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Effects of swing-weight on swing speed and racket power

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Abstract

Measurements are presented of the speed at which six different rods could be swung by four male students. Three of the rods had the same mass but their swing-weight (i.e. moment of inertia) differed by large factors. The other three rods had the same swing-weight but different masses. Our primary objective was to quantify the effects of mass and swing-weight on swing speed. The result has a direct bearing on whether baseball, tennis, cricket and golf participants should choose a heavy or light implement to impart maximum speed to a ball. When swinging with maximum effort, swing speed (V) was found to decrease as swing-weight (I_o) increased, according to the relation $V = C/I_o^n$, where C is a different constant for each participant and $n = 0.27$ when $I_o > 0.03$ kg · m^2. Remarkably similar results were obtained previously with softball bats (where $n = 0.25$) and golf clubs (where $n = 0.26$). Swing speed remained approximately constant as swing mass increased (when keeping swing-weight fixed). The implications for racket power are discussed.

Keywords: *Swing-weight, tennis, baseball, golf, racket, power*

Introduction

Many attempts have been made to describe the power of bats, clubs and rackets, especially in relation to the perceived increase in ball speed resulting from the use of modern materials and construction methods (Brody, Cross, & Lindsey, 2002; Nathan, 2003). For example, it is widely believed that modern tennis rackets are more powerful than old wood rackets, although the evidence for this is weak. The game of tennis is clearly played at a faster pace than in the past, but that can be attributed in large part to the increased fitness of modern players and a larger racket head that allows players to hit the ball faster while still maintaining good ball control. Modern rackets are generally lighter than old wood rackets and have a lower swing-weight (defined below), allowing modern players to swing the racket faster. A light racket will generate a lower ball speed than a heavy racket if both rackets are swung at the same speed. A question of interest in terms of racket power is whether the increased speed of a light racket is sufficient to compensate for the reduction in momentum and kinetic energy accompanying the decrease in mass. A weak link in this type of discussion has been the lack of reliable information on the effects of changes in mass and swing-weight on swing speed.

The effects of implement mass and swing-weight on the so-called "power" of an implement is still an open question. Swing speed appears to depend more on swing-weight than on implement mass, but the reasons for this are not properly understood. Previous research on the effects of swing-weight and implement mass on swing speed has used baseball bats (Fleisig, Zheng, Stodden, & Andrews, 2000), golf clubs (Daish, 1972) and tennis rackets (Mitchell, Jones, & King, 2000). However, in most of these studies the effects of mass and moment of inertia (MOI) have not been determined separately. An exception was the recent study by Smith, Broker and Nathan (2003), who analysed the swing speeds of 16 softball players swinging 20 different bats modified so that 10 of the bats had the same mass (but different MOI) and 10 had the same swing-weight (but different mass). They found that swing speed depends on swing-weight but it does not depend on bat mass. Swing-weight itself depends on bat mass, as well as on bat length and mass distribution, but swing speed was found to be essentially independent of bat mass provided that swing-weight was held constant.

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The study by Smith *et al.* (2003) was somewhat restricted in that bat mass and MOI were each varied over a relatively small range, from 0.695 to 0.879 kg (24.5 to 31.0 oz) and from 0.128 to 0.200 $kg \cdot m^2$ (7000 to 11000 $oz \cdot inch^2$), respectively. In the present study, we chose to study a set of rods having much larger variations in mass and MOI, by factors of 2.7 and 11, respectively. To achieve such large variations, we allowed the rod lengths to vary, whereas the bats studied by Smith *et al.* (2003) were of equal length. Very large variations in implement mass and MOI are not normally encountered in any given sport, with the result that variations in swing speed are usually quite small and difficult to measure. Much clearer effects are observed by varying mass and MOI well outside the normal range. In a recent study of overarm throwing (Cross, 2004), it was found that throw speed decreased by a factor of only 2.4 when the mass of the thrown object was increased by a factor of 60. Smith *et al.* (2003) reported that bat swing speed decreased by only 10% when the bat MOI was increased by a factor of 1.6. By restricting this study to a series of relatively light rods, each swung by one hand, our results are of greater relevance to the swing of a tennis racket rather than a bat or a club. Nevertheless, our results are of general relevance to all types of swinging.

The term "swing-weight" is commonly used by the sporting community to describe the moment of inertia (MOI) of an object. However, there is a noteable exception in the golf literature where the term swing-weight is used to refer to the first moment of a golf club about an axis 35.7 cm (14 inches) from the end of the handle (Jorgensen, 1999). In our paper, swing-weight refers to the second moment or the moment of inertia, as is more appropriate when an implement is swung. We use the colloquial term "swing-weight" rather than moment of inertia because all objects have different moments about the three principal axes, only one of which (i.e. the swing-weight) is usually relevant in determining swing speed.

For practical reasons, swing-weight is usually measured under laboratory conditions with respect to a defined axis of rotation close to the handle end. For example, the swing-weight of a tennis racket is usually quoted with respect to an axis of rotation 10.16 cm (4 inches) from the butt end of the handle. The actual axis of rotation may be quite different in practice. Furthermore, the moments of inertia about axes through the wrist, elbow and shoulder are all different and they may each affect the dynamics of arm motion in different ways. Since different players tend to use different swing techniques, some with more wrist action or with more internal rotation of the arm than others, the effects of swing-weight and implement mass on swing speed may vary substantially between players.

Experimental procedure

The rods chosen for swinging are shown in Figure 1. The two wooden rods were modified by the addition of a 0.036 kg bolt and nut inserted through a hole near the far end of each rod to increase the mass and MOI. The six rods varied in mass from 0.208 to 0.562 kg. The MOI for rotation about an axis through the handle end varied by a factor of 11, from 0.0089 to 0.0992 $kg \cdot m^2$. However, in all subsequent references to these rods, we will quote the mass and MOI of each rod by including the mass of one hand. The purpose of this is to recognize that the hand and the rod act dynamically as a single segment, since both have the same angular velocity and both are free to rotate about a common axis through the wrist. In other words, the rod will be regarded simply as an extension of the hand. In this way, the effect of a change in rod mass of, say, 0.1 kg can be seen to represent a relatively small increase in overall mass of the final segment in the kinetic chain, as opposed to a relatively large increase in rod mass alone.

We assume that the hand mass is 0.53 kg for all four participants, based on a figure of hand mass being 0.65% of total body mass for males (Kreighbaum & Barthels, 1996). All four participants had a total body mass of between 76 and 85 kg. Assuming that the hand extends over the last 10 cm of the handle, the resulting values of I_{cm} and I_o are as shown in Figure 1, where I_{cm} is the moment of inertia of the rod–hand system about an axis through its centre of mass, and I_o is the moment of inertia of the rod–hand system about an axis through the end of the handle. Each rod was held with the handle end of the rod adjacent to the wrist. The end of the handle was displaced by about 3 cm from the wrist axis, which had the effect of increasing the MOI about an axis through the wrist by less than 1% compared with the corresponding MOI about an axis through the end of the rod. The corresponding mass, M, of the rod–hand system is given by $M = m + 0.53$, where m is the mass of each rod as indicated in Figure 1. Even though the mass of the hand was greater than the mass of most of the rods, the hand made an almost negligible contribution to the value of I_o (0.0013 $kg \cdot m^2$). However, the hand makes a significant contribution to the moments of inertia of the rod–hand system about other axes of rotation, including axes through the elbow and shoulder.

Each rod was swung by four male students, primarily in the sagittal plane and using only one arm. For identification purposes, the students will be called Bob, Joe, Ken and Tom. Motion of various

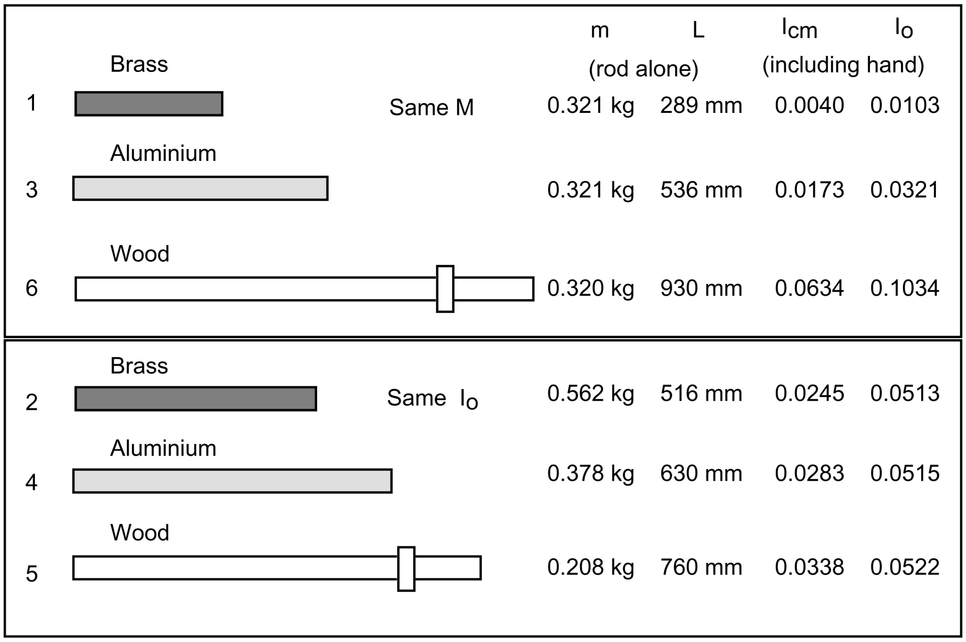

Figure 1. Rods used in the experiment. The moments of inertia I_{cm} and I_o (units $kg \cdot m^2$) refer to axes through the centre of mass and the handle (left) end, respectively.

body segments was restricted as far as possible so that the rods could be swung using only the upper arm, forearm and wrist. The students were specifically instructed not to use any other part of the body, such as the legs, hips or torso. Measurements were also made under conditions where the rods were swung using only the forearm and wrist. In this manner, it was anticipated that the effects of rod mass and MOI on arm motion would be clearly identified without the added complicating effects of rod mass and MOI on motion of body segments beyond the shoulder or elbow. Furthermore, by restricting motion of the arm and the rod to the sagittal plane, the problem was reduced essentially to two dimensions. Internal rotation of the upper arm plays an important role in many swinging styles, but it did not play any significant role in the present study. Given that the hand, forearm and upper arm can each rotate in various planes, there are many different ways to swing a rod. The present study was restricted in its scope to just one of those ways (or two if we include the case where the upper arm remained at rest), but it was one where each rod could be swung at near maximum possible speed.

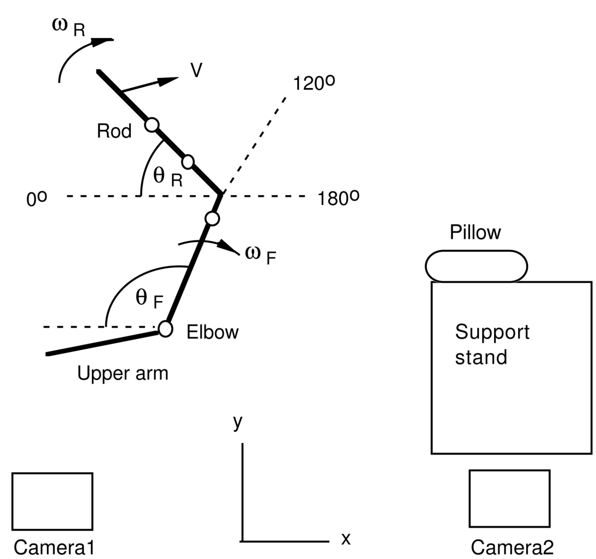

Figure 2. Geometry of the experiment. The four open circles show locations of markers used to record (x,y) coordinates on the forearm and rod. V denotes the velocity of a point on the rod 60 cm from the handle end.

Swing speed was recorded using the arrangement shown in Figure 2. Each participant was asked to sit or stand at a selected location and to swing each rod as fast as possible so that the rod would impact a pillow located anteriorly. The height of the pillow was adjusted so that the forearm would be approximately horizontal at impact. Each swing was recorded at 240 frames per second using two Qualisys infrared cameras located approximately 3 m apart in a plane parallel to the sagittal plane and about 4 m from the sagittal plane. Two cameras were used to obtain three-dimensional images, although subsequent analysis showed that motion of the forearm and rod out of the sagittal plane was negligible in all cases. Reflective tape was placed on the elbow, around the wrist, and around each rod at two points. For the shortest rod, the tape was located at points 15 and 27 cm from the end of the handle. For all other rods, the tape was located 15 and 30 cm from the end of the handle. Subsequent analysis showed that the image quality was excellent, with the tape separation remaining equal to the known separation within 0.5 mm throughout each swing.

Each participant swung each rod (in random order) three times, at intervals of about 30 s. Two participants completed their standing swings before sitting down to swing the rods, while the other two reversed this procedure. In the seated position, the participants held each rod behind their back with their upper arm resting horizontally along the edge of a table. They were asked to lean slightly towards the table to help ensure that their upper arm remained at rest during each swing and that their elbow also remained on the table. The starting angle of each rod was not standardized. Each participant was instructed to swing each rod in a manner that felt natural and comfortable. All four participants started each swing with the rod behind their back and below the horizontal position. All rods were swung through an angle of at least 150° and sometimes up to 240° before striking the pillow. In total, 144 separate rod swings were analysed (6 rods × 4 participants × 3 swings × 2 techniques). Increasing the number of rods and swings further would have entailed a very large number of maximum effort rod swings by each participant, and fatigue would then have been a significant factor.

Data analysis

Data files from the two cameras were processed to output centred (x, y, z) coordinates of each marker over selected intervals of time starting from the beginning of each swing and ending just before each rod hit the pillow. The z axis data (towards the camera) were subsequently discarded since motion out of the sagittal plane was negligible and since our primary interest was motion within the sagittal plane. The y axis is vertical and the x axis is horizontal throughout this paper. The raw data were processed to determine the angle of inclination of the forearm, θ_F, and the rod, θ_R, as a function of time, t. The θ_F versus time and θ_R versus time data sets were each fit by a sixth-order polynomial to calculate the angular velocity of the forearm, ω_F, and the angular velocity of the hand and rod, ω_R. Even though the fits were excellent, with a regression coefficient greater than 0.9999 in all cases, greater accuracy towards the end of each swing was obtained by fitting a sixth-order polynomial to the last 20 or 30 data points. Additional accuracy checks were performed by fitting fifth-order polynomials to the complete data set as well as the last 20 or 30 data points. Significant discrepancies were found in some cases, arising from the fact that the last data point sometimes included the initial impact of a rod with the pillow. When this data point was excluded, all polynomial fits for any given swing generated angular velocities that agreed within 1% and angular accelerations that agreed within 5%. Fits over an even smaller subset of data near $\theta_R = 120°$ also provided accurate data on the angular velocity at $\theta_R = 120°$, but the complete data set needed to be analysed to determine an appropriate angle at which to present the angular velocity data. The complete data set also provided interesting information on the time history of the wrist and elbow forces and torques applied by each participant. We intend to report on the effects of swing-weight on swing style in another paper.

Doubt has previously been cast on the value of the polynomial fit technique when processing kinematic data (Winter, 1990). A problem can arise if the order of the polynomial is too low to reproduce rapid changes in the data, in which case the polynomial will simply smooth the data. Conversely, a high-order polynomial fit to a small number of data points can artificially generate large excursions in slope between data points even if the regression coefficient is close to unity. Consequently, we undertook the additional tests described above to ensure that the fits were valid and that estimates of the velocity and acceleration based on those fits were reliable.

The same curve fit procedures were used to calculate the x and y components of the velocity of each marker, from which the velocity components and absolute speeds of selected points along each rod could be determined by linear extrapolation. Linear and angular velocity errors were both estimated to be less than 1%. The quantity of greatest interest in this paper is the linear speed, V, of any given impact point on a rod, since this is the speed that determines the outgoing speed of a struck ball. Many such speeds could be quoted from our study, but we have chosen to present primarily the data pertaining to the speed, V_{120}, of a point 60 cm from the handle end of each rod at a time when each rod was inclined at an angle of 120° to the horizontal (30° past the vertical position). The 120° position was chosen in part because all rods reached this point before striking the pillow but not all rods were swung past the 130° position. In addition, the maximum speed of each rod was reached at a rod angle of about 120° or at a slightly larger angle. As a result, rod speeds quoted at a rod angle of 120° are almost the same as the rod speeds at other nearby angles.

Despite the fact that half the rods were shorter than 60 cm, we have extrapolated data beyond the end of these rods as if the rods were actually longer than 60 cm. The 60 cm point was chosen as a typical impact point for a tennis racket, given that most rackets are about 69–70 cm long. An assumption here is that the swing speed of any given rod will depend primarily on its mass, M, or its moment of inertia, MOI (about an axis through the end of the handle), since these are the only two quantities that determine the dynamics of a rod when it is subject to a given force and a given torque. Consequently, swing speed should depend only very weakly on the length of the rod, L, or on the location of its centre of mass (commonly known as the balance point). In other words, for any given M and MOI, a change in the rod length or the balance point should not have any significant affect on swing speed. This assumption is justified in the Appendix.

Results and discussion

From each of the three swing trials for each rod, we selected the maximum linear speed, V_{120}, as representing the maximum effort swing. The results are shown in Figure 3 for both the standing and sitting swings. Apart from one small discrepancy for Tom, the results show clearly that swing speed decreases as MOI increases when M is held constant, regardless of whether the rod is swung by both the upper arm and the forearm or by the forearm alone. The one small discrepancy for Tom indicates that the rod with the lowest MOI was not swung with maximum effort in the standing position. Indeed, Tom managed to swing this rod faster using the forearm alone. The reason is that Tom swung this rod so fast in the seated position that he felt a significant sting in his hand, presumably associated with the fact that the impact point was close to the hand for this, the shortest rod. When he then swung this rod in a standing position, he appeared reluctant to swing it with maximum effort. The results for Ken are surprising since, for any given rod, the swing speeds in the seated and standing positions were approximately the same. Results for Joe using the constant MOI rods when seated are not included here, because the wrist marker worked loose during this series of swings, which did not become apparent until well after the measurements were completed.

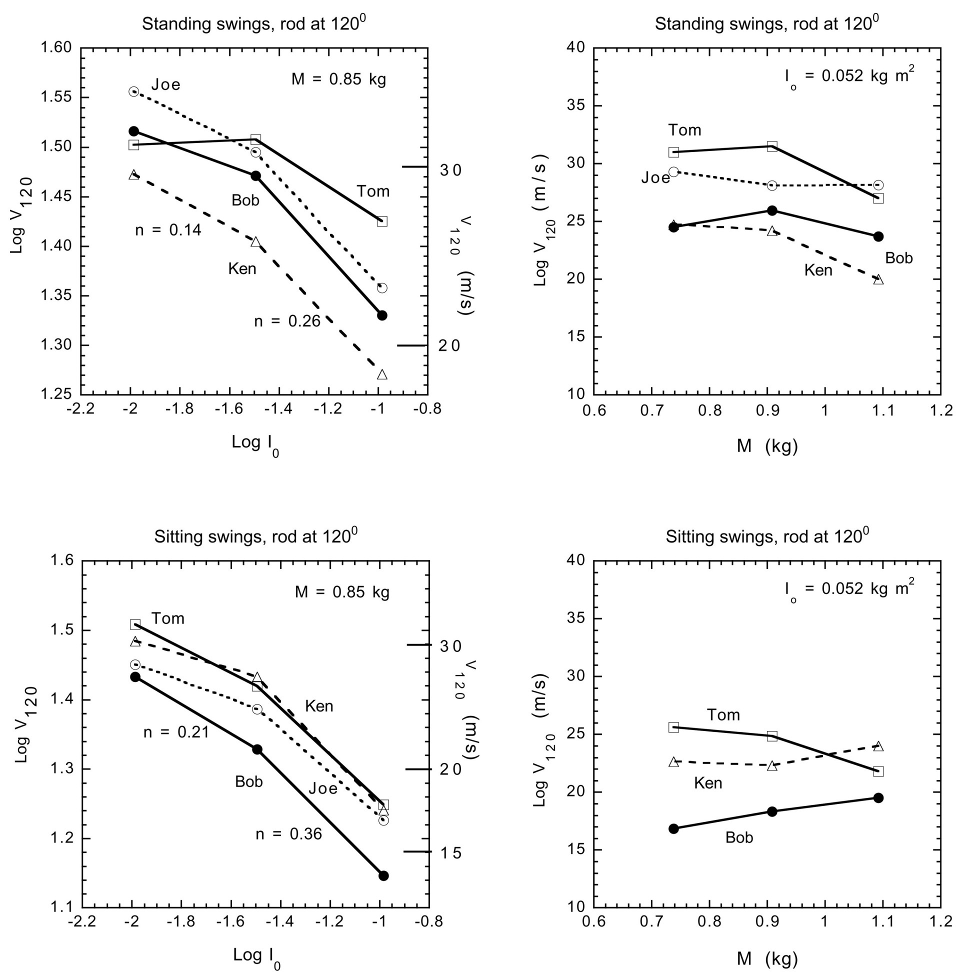

Figure 3. V_{120} results for both standing and seated positions. For the constant mass rods, log V_{120} is plotted on the left vertical scale with a translation to V_{120} on the right vertical scale.

Figure 3 shows log V_{120} versus log I_o in order to test for a power law dependence of the form $V_{120} = C/I_o^n$, where C is a constant for each participant depending on strength and technique. Since log V_{120} = log C - n log I_o, n is given by the slope of the log–log graph. This type of power law provides good fits to the swing speed data for softball bats where $n = 0.25$ (Smith *et al.*, 2003), and it also provides a good fit to golf club data (Daish, 1972). Daish quotes $n = 1/5.3 = 0.19$ using clubhead mass rather than MOI as the variable. If one assumes that the shaft mass was approximately 0.4 kg in his experiment, then it is a simple matter to calculate the MOI for his clubs about an axis through the end of the handle. In that case, a good fit to his clubhead speed versus MOI data is obtained with $n = 0.26$ (over the range $I_o = 0.166$ to $0.412\ \text{kg}\cdot\text{m}^2$). In our case, we found that the above power law did not provide particularly good fits to the data. As indicated in Figure 3, n was relatively small at low I_o, increased at high values of I_o and was larger for sitting swings than for standing swings. Separating the data into low and high I_o regions defined by the linear segments in Figure 3, and ignoring the discrepancy for Tom, we found n values (mean $\pm s$) as follows: (a) standing, $n = 0.12 \pm 0.02$ at low I_o and $n = 0.27 \pm 0.01$ at high I_o; (b) sitting, $n = 0.16 \pm 0.05$ at low I_o and $n = 0.35 \pm 0.02$ at high I_o.

The variation of n with I_o in our experiment is not surprising, considering the large range of I_o values studied. In the limit where I_o tends to zero, n must also approach zero, since swing speed will then depend only on the MOI of the arm. Swing speed will not become infinite when $I_o = 0$. The above power law is therefore appropriate only over a restricted range of I_o values. Values of I_o less than about $0.015\ \text{kg}\cdot\text{m}^2$ are of little practical interest given that all badminton, squash and tennis rackets have larger I_o values. Considering only the high range $I_o > 0.03\ \text{kg}\cdot\text{m}^2$ standing data where $n = 0.27$, our results are surprisingly similar to those obtained previously with softball bats (where $n = 0.25$) and golf clubs (where $n = 0.26$). Both one-handed and two-handed swing speeds therefore seem to obey the same or very similar power laws over a wide range of swing styles and swing-weights. Swinging with the forearm only is a somewhat different swing technique, as indicated by the higher values of n quoted above. Changes in rod MOI have a greater influence on forearm-only swing speeds, presumably because the rod MOI is then a larger fraction of the total moment of inertia about the most proximal axis.

It is emphasized that the power laws described above have somewhat limited validity. In our own case, the three data points for each participant were sufficient to show that a simple power law cannot be used to describe the variation of swing speed with swing-weight over an extended range of I_o values. Furthermore, one cannot establish the existence of a simple power law from only the two highest I_o values. Similarly, one cannot establish the existence of a universal power law dependence on I_o, even with many data points, if the range of I_o is limited. Nevertheless, a simple comparison between our results and those of previous studies can be made by fitting a power law function to the high I_o data, in which case we find that the exponent is very similar to that found by previous authors. From a theoretical point of view, a power law exponent of $n = 0.5$ would imply that the kinetic energy of a swung implement is independent of I_o, while an exponent of $n = 0.33$ would imply that the power input is constant (Brody *et al.*, 2002; Nathan, 2003).

The results in Figure 3 showing V_{120} versus M when MOI is held constant are not as easy to interpret, since in some cases there is a slight increase in swing speed with M, in other cases there is a slight decrease, and in some cases V_{120} is maximized for the intermediate mass rod. One result that is clear is that the lightest rod is not necessarily swung the fastest. In all cases, the lightest rod is swung at approximately the same speed as the intermediate weight rod. An even simpler result is obtained if one averages the swing speeds over all four participants, in which case V_{120} is essentially independent of M. Averaging results over different participants is a valid procedure when sampling population averages, but in this case it hides information about individual differences in swing techniques. The results suggest, for example, that Bob and Tom might each benefit by using an intermediate weight racket, but on average there is no benefit for any player in using a light, heavy or intermediate weight racket.

Implications for bat and racket power

Given the apparently universal n $\approx$ 0.26 power law for maximum effort swinging, it is appropriate to examine the consequences in terms of bat and racket power, in a manner similar to that described by Daish (1972). Daish considered the impact of a clubhead and a golf ball as a simple head-on collision between two point masses and found that the optimum mass, M, of the club head is given by $M = m(1/n - 1)$, where $m = 0.0459$ kg is the ball mass. Taking $n = 1/5.3$, he concluded that the optimum mass was 0.197 kg, regardless of the strength of the player. However, he also noted that the mass of the clubhead could be varied over a range of 0.136–0.283 kg with only a 1% reduction in ball speed. A similar calculation can be performed for a bat or a racket, even though the mass is distributed over the length of the implement rather than being concentrated at the impact point. In this case, one can define an effective mass, M_e, at the impact point given by

$$M = \frac{M}{(1 + Mb^2/I_{cm})} \quad (1)$$

where M is the implement mass, I_{cm} is the moment of inertia for rotation about the centre of mass, and b is the distance from the impact point to the centre of mass (Brody *et al.*, 2002). The collision between a ball and the implement can then be treated as one between two point masses. For a bat or a racket, the rebound speed of the ball depends on the location of the impact point, and is at its maximum typically about 0.15 m from the tip where M_e/M is typically about 0.4 for a racket and about 0.8 for a baseball bat. M_e is equal to M at the centre of mass where $b = 0$ but it is less than M at other impact points.

Consider a ball of mass m that is initially at rest and struck by a mass M_e incident on the ball at speed V. The speed, v, of the ball after the collision is given by

$$v = \frac{M_e V(1 + e)}{M_e + m} \quad (2)$$

where e is the coefficient of restitution. Given that all wooden bats and all rackets vary only slightly in length and mass distribution, the swing-weight I_o will be directly proportional to the actual mass M to a good approximation. Hence $V = C_1/M^n$, where C_1 is a constant. Let $M_e = M/x$, where x depends on the actual impact point. Then

$$v = \frac{C_2 M^{(1-n)}}{(M + xm)} \quad (3)$$

where $C_2 = (1 + e)C_1$. v is at its maximum when $dv/dM = 0$, giving

$$M = xm(1/n - 1) \quad (4)$$

which is a more general version of the result obtained by Daish. For example, if $x = 2$ and $n = 0.27$, then v is a maximum when $M = 5.41m$, but if $x = 3$ (impact closer to the tip) $M = 8.11m$. Alternatively, if $n = 0.25$ for a particular player, then $M = 6m$ when $x = 2$ and $M = 9m$ when $x = 3$. For a 0.057 kg tennis ball, there is a range of optimum M values from about 0.308 to 0.513 kg depending on the impact point and the particular power law for any given player. Similarly, for a 0.145 kg baseball, the optimum bat mass varies from 0.783 to 1.305 kg for the same values of x and n. However, the plot of v versus M in Figure 4 (for a tennis ball and racket) shows that there is a broad maximum in v, in which case there is no significant advantage to a player in choosing the bat or racket mass that maximises v precisely.

Conclusion

A series of six different rods were swung by four male students with maximum effort. Three of the rods had the same mass but their swing-weight differed by large factors. The other three rods had the same swing-weight but different masses. Swing speed (V) was found to decrease as swing-weight (I_o) increased. Assuming a power law relation of the form $V = C/I_o^n$, where C is a different constant for each participant, it was found that $n = 0.27 \pm 0.01$ when $I_o > 0.03\ \text{kg} \cdot \text{m}^2$ and when the participants swung each rod using only the upper arm, forarm and wrist. Remarkably similar results have been obtained previously with softball bats (where $n = 0.25$) and golf clubs (where $n = 0.26$). Swing speed remained approximately constant as swing mass increased (when keeping swing-weight fixed).

The results presented here should add substance to further work on the subject of racket or bat power. The elementary calculations presented above show that changes in racket mass or bat mass have only a small effect on the power of these implements. Nevertheless, small changes in power can make a significant difference to the result of a particular shot or in the manner in which an implement is used.

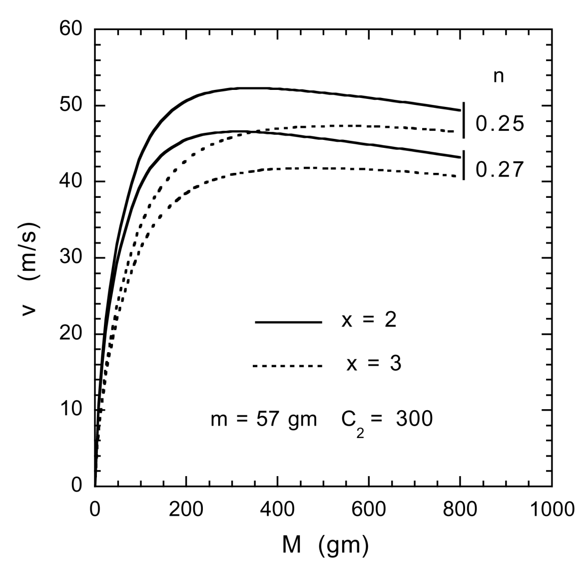

Figure 4. Ball speed v versus racket mass M from equation (3) for two values of n and two values of x. $x = 2$ corresponds to an impact near the middle of the strings and $x = 3$ corresponds to an impact closer to the tip. In practice, C_2 will vary slightly with impact point, since C_1 increases and e decreases towards the tip, but C_2 is assumed to remain constant here.

Detailed calculations and measurements are needed to determine the effects of even small changes in mass or moments of inertia. For example, an extra 0.001 kg added to a racket can change the frame stiffness, balance point, sweet spot location, centre of percussion, swing-weight, polar moment of inertia, vibration frequency and coefficient of restitution, all of which may affect racket power and swing technique in subtle ways but in ways that elite players may be able to detect (Cross, 2001).

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Appendix: Effect of balance point on swing speed

The dynamic behaviour of a rod that is swung about any given axis depends only on its mass and its moment of inertia (MOI) about that axis. Consider two rods with the same mass M and the same MOI, I_o, about an axis through the end of the handle. We now show that the moment of inertia of the two rods will be essentially the same about any other axis beyond the end of the handle even if the two rods have distinctly different balance points. Suppose one rod has a uniform mass distribution along its length and the other consists of a massless rod with a point mass M at the far end. $I_o = ML^2/3$ for the uniform rod, where L is the length of the implement. The MOI for the massless rod is $I_o = MR^2$, where R is the radius of the point mass. If the I_o values are the same, then $R = 0.577L$. Both rods have the same mass and MOI, but the overall lengths are quite different and the balance points are at distances $0.5L$ and $0.577L$ respectively from the end of the handle.

Now consider rotation of each implement about a different axis, say a distance $L/2$ beyond the handle end. The MOI of the uniform rod is then $ML^2/12 + ML^2 = 1.083ML^2$ and the MOI of the slender rod is $M(R + L/2)^2 = 1.160ML^2$. The two very different models of rackets differ by 7% in MOI, which corresponds to less than a 2% difference in swing speeds if one assumes a power law dependence with an exponent $n = 0.27$. If $V = C/I_o^n$, then the percentage change in V is n times the percentage change in I_o. Consequently, a shift in balance point should not affect the swing speed significantly, provided the relevant axis of rotation does not lie close to the middle of the implement. In practice, the axis of rotation of a swung implement is typically close to the handle end or a short distance beyond the end of the handle (Brody *et al.*, 2002; Mitchell *et al.*, 2000; Smith *et al.*, 2003). In that case, any two rods having the same M and I_o will have exactly the same M and essentially the same MOI, regardless of the location of the balance point or the length of the rods and regardless of the axis of rotation.

The change in MOI for the two racket models is even smaller if one includes the mass of the hand. Since the hand has a large effect on the total moment of inertia for axes remote from the hand, any effect due to a change in the mass distribution of the rod is even smaller than the above estimate.