Today – speed of wave on stretched string
– the wave equation.

Assignment extension – due tomorrow

Stretched rope/string

$$v = \sqrt{\frac{F}{\mu}}$$

F ← tension in rope

μ ← mass per unit length

does not depend on amplitude
– freq/wavelength

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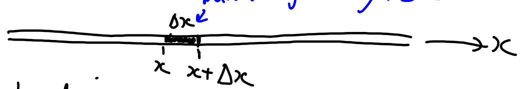
Wave equations occur often in physics

Consider a rope. A small segment

mass of segment = $\mu \Delta x$



Goal: derive eqn ~~for~~ for $y(x,t)$

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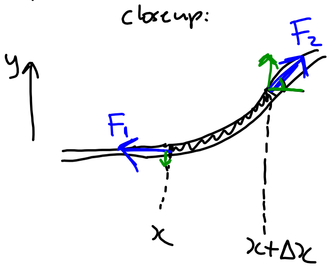
This is a D.E.

closeup:



note two forces on our segment.

We assume segment moves vertically but not horizontally
⇒ horiz. components of F_1 & F_2 are equal

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We identify this with F, the tension in rope in its equil. posn.

This will be true provided amplitude of wave is not too big.

The vertical ~~horiz~~ components of F_1 & F_2 are different. Call them F_{1y} and F_{2y} with +ve y being upwards.

⇒ segment will accelerate.

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Notice: $\left(\frac{\partial y}{\partial x}\right)_x$ = slope of rope at x $= \frac{-F_{1y}}{F}$

$\left(\frac{\partial y}{\partial x}\right)_{x+\Delta x}$ = slope of rope at $x+\Delta x$ $= \frac{F_{2y}}{F}$

Note $\frac{\partial y}{\partial x}$ means partial der. of y wrt x (keeping t constant)

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What is total force on segment?
It's in y dirn.

$$F_{total} = F_{1y} + F_{2y}$$

$$= F\left[\left(\frac{\partial y}{\partial x}\right)_{x+\Delta x} - \left(\frac{\partial y}{\partial x}\right)_x\right]$$

Apply Newton's 2nd Law to segment

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$$F_{total} = m \frac{\partial^2 y}{\partial t^2}$$

mass of segment → $\mu \Delta x$; accel. of segment

$$\Rightarrow F\left[\left(\frac{\partial y}{\partial x}\right)_{x+\Delta x} - \left(\frac{\partial y}{\partial x}\right)_x\right] = \mu \Delta x \frac{\partial^2 y}{\partial t^2}$$

divide by Δx & F

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$$\frac{\left(\frac{\partial y}{\partial x}\right)_{x+\Delta x} - \left(\frac{\partial y}{\partial x}\right)_x}{\Delta x} = \frac{\mu}{F} \frac{\partial^2 y}{\partial t^2}$$

as $\Delta x \to 0$

$$(15\text{-}12) \quad \boxed{\frac{\partial^2 y}{\partial x^2} = \frac{\mu}{F} \frac{\partial^2 y}{\partial t^2}}$$

wave eqn for stretched rope (assume low amplitude)

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Let's solve this PDE

One soln is

$$(15\text{-}7) \quad y(x,t) = A\cos(kx - \omega t)$$

sinusoidal wave traveling in the +x dirn

Easy to show 15-7 is a soln. of (15-12)

$$\frac{\partial y}{\partial x} = -kA\sin(kx - \omega t)$$

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$$\frac{\partial^2 y}{\partial x^2} = -k^2 A\cos(kx - \omega t)$$
$$= -k^2\, y(x,t)$$

while $\frac{\partial^2 y}{\partial t^2} = -\omega^2 y(x,t)$

using $v = \frac{\omega}{k} = f\lambda$ (15-6)

$$\Rightarrow \frac{\partial^2 y}{\partial x^2} = +\frac{k^2}{\omega^2}\frac{\partial^2 y}{\partial t^2} \Rightarrow \boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}}$$

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