Today — intro to ~~ch~~ chaos (cont.)

Ⓐ describe — defining feature

sensitive dependence on initial conditions (exponentially)

- starting with two nearly-identical configurations, they diverge exponentially with time

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$\Delta x(t) \propto e^{\lambda t}$

difference bet. two examples (e.g. angle)

Lyapunov exponent

Ⓑ when might ~~to~~ we expect chaotic behavior?

observation: given system may behave chaotically

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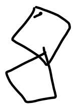
at some energies & not others

chaotic at intermediate energies



- low ~~E~~ energy - just a pendulum
- high " - swings in a circle

Turns out there are two necessary conditions:

① system must be sufficiently complicated

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must have at least 3 degrees of freedom

- pendulum (single), mass on spring } only have 2 d.o.f.

e.g. initial disp and time of passage through equilibrium

- double pendulum has 4 d.o.f.

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② the equations must be non-linear

- $m\frac{d^2x}{dt^2} = -kx$ is linear

  i.e. $x, \frac{dx}{dt}, \frac{d^2x}{dt^2}$ only appear in 1st power (or not at all)

  will never exhibit chaos

- a pendulum at large amplitude is non-linear:

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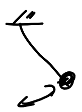

$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta$ non-linear

$\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \dots$

$\boxed{\text{Demo: Rubens flame tube}}$

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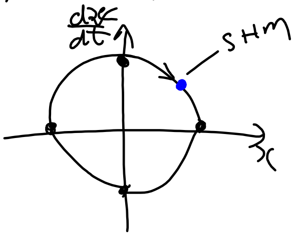
Phase diagrams

plot velocity vs displacement



• is "Poincaré section" of this system.

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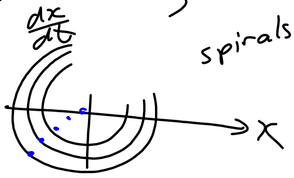
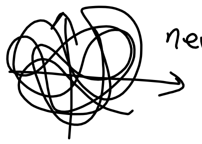
Now add damping



spirals in

Poincaré section

Chaotic?



never ever ever repeats exactly.

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So phase diagram of chaotic system is messy.

Next step. (Poincaré)

blue dot at regular intervals

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Turns out that Poinc. section of chaotic system looks regular

see notes!

fractals

this pattern is self-similar (looks similar on close-up scale)

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