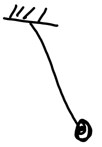
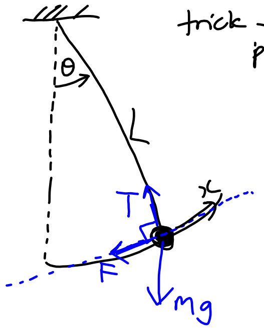
Lecture 4 – pendulum
- damped oscillations
- resonance

Pendulum



Apr 24-2:00 PM



trick – resolve forces parallel & perpendicular to motion

$$F = -mg\sin\theta$$

restoring force not linearly prop. to displ.

Apr 24-2:08 PM

Newton's 2nd Law

$$F = m\frac{d^2x}{dt^2} \quad — ②$$

Need to write in terms of θ

Trick: $\frac{dx}{dt} = L\frac{d\theta}{dt}$

linear speed   angular speed

~~Aside $x \approx L\theta$ only approx~~

Apr 24-2:13 PM

differentiate

$$m\frac{d^2x}{dt^2} = mL\frac{d^2\theta}{dt^2}$$

Combine with eq ①

$$\Rightarrow \quad \frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta$$

notice m cancelled   v. hard D.E. !!
– no soln in analytic form

Apr 24-2:18 PM

Assume θ is small
(amplitude is small)
Then D.E. becomes

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta$$

Taylor series: $\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots$

This is same DE as before.

Soln $\theta(t) = A\cos(\omega t + \phi)$

Apr 24-2:22 PM

where $\omega = \sqrt{\frac{g}{L}}$

Period $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$

If θ is not small, solution in terms of elliptical integrals $\rightarrow$ T slightly larger

Apr 24-2:27 PM

Physical pendulum

mass is distributed
read yourself
-can show that for small θ
it is SHM with

$$\omega = \sqrt{\frac{mgd}{I}}$$ ← about pivot

Apr 24-2:30 PM

Damped Oscillations

- maths more difficult
- extra force due to friction
turns out sometimes that friction is prop. to speed. (approx).
- in that case, maths is ok.

Apr 24-2:35 PM

consider oscillation with damping

$$F = -kx - b\frac{dx}{dt}$$

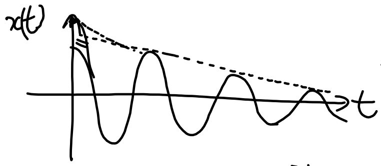
Now make D.E. using Newton's 2nd

$$F = m\frac{d^2x}{dt^2}$$

damping

$$\Rightarrow m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$$

Apr 24-2:39 PM



cosine multiplied by negative exponential

$$x(t) = Ae^{-\beta t}\cos(\omega' t + \phi)$$

can subst into DE - wait till we cover complex notation

Apr 24-2:42 PM

it is a soln provided

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

(~~but~~ slightly lower frequency)

and $\beta = \frac{b}{2m}$

Apr 24-2:46 PM