Workshop Tutorials for Physics

Solutions to WR9: **Optical Instruments**

A. Qualitative Questions:

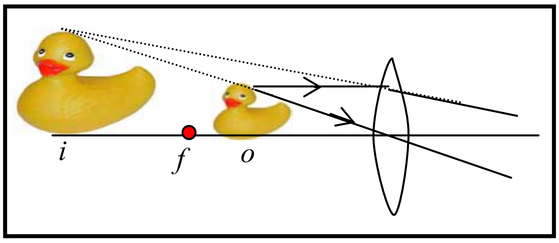
1. The resolving power of an instrument is governed by the Rayleigh criterion: $\sin\theta = 1.22\lambda/d$, where θ is the angular separation of the sources (e.g. stars viewed through a telescope) or features (fine detail viewed through a microscope) to be resolved, λ is the wavelength of the light and d is the aperture of the instrument. The Rayleigh criterion says that two features are just resolvable when the first minima of the diffraction patterns produced by the respective sources/features overlap. The larger the wavelength, the greater $\sin\theta$ must be for the objects to be resolvable, and hence the longer the wavelength the lower the resolution – resolution improves with decreasing wavelength.

2. When discussing the ability of an optical instrument to aid our ability to see we discuss the angular magnification of the instrument. The angular magnification is $M_\angle = \theta_i/\theta_0$ where θ_i and θ_o are the angular sizes of the image and the object, respectively, at the standard eye near point (25cm). Dealing with angular size rather than physical object and image sizes therefore gives a definition of M that is unambiguous.

B. Activity Questions:

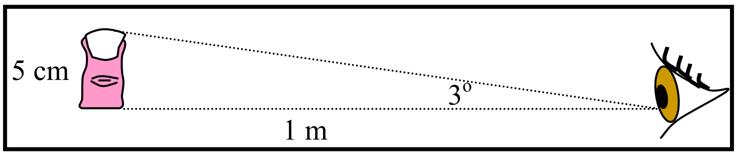
1. Magnifying glass

A magnifying glass uses a convex lens.
The image is upright, magnified and virtual.
The object must be at or within the focal length of the lens.



2. Thumb in your eye

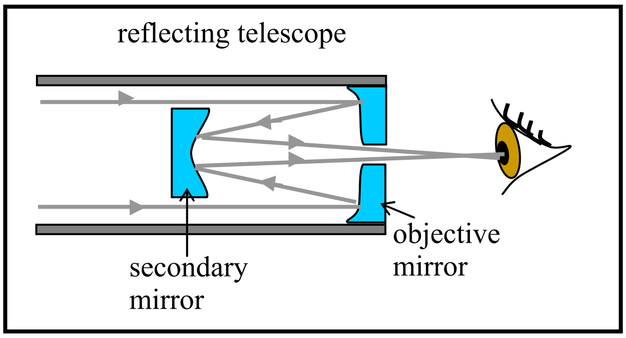
When you hold your thumb at arms length it is approximately a metre away, and is about 5 cm tall. The angle subtended is only 3^o.

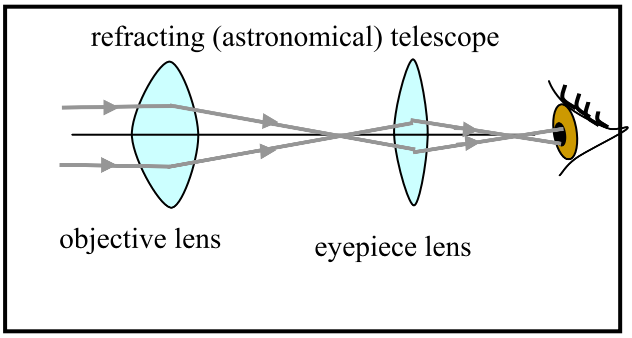


The resolving power of the eye is very good, and it is easy to resolve small features on your thumb which have an angular separation much less than this.

3. Telescopes

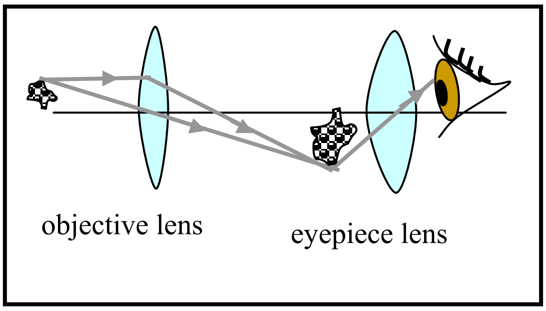
A telescope is used to look at (relatively) large objects a long way away. The telescope on the left is a reflecting telescope. It uses concave mirrors to produce a real, enlarged image at the eye. Rays from a distant object come in approximately parallel and are converged by the mirrors. The telescope on the right is a refracting telescope, also called an astronomical telescope. This telescope uses a pair of convex lenses to produce a real, enlarged, upright image. The focal length of the eyepiece should be much smaller than the focal length of the objective lens.
The telescope that you used was a refracting telescope. This is the common sort of simple astronomical telescope or spotting telescope.





4. Microscope

A microscope is used to look at small objects close up. A simple microscope uses only one convex lens, and is really just a magnifying glass. A compound microscope which is the most commonly used type, has a pair of convex lenses, as shown. The objective lens has a small focal length compared to the eyepiece lens. The image formed by the objective is viewed by the eyepiece which acts as a magnifier.



C. Quantitative Questions:

1. A compound microscope has an objective lens of focal length 5 mm. An object is placed 5.2 mm from the objective lens. The ocular (eyepiece) lens has focal length of 4 cm. The lenses are separated by a distance of 24.5 cm.

a. The object should be placed at the secondary focal point (at distance $-f$ from lens) to give an image at infinity.

b. The objective magnification $m_o = \frac{i}{o}$ where i is the image point and o is the object point.

In this example, $m_o = \frac{i_o}{o_o} = \frac{i_o}{f_o}$. We have $\frac{1}{f_o} = \frac{1}{o_o} + \frac{1}{i_o}$, rearranging gives:

$\frac{1}{i_o} = \frac{1}{f_o} - \frac{1}{o_o} = \frac{1}{i_o} = \frac{1}{5.0\,\mathbf{mm}} - \frac{1}{5.2\,\mathbf{mm}}$, so i = 130 mm. So now we have $m_o = \frac{i_o}{o_o} = \frac{130\,\mathbf{mm}}{5.2\,\mathbf{mm}} = 25$.

c. The magnification produced by the ocular or eyepiece lens is $m_e = \frac{i_e}{o_e}$ where i_e and o_e are the eyepiece image and object distances respectively. We have i_e = 245 mm – 130 mm = 115 mm. Using the lens formula again gives: $\frac{1}{f_e} = \frac{1}{o_e} + \frac{1}{i_e}$, rearranging gives:

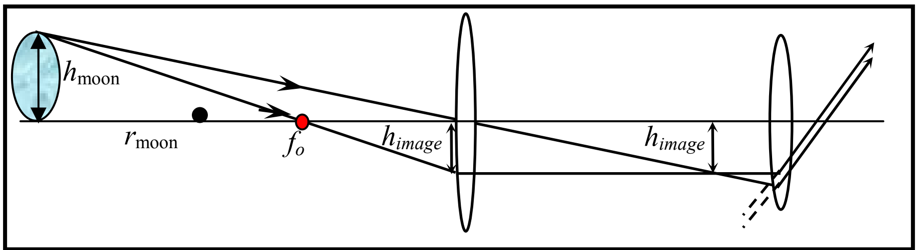
$\frac{1}{i_e} = \frac{1}{f_e} - \frac{1}{o_e} = \frac{1}{i_e} = \frac{1}{40\,\mathbf{mm}} - \frac{1}{130\,\mathbf{mm}}$, so i_e = - 58 mm. So now we have $m_e = \frac{i_e}{o_e} = \frac{130\,\mathbf{mm}}{58\,\mathbf{mm}} = -2.2$.

d. The total magnification of the microscope is $m_{total} = m_o \times m_e = 25 \times -2.2 = -56$.

2. Astronomical telescopes use two lenses to form an image of distant objects, such as stars.

a. See diagram opposite. The lens with the longer focal length (less convex) goes on the object side.

b. The magnification of a telescope is the –ve of the ratio of the focal length of the objective to the eyepiece.



c. The moon is about 3.8×10^5 km away from the telescope and has a diameter of 3.16×10^3 km, and is viewed with an objective lens with focal length 16 m. Using similar triangles and looking at the diagram above, you can see that $h_{moon}/r_{moon} = h_{image}/f_o$. Rearranging for h_{image} gives:

$h_{image} = (h_{moon}/r_{moon}) \times f_o = (3.16 \times 10^6\text{ m} / 3.8 \times 10^8\text{ m}) \times 16\text{ m} = 0.13$ m or 13 cm.

(Note that $r_{moon} >> f_o$, so we can use r_{moon} when really we should use $r_{moon} - f_o$)