Lec. 3 - energy in SHM
- vertical mass on spring
- angular oscillations
- pendulum

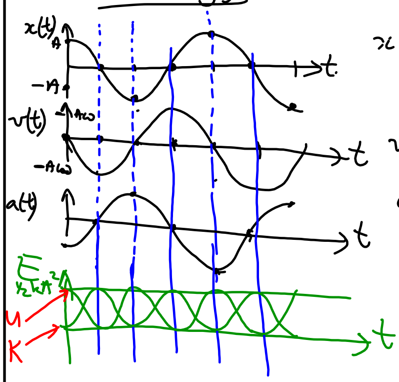
Recall: horizontal mass + spring

$$m\frac{d^2x}{dt^2} = -kx \quad \text{(D.E.)}$$

soln. is $x(t)=A\cos(\omega t+\phi)$ where $\omega=\sqrt{\frac{k}{m}}$

Apr 13-1:57 PM

Energy



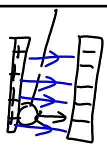
$x(t)=A\cos(\omega t+\phi)$

$v(t)=-A\omega\sin(\omega t+\phi)$

$a(t)=-A\omega^2\cos(\omega t+\phi)$
$= -\omega^2 x(t)$

Apr 13-2:06 PM

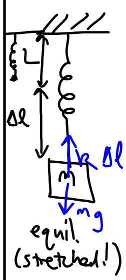
H/W



(a) is this SHM? No
- ball moving in uniform electric field
- uniform accel until hits the other plate
- not sinusoidal.
- no equil. position or restoring force that is prop to disp.

Apr 13-2:25 PM

- vertical mass on spring

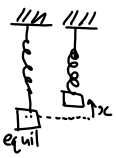


- Δl is amount by which spring is stretched when at equilibrium
- what is total force on object? zero, and so $mg = k\Delta l$

Apr 13-2:28 PM

If we now displace the object from equil, it will oscillate.
Is this SHM? Yes, provided spring obeys Hooke's "Law"

Proof: suppose we displace upwards by a distance x



Force on object

$$F = k(\Delta l - x) - mg$$
$$= \cancel{k\Delta l} - \cancel{mg} - kx$$
$$= -kx$$

Apr 13-2:34 PM

We already did this problem.

We have $F=-kx$

together with Newton's 2nd Law $F=m\frac{d^2x}{dt^2}$

$\Rightarrow$ DE $m\frac{d^2x}{dt^2}=-kx$

soln. $x(t)=A\cos(\omega t+\phi)$
where $\omega=\sqrt{\frac{k}{m}}$

Apr 13-2:40 PM

Total energy constant in time

but now $E = K + U_{grav.} + U_{spring}$

$= \frac{1}{2}mv^2 + mgx + \frac{1}{2}k(\Delta l - x)^2$

(optional – graph this)

Apr 13-2:42 PM

Another type of oscillation

Torsional pendulum

when twisted, there is a restoring torque

Turns out (small amplitudes) to be linearly prop. to angular displacement (approx.)

$\tau = -\kappa\theta$ $\quad (F = -kx)$

Apr 13-2:45 PM

Use Newton's 2nd Law ← moment of inertia

$\tau = I\alpha$

$\tau = I\frac{d^2\theta}{dt^2}$ $\quad (F = ma)$

Combine:

$I\frac{d^2\theta}{dt^2} = -\kappa\theta$ (D.E.)

Soln same as before (do it!)

Soln. $\theta(t) = \Theta\cos(\omega t + \phi)$ $\quad \| \; x(t) = A\cos(\omega t + \phi)$

capital theta

Apr 13-2:49 PM

provided $\omega = \sqrt{\frac{\kappa}{I}}$ $\qquad \omega = \sqrt{\frac{k}{m}}$

Richard Feynman "the same eqns have the same solutions"

Lectures on Physics Vol I

Apr 13-2:53 PM

Energy $E = K + U$ kinetic + potential

$= \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

elastic potential energy in spring, where x is length relative to relaxed spring

$= \frac{1}{2}mA^2\omega^2\sin^2(\omega t + \phi)$
$+ \frac{1}{2}kA^2\cos^2(\omega t + \phi)$

But $\omega^2 = k/m$

Apr 13-2:13 PM

$\Rightarrow E = \frac{1}{2}kA^2$ ← const. in time

← equals U when spring is maximally stretched.

Apr 13-2:19 PM