Workshop Tutorials for Physics

Solutions to MR2: **Using Vectors**

A. Qualitative Questions:

1. Barry is running around the yard chasing birds.

a. Barry's displacement can be less than the distance he has traveled. Imagine if Barry ran backwards and forwards across the yard twice and finished back in the same place. His displacement (the vector quantity representing the difference between the initial and final position) would be zero but he would have run a fair distance.

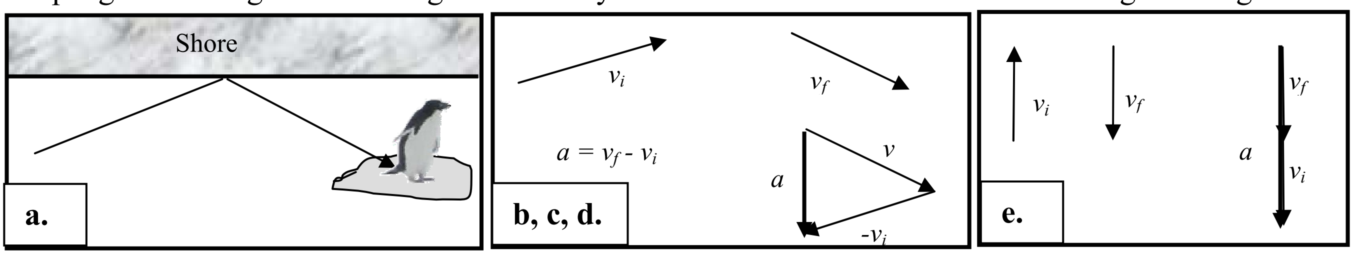
b. The displacement can never be more than the distance traveled. To travel from one point in space to another, the minimum distance Barry can travel is the straight line joining the two points. He can never travel less distance than that.

Barry comes to rest in the yard, some distance from where he started.

c. No component of his displacement vector can be greater than the magnitude of the vector itself. Components are defined in two directions at right angles to each other. Hence the displacement is the hypotenuse of a right angle triangle. The hypotenuse will be greater than either of the two sides.

d. If the directions of the components were taken such that one was in the same direction as the displacement then one component would have the same magnitude as the displacement vector and the other would be zero.

2. A penguin floating on an iceberg drifts slowly towards the shore and bounces off at a gentle angle.



B. Activity Questions:

1. Battleship

Battleship and similar games use vectors to determine the position of a ship. The vectors are usually written in terms of letter and number axes, rather than *x, y* axes, but are otherwise identical to vectors used in physics and mathematics. One way of describing the position of a pin is to give the lengths of perpendicular components, for example horizontal (numbers) and vertical (letters). Another way is to give the length of the vector and its angle to the horizontal. For example a pin at position C4 is also 5 units from the origin on a line 49^o above the horizontal.

2. Maps

Vectors are used to define positions on the maps via a letter/number grid. Most maps will also show a vector pointing north to define compass directions on the map.

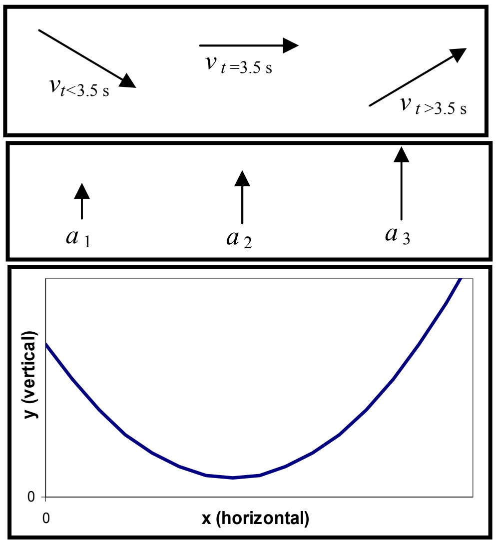
3. Vector Game

The axes are chosen in advance and marked, so you know which direction is $+x$ and which direction is $+y$. For example, forward may be $+x$ and right may be $+y$. If the caller says "$5x + 3y$" you take 5 steps forwards and three steps to the right. If the caller says "$-5x - 3y$" you take five steps back and three steps left.

4. Mirrors and reflections

In your reflection left and right seem to be reversed, but not up and down. This is because of the way we define left and right as relative to ourselves, not our surroundings. For example, "towards the wall" and "away from the wall" are not reversed, just as up and down are not reversed. Up and down are defined externally, usually relative to the ground. It is important to know how your coordinate systems are defined, and whether they change as you move!

C. Quantitative Questions:



1. A magpie has a position vector given by $\vec{\mathbf{R}} = 5t.\,\hat{\mathbf{i}} + (14 - 7t + t^2)\,\hat{\mathbf{j}}$.

a. The instantaneous velocity is found by taking the derivative of $\vec{\mathbf{R}}$, $\vec{\mathbf{v}} = d\vec{\mathbf{R}}/dt = 5.\hat{\mathbf{i}} + (-7 + 2t)\,\hat{\mathbf{j}}$.

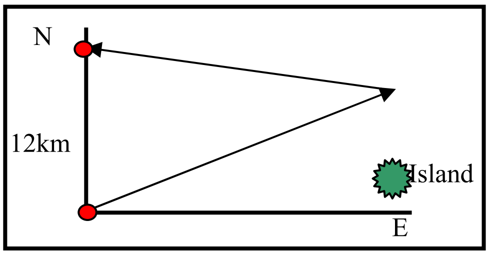
This is a constant velocity in the horizontal or $\hat{\mathbf{i}}$ direction, and a steadily increasing vertical velocity. The vertical component is initially negative and increases over time, becoming positive for times $t > 3.5$ s. See diagram opposite.

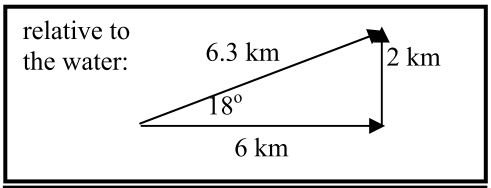
b. The instantaneous acceleration is found by taking the derivative of $\vec{\mathbf{v}}$, $\vec{\mathbf{a}} = d\vec{\mathbf{v}}/dt = 2t\,\hat{\mathbf{j}}$. This is always upwards and increasing.

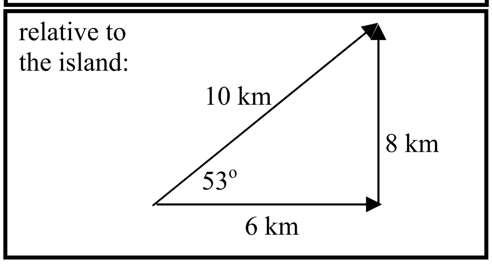
c. The bird has constant velocity horizontally, and is coming down and then going up again. The bird may be swooping after prey. See plot opposite.

2. Brent is taking Rebecca for a ride in his new boat. He shows Rebecca how he can program in the coordinates. "I just punch in 6 km east and 2 km north, and we'll be there in no time!"

An hour and a half later the navigator beeps to say they are about to arrive. Rebecca looks around and sees the island due south of them.







a. The current takes them north of the island on the way out and takes them further north on the way back. See diagram opposite.

b. They are a total of 12 km north over the whole trip which takes 3 hours. The velocity of the current is thus 4 km.h^{-1} due north.

c. The velocity of the boat relative to the water (ie ignoring the current) is

$v = x/t = \sqrt{(2\text{ km})^2 + (6\text{ km})^2}\,/1.5\text{ h} = \sqrt{40\text{ km}^2}\,/1.5\text{ h}$

$v = 4.2\text{ km.h}^{-1}$.

It is in a direction $\theta = \tan^{-1}(2\text{ km}/6\text{ km}) = 18.4^\circ$ north of east.

d. The boat travels 6 km east and (2 km +6 km) north relative to the island. Remember Brent punches in 6 km east and 2 km north and an extra 6 km north is provided by the current. This journey takes 1.5 h.

Total displacement = $\sqrt{(6\text{ km})^2 + (8\text{ km})^2} = 10$ km.

$v = 10\text{ km}/1.5\text{ h} = 6.7\text{ km.h}^{-1}$.

The direction of v is $\theta = \tan^{-1}(8\text{ km}/6\text{ km}) = 53.1^\circ$ north of east