

Workshop Tutorials for Physics

Solutions to WR10: Physical Optics

A. Qualitative Questions:

1. Consider light from a laser incident on two narrow slits.

a. If you treated light as rays moving in straight lines, you would expect to get two illuminated rectangles (essentially images of the two slits) on the screen in line with the slits – in this case the slits behave as a ‘shadow mask’ for the light.

b. You need to treat light as a wave to explain the interference pattern that is formed. A wave is a succession of crests and troughs. The alignment at a given point on the screen of crest-with-crest or crest-with-trough leads to bright (maxima in intensity) or dark (minima) ‘fringes’ respectively. This is called an interference effect and is characteristic of interacting waves.

The angular position θ of the fringes at the screen is given by $\sin \theta = \lambda/d$ for wavelength λ and distance between slits d . The actual distance between any fringe and the centre of the pattern is $y_m = m\lambda L/d$ where m is the order of the fringe ($m = 0, 1, 2, \dots$) and L is the distance from the screen to the slits.

c. If the distance between slits was decreased, $\sin \theta$ would be bigger, and the fringes further apart.

d. $\theta = y/L$ where y is height of a fringe above the centre of screen and L is slits-screen separation. If the screen is moved closer to the slits, decreasing L , this will again lead to an increase in fringe spacing.

e. Diffraction minima occur at $a \sin \theta_d = m\lambda$, slit width a , integer m . For fixed d but decreased a , θ_d increases so the fringes become broader.

f. If one of the slits is covered the interference effect will disappear. However a diffraction pattern due to a single slit will be observed on the screen.

2. A light source that shines on a slit. The slit is twice as long as it is wide. The angle between the center of a diffraction pattern and the first minimum is given by $\sin \theta = \lambda/a$ where a is the width of the slit. We will see a diffraction pattern – a bright central fringe with successively fainter subsidiary maxima either side – that is exactly twice as big vertically as it is laterally.

B. Activity Questions:

1. 2 source interference patterns

For a trough from one source to meet a crest from the other requires a path difference of $\frac{1}{2} \lambda$ between the two waves. Hence a nodal line first appears along the axis joining the two sources when the sources are $\frac{1}{2} \lambda$ apart. At any separation smaller than this there are no lines along which troughs and crests meet to give destructive interference.

2. Single Slit Diffraction

Diffraction patterns occur when light passes through a single slit. A diffraction pattern is an interference pattern due to the path difference between light arriving at a point from the left hand end of the slit and light arriving from the right hand end of the slit. A slit of width a will give a path difference of $a \sin \theta$ for the two rays. The diffraction minima occur when the path difference is equal to a multiple of $\frac{1}{2} \lambda$, i.e. where $a \sin \theta = \frac{1}{2} m \lambda$, so $\sin \theta = \frac{1}{2} m \lambda / a$. The bigger the slit, the smaller the spacing between fringes.

3. CD

Compact discs behave like diffraction gratings. This is because the data is stored on a CD using pits, and each pit is around 500 nm wide – within the wavelength range of visible light. When light is incident on the CD it is reflected from the pits and interferes. The intensity of the resultant light depends on the path difference, which is a function of wavelength. Different wavelengths hence give constructive or destructive interference at a given point, giving a particular colour at that point.

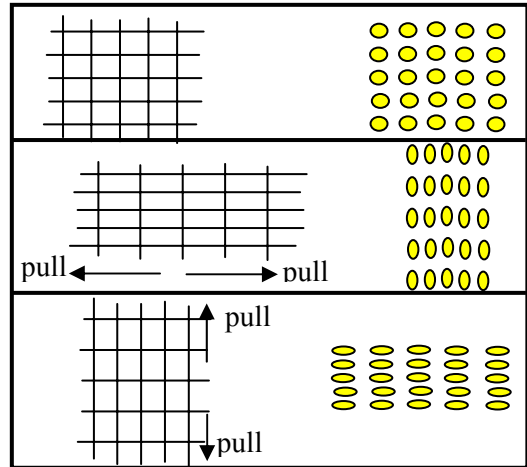
4. Diffraction patterns

The network of fine threads in the fabric forms a grating. When you shine the laser light through the fabric you see a diffraction pattern.

The spacing between the maxima in the pattern (bright spots) is inversely proportional to the grid spacing; $d \sin\theta = m\lambda$, can be used to find the grid spacing, d , given the angular separation, θ , of the maxima.

The diagrams show the fabric to the left and the diffraction pattern to the right.

When you stretch the fabric horizontally it also squeezes in vertically, the pattern will do the reverse of this, squeezing in horizontally and stretching vertically. When you stretch it vertically it will squeeze in vertically and stretch horizontally.



C. Quantitative Question:

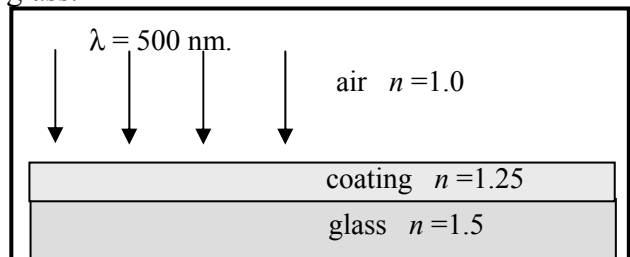
1. Consider a lens as shown below with a coating on the glass.

a. If a ray of sunlight is reflected from the air-coating interface. A phase change occurs upon reflection at an interface when the reflecting medium has higher index of refraction. Here, $n_{\text{coating}} > n_{\text{air}}$, so a phase change of π rad occurs.

b. It would also undergo a phase change on reflection from the coating-glass interface, as $n_{\text{glass}} > n_{\text{coating}}$.

c. The required condition for destructive interference is $2d = (m + \frac{1}{2}) \lambda/n$, where d is the coating thickness and $m = 0, 1, 2, \dots$. There is no phase change of the *transmitted* wave.

d. The minimum thickness of coating required to provide anti-reflection for sunlight is when $m = 0$, so: $d = \frac{1}{2} (\frac{1}{2}) \lambda/n = \frac{1}{4} \times (500 \times 10^{-9} \text{ m} / 1.25) = 1.0 \times 10^{-7} \text{ m} = 10 \mu\text{m}$.



2. An interference pattern is formed by shining light of wavelength $\lambda = 550 \text{ nm}$ through twin slits. The slits have a width of 0.03 mm and are spaced 0.15 mm apart.

a. We know $I = I_{\text{max}} (\cos^2 \beta) \left(\frac{\sin \alpha}{\alpha} \right)^2$ where $\beta = \frac{\pi d}{\lambda} \sin \theta$ and $\alpha = \frac{\pi a}{\lambda} \sin \theta$.

For the interference pattern: $\beta = (2m + 1) \frac{\pi}{2}$ where $m = 0, 1, 2, \dots$ gives zeros in intensity.

For the diffraction pattern: $\alpha = m\lambda$ gives zeroes in intensity. The first diffraction minimum is when $m = 1$ so $\sin\theta = \lambda/a$. So we can find the number of interference minima before the first diffraction minima

using: $(2m + 1) \frac{\pi}{2} = \frac{\pi d}{\lambda} \sin \theta = \frac{\pi d}{\lambda} \left(\frac{\lambda}{a} \right)$ and $2m + 1 = 2 \left(\frac{d}{a} \right) = 2 \left(\frac{0.150 \text{ mm}}{0.030 \text{ mm}} \right) = 10$ so $2m + 1 = 10$, $m = 4.5$,

but m must be an integer, hence $m = 4$.

b. The pattern formed by this arrangement of slits is shown opposite.

