Bypassing shake, rattle and roll

The Tacoma Narrows Bridge is perhaps the most famous example of a bridge that collapsed unexpectedly in response to external forces. But new "wave bypass" technology – similar to that underpinning invisibility cloaks – could help avoid such disasters, as **Michele Brun**, **Alexander Movchan**, **Ian Jones** and **Ross McPhedran** explain

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William McGonagall is widely recognized as being the worst poet in the English language. His poetic skills are in full evidence in his epic "The Tay Bridge Disaster", in which he recounts the events that occurred when a speeding railway train brought down a newly erected bridge near the town of Dundee. Both the design of the bridge and its interaction with the strong emanations of the mythical wind god Boreas were factors in the disaster, as described by McGonagall:

But when the train came near the Wormit Bay,
Boreas he did loud and angry bray,
And shook the central girders of the Bridge of Tay
On the last Sabbath day of 1879,
Which will be remember'd for a very long time.

The Tay disaster is one incident in the long history of bridges not only as a means of transporting people, vehicles and goods, but also as occasional provokers of disastrous loss of life. Indeed, many famous bridges are best remembered in their ruined state, such as the edifice immortalized in the children's nursery rhyme "London Bridge is falling down". Two other examples of failed bridges are the Angers Bridge on the Maine river in western France and the Tacoma Narrows Bridge in Washington State, US. The former collapsed in April 1850 when a group of 483 soldiers started marching in precise formation across the bridge. Their pounding excited a catastrophic vibration, leading to the troops falling into the river below and some 226 people losing their lives; since then soldiers crossing a bridge have always been ordered to break step and to space themselves farther apart than normal. As for the Tacoma Narrows Bridge – also known as "Galloping Gertie" – it suffered the angry brays of Boreas in 1940 when a rhythmic excitation by a strong cross-wind provoked a coupled flexural-torsional vibration in the elegantly designed bridge, which famously shook itself into pieces.

Most modern, 21st-century structures are designed so that damaging low-frequency, elastic vibrations can be avoided. But unexpected external loads can still trigger unwanted shake and rattle, with some dangerous consequences. Consider, for example, the Millennium Bridge in London and the Volga Bridge in the Russian city of Volgograd. The first of these opened as a footbridge across the Thames in 2000 but had to be shut soon after for a major redesign after members of the public complained about it moving excessively when they walked across it. The 7.1 km Volga Bridge – a road bridge over the river Volga – had similar problems when a long-wavelength resonance vibration caused sections of the bridge to bend in May 2011, less than a year after it had opened. It remained shut for some time as engineers at the German firm Maurer Söhne designed a series of hydraulic "mass dampers", each weighing just over 5 tonnes, that were then attached at various points to the main roadway, or "deck", of the bridge.

Thankfully there are some simple principles that those designing bridges can use to stop disastrous vibrational resonances from occurring in the first place. These design principles, to which the current authors are contributing (arXiv:1107, 1788), do not supplant the need for the sophisticated computer-aided design packages that structural engineers often use to make precise plans for major constructions. Rather, the design principles avoid the possibility that some overlooked vibrational mode could provoke a disastrous or unsatisfactory level of performance. Even though the Volga and Millennium bridges were designed using industry-standard packages, the fact that problems arose shows that it is all too easy with large and complicated structures to overlook vibrations that may cause structural problems under practical conditions.

Our approach for avoiding the problem of unwanted vibrations in bridges involves first analysing a simplified model of the bridge's structure, which provides a realistic estimate of the range of frequencies where problems will occur. Knowing these troublesome frequencies, we have shown it is possible to design a lightweight "wave bypass" structure that would – if attached to the bridge – divert the vibrations away from load-bearing elements and then be damped out easily. The bypass structure is essentially a highly directive system that re-routes the waves around the

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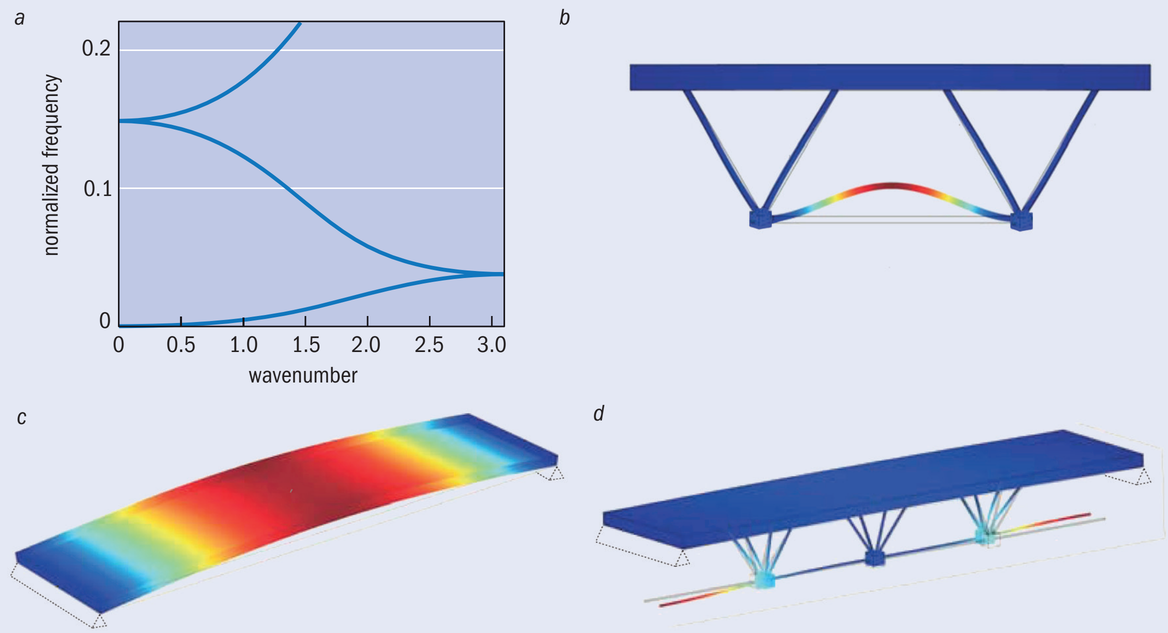
Scene of destruction
The Tacoma Narrows Bridge, which shook itself to pieces in 1940.

1 Modelling the Volga Bridge



In May 2010 – less than a year after opening – the Volga Bridge in Russia had to be closed after it sustained a damaging and unexpected long-wave resonance vibration. We sought a solution to the bridge's problems by modelling it as a periodically constrained slender elastic solid and then considering what happens when a lightweight periodic system of resonators – consisting of a series of linked masses – is attached underneath the main deck of the bridge. (*a*) Shown here in a 2D version of our model is one unit cell of the periodic structure (i.e. between a pair of pillars) with a pair of masses bolted on, with the colours indicating the bridge's normalized displacement ranging from blue (small) to red (large). The bridge deck has a negligibly small displacement amplitude, whereas the added resonators act as a bypass waveguide, diverting unwanted vibrations away from the main structure. (*b*) An illustrative dispersion diagram for the 2D periodic waveguide shows the normalized frequency, *F*, of a flexural wave as a function of wave number. $F = 0.0358$ corresponds to a standing wave replicating the vibration of the model bridge. (*c*) When the bridge is modelled in 3D – one "unit cell" of which is shown here between two pillars – the low-frequency flexural modes experienced by the real bridge are clearly revealed. (*d*) When the resonators are added (three per unit cell in this case), they redirect the energy so that it flows away from the main deck of the bridge, which then has minimized the displacement.

bridge deck, which is then shielded from vibrations within the unwanted frequency range.

The design principles that underpin these wave bypass structures are not new – being similar to the waveguides that are already used in many areas of science and engineering to funnel or direct waves along particular paths. Waveguide technology most notably underpins the operation of the fibre-optic cables that are the backbone of the Internet, while the principle is also used by researchers studying metamaterials – artificial structures that can be used for cloaking purposes by guiding waves around an object, which therefore remains hidden from view. These wave bypass principles are also found widely in natural systems (see box on p36).

An elementary approach

Guiding light through a bypass and controlling bridge vibrations both rely on the same underlying physics principles, even though one involves electromagnetic waves and the other mechanical vibrations; indeed, the mathematics of some classes of formulations in electromagnetism and elasticity are quite similar. But how easy is it in practice to suppress shake, rattle and roll in a long bridge? Of course, engineers could completely alter the structure of pre-existing bridges to remove these unwanted effects, but that is unlikely to be a realistic option in most cases. Our approach, instead, is to see if we can fix the problem by examining a small, representative part of the structure only and making lightweight changes.

Our approach is to fix the problem by making only lightweight changes

Inspired by the problems suffered by the Volga Bridge, our work involves considering the deck of a bridge as a slender 2D solid lying on pillars that are fixed to it underneath at regularly spaced intervals. Such a periodically constrained system approximates at low frequencies an elastic beam, but also has additional, long-wavelength "flexural" modes. By analysing the vibration of each repeating element or "unit cell" of the bridge – i.e. the section between each pair of supporting pillars – we have shown that it is possible to deflect these unwanted modes away by adding to the bridge a carefully designed system of linked resonators in the form of heavy, concentrated cubic masses. Situated beneath the bridge deck, the masses would be connected by bars to the main structure (figure 1).

The key parameter in our analysis is the normalized frequency, $F = fd/v$, where f is the frequency of

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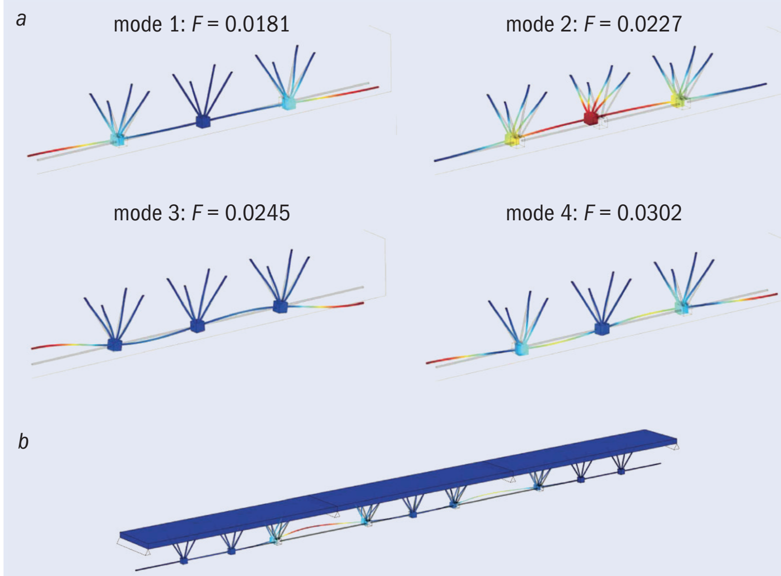
Before it all went wrong The Volga Bridge under construction with only half the bridge deck in place.

2 Light but strong



(a) We propose a design for the lightweight resonant structures that – if they were fitted to the Volga Bridge – would eliminate unwanted vibrations. Each image shows the normalized frequency, F, of the first four vibration modes and the displacement ranging from blue (small) to red (high). The first mode (F= 0.0181) eliminates the "flexural" vibration of the bridge deck, while the other three modes of the lightweight resonators, which are at normalized frequencies of 0.0227, 0.0245 and 0.0302, improve the structural response of the bridge over the frequency interval near the resonance flexural mode affecting the bridge. (b) If the complete bridge is modified, the flexural vibration of the main body is suppressed (the normalized frequency is 0.0182), with the built-in resonators having taken on the vibrational motion.

vibration, d is the distance between the pillars on the bridge and v is the speed of the shear wave in the upper deck of the bridge. (The mass of the bridge is embedded within v, which is the square root of the ratio of the bridge's shear modulus to its density.) In the absence of the linked resonators, our analysis reveals that the flexural mode, similar to that which damaged the Volga Bridge, corresponds to a standing wave with a normalized frequency of $F = 0.0358$. But with the resonators in place, the vibration of the main structure reduces almost to zero if we fine-tune the resonators so that their resonant frequencies are similar to $F = 0.0358$. The troublesome vibrations are redirected into these resonators, making the bridge safe once again.

The advantage of this approach is that the total mass of each resonator would be several orders of magnitude less than the bridge itself, while the bars linking the resonators would have a relatively low stiffness. The structures could also be easily predesigned by evaluating their frequencies of vibration when they are isolated from the bridge. Moreover, there would be no need to change the way the main deck is attached to the existing pillars or to adjust the stiffness of the deck.

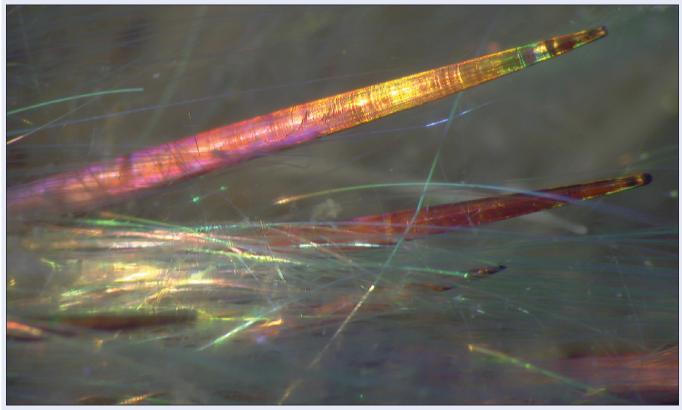
Working in three dimensions

Of course, if we are to understand properly the problems faced by the Volga Bridge, we need to consider it in not just two spatial dimensions, but all three. As with the simpler 2D case, however, our approach is to focus on just one unit cell. We first have to analyse the resonator structures in isolation – in other words, to work out what frequencies they vibrate at when unattached to the bridge. Given these numbers, our analytical model yields accurate estimates of the frequencies of standing waves within the bridge if these resonators were then attached to each repeating unit (as shown in figure 1d). We then select the physical and stiffness parameters of the resonators so that one of the eigenfrequencies of the lightweight resonator structure matches the frequency of the standing wave of the unmodified bridge.

When the two frequencies in question are sufficiently close to each other, the combined structure will change its dynamic response within the required low-frequency range as a result of the resonance of the lightweight structure. In other words, embedding a periodic system of low-frequency lightweight resonators will create a cluster of standing waves within these resonators near the resonant frequency, with the amplitude of vibration of the bridge being negligibly small. We can also fine-tune the lightweight structures attached to the bridge by simply tweaking the stiffness of the horizontal bars connecting the resonating masses.

Although our approach was only inspired by the problems that the Volga Bridge experienced, we believe our generic approach would – if implemented for real – be a practical solution to this bridge's difficulties. As our numerical model of the redesigned bridge reveals (figure 2), the unwanted waves would be channelled through the chain of resonators, away from the upper deck of the bridge. Indeed, such valuable insights along with real data could help us design a full-scale engineering system. The same method of using specially designed systems of resonators could

Wave bypasses in nature



Greg Rouse, University of Sydney

The wave bypass structures that could help engineers design bridges to cope better with unexpected external forces are, in fact, found widely in natural structures that guide light rather than mechanical vibrations. Certain moths and butterflies, for example, have some amazing nanostructures that can create beautiful optical effects. These "structural colours" are not produced by chemical pigments but are created instead by light being moulded through diffraction and interference in a way that often surpasses any existing human efforts to do so. Indeed, as zoologists Andrew Parker from London's Natural History Museum and Helen Townley from the University of Oxford in the UK have pointed out, these natural advanced photonic structures are sometimes so complex that the only feasible way of making them in the laboratory is to let nature do the job for us using special "cell-culture techniques" (2007 *Nature Nanotechnology* **2** 347). The resulting wave bypasses use nanostructures to create diffracting mirrors that funnel light of the desired colour into the vivid optical display, such as that observed in the hair of a "sea mouse" (pictured) – a worm living at depths of 10–1000 m. Its hairs consist of many thin hollow cylinders – just four-tenths of a micron in diameter – arranged in a hexagonally close-packed structure exactly like the cylinders in a photonic-crystal fibre (the only difference being that the material between the cylinders is the biological substance chitin in a sea mouse, but glass in the fibre). Rays of light incident on the sea mouse's hair obey the rules for photonic-crystal fibres, with only certain wavelengths being reflected. The peak in reflectance occurs at a wavelength of 633 nm (red/pink), producing this vivid and unusual colouration.

also be used to suppress low-frequency lateral vibrations in skyscrapers and other tall buildings.

It is, to us, remarkable that the principles lying behind structured mirrors, waveguides and bypass structures can be used not just by the humble sea mouse for its iridescence but also by engineers in the quest for more resilient and robust bridges.

We return to the wise – if not eloquent – words of McGonagall:

I must now conclude my lay
By telling the world fearlessly without the least
dismay,
That your central girders would not have given way,
At least many sensible men do say,
Had they been supported on each side with buttresses,
At least many sensible men confesses,
For the stronger we our houses do build,
The less chance we have of being killed.