entire flight. The vertical component of the initial velocity is  $v_y = v_0 \sin\theta$ , and in the vertical direction the ball has a constant acceleration due to gravity. Hence at any time  $v_y = v_0 \sin\theta - gt$ .

a. The ball is  $x = 10 \text{ m}$  away from the wall to begin with. The time taken for the ball to reach the wall is

$$t = x/v = x/(v_0 \cos\theta) = 10 \text{ m} / (15 \text{ m.s}^{-1} \cos 40^\circ) = 0.87 \text{ s}.$$

b. The vertical position of the ball at any time is  $y = (v_0 \sin\theta)t - \frac{1}{2} g t^2$ .

The ball reaches the wall at time  $t = 0.87 \text{ s}$ , hence it hits at a height

$$y = (v_0 \sin\theta)t - \frac{1}{2} g t^2 = (15 \text{ m.s}^{-1} \sin 40^\circ) \times 0.87 \text{ s} - \frac{1}{2} \times 9.8 \text{ m.s}^{-2} \times (0.87 \text{ s})^2 = 4.7 \text{ m}.$$

This is the height above the height at which the ball was released.

c. As the ball hits the wall the horizontal component of its velocity is still  $(v_0 \cos\theta) = 11.5 \text{ m.s}^{-1}$ .

The vertical component is  $v_y = v_0 \sin\theta - gt = (15 \text{ m.s}^{-1} \sin 40^\circ) - (9.8 \text{ m.s}^{-2} \times 0.87 \text{ s}) = 1.1 \text{ m.s}^{-1}$ .

Note that this is positive so the ball is going upwards. The total velocity is  $v = (v_y^2 + v_x^2)^{1/2} = 12 \text{ m.s}^{-1}$ .

d. The ball cannot have passed the highest point of its trajectory as it is still going upwards.

