Sound speed in ideal gas

$\gamma = \frac{7}{5}$ for diatomic e.g. N_2, O_2

$\frac{5}{3}$ for monatomic e.g. He

Today – Doppler effect
- shock waves
- Complex algebra
- Intro to chaos

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Doppler Effect

change in observed frequency of wave due to relative motion between source and observer.
S (listener) L

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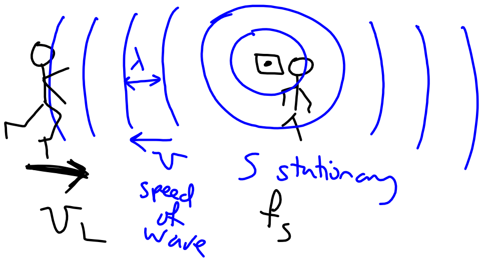
Case 1 source stationary



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what freq f_L does listener observe?

Trick use $v = f\lambda$

$$f_L = \frac{v + v_L}{\lambda} = \frac{v + v_L}{v/f_s}$$

$$\boxed{f_L = \left(1 + \frac{v_L}{v}\right) f_s} \quad (16\text{-}26)$$

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This is for movement towards S.

If L is moving away from S

use $f_L = \left(1 - \frac{v_L}{v}\right) f_s$

Case 2 – General case – both moving

Fig 16-27 drawn in frame in which medium is stationary.

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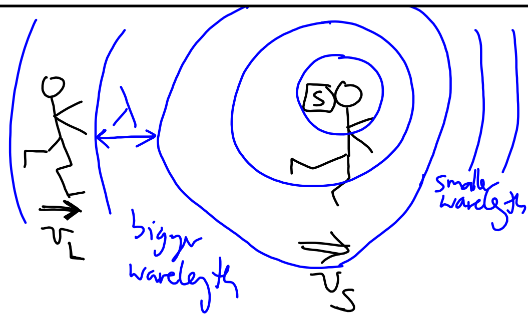

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observed wavelength is bigger

$$\lambda = \frac{v+v_s}{f_s}$$

As before $f_L = \frac{v+v_L}{\lambda}$ where λ is above

$$\Rightarrow \boxed{f_L = \frac{v+v_L}{v+v_s} f_s} \quad (16\text{-}29)$$

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- Note this reduces to 16-26 if we set $v_s = 0$, as it should.
- sign convention

  $v_L > 0$ if L moving to right

  $v_s > 0$ if S moving to right
- Useful way to write 16-29

  define $\Delta f = f_L - f_s$ (difference)

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Then $\frac{\Delta f}{f_s} = \frac{f_L - f_s}{f_s}$

$= \dots$

$= \frac{v_L - v_s}{v + \cancel{v_s}}$ ← relative velocity

Often, $v \gg v_s$

source moving much slower than wave

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$\frac{\Delta f}{f} \doteq \frac{v_L - v_s}{v}$ ← rel. speed, v ← wave speed.

e.g. travel at 1% of speed of sound

→ Doppler shift of freq is ~1%

Light is different!

$f_R = \cancel{f_L} = \sqrt{\frac{c-v}{c+v}}\, f_s$ (C-37)

rel. velocity

receiver

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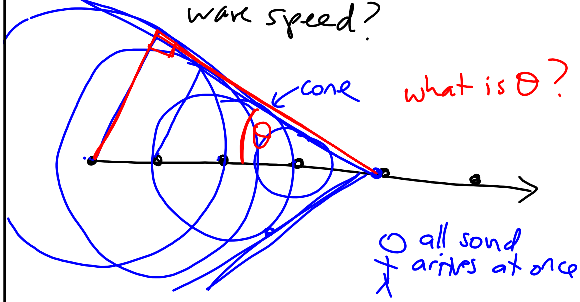
Shock waves

What if move faster than wave speed?



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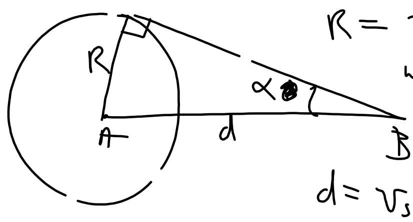
If source takes time t to move from A to B



$R = vt$

wave speed

$d = v_s t$

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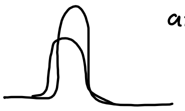
$$\boxed{\sin\alpha = \frac{v}{v_s}}$$

only works if $v_s > v$ (supersonic)

Demo on interference

Quincke's tube



air splits & recombines

destructive interference if extra length is $\frac{\lambda}{2}$

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Two topics left (Adv.)

- complex algebra
- chaos

"sensitive dependence on initial conditions"

Complex Algebra

very useful in solving DEs

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If z is a complex number

write $z = x + iy$

$(i^2 = -1)$ x real, y imaginary

Modulus of z

$$|z| = \sqrt{x^2+y^2}$$

Complex conjugate of z

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Euler's formula

$$\cos\theta + i\sin\theta = e^{i\theta}$$

Proof uses Taylor series

$$e^z = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \ldots$$

$$\sin z = z - \frac{z^3}{3!} + \frac{z^5}{5!} \ldots$$

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$$\cos z = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} \ldots$$

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$$\bar{z} = x - iy$$

Easy to show $z\bar{z} = |z|^2$

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