

Lecture 11

Conservation of mechanical energy

Conservation of Mechanical Energy

Under the influence of **conservative** forces only
(i.e. no friction or drag etc.)

$$\text{M.E.} = K + U = \text{constant}$$

Note that U and K can include such things as elastic potential energy, rotational kinetic energy, etc.

Example: simple pendulum or slippery dip
(if friction & air resistance are negligible).

KJF §10.7

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Simple Pendulum

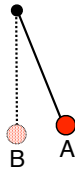
The system is (pendulum + earth).

F_T (tension in string) is always perpendicular to motion so does no work.

Weight (gravity) does all the work.

Because gravitational force is conservative, if drag & friction negligible then,

$$\text{M.E.} = K + U = \text{constant, i.e. } (K + U)_A = (K + U)_B$$



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Example: Tarzan



Tarzan who weighs 688N swings from a cliff at the end of a convenient vine that is 18m long. From the top of the cliff to the bottom of the swing he descends by 3.2m.

- What is his speed at the bottom of the swing? Neglect air resistance.
- The vine will break if the force on the vine exceeds 950N. Does it break at the bottom of the swing?

[7.9 m.s⁻¹, no]

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Non-Conservative Forces

These are **dissipative** forces such as drag and friction.

Mechanical energy is not conserved when non-conservative forces are acting because friction (and other dissipative forces) convert work or ME directly into thermal energy.

Thermal energy is just the sum of all the kinetic & potential energies of the molecules of a body.

Include this in our expression for conservation of energy:

$$K_i + U_i + W = K_f + U_f + \Delta E_{th}$$

KJF §10.3, eqn. 10.6

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Work and Friction (1)

Example 1:

Block on horizontal surface slides to rest due to kinetic friction. Work done **by** friction is

$$\Delta \text{ME} = \Delta K = -F_k d$$

Example 2:

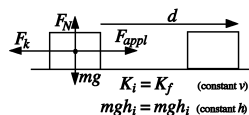
Block sliding along a horizontal surface at constant velocity. If work is done **AGAINST** friction by an applied force F_{app} and ΔK & $\Delta U = 0$ then;

The amount of thermal energy produced must be exactly equal to the amount of work done, in other words...

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Work and Friction (2)

$$W = F s \cos\theta$$



Force working AGAINST friction is $F_{\text{appl}} = -F_k$ (why?) but $F_k = \mu_k F_N$, $\cos\theta = 1$, and $s = d$, so the amount of thermal energy produced is

$$\Delta E_{\text{th}} = F_k d = \mu_k F_N d$$

Clearly, here work is **not** reversible. (Why not?)

Work done BY friction - same magnitude, opposite sign

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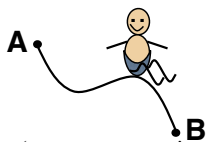
If friction is present, include the surface in the system. Then the frictional force is an internal force so

$$K_i + U_i = K_f + U_f + \Delta E_{\text{th}}$$

KJF Tactics Box 10.1

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Example: child on a slide with friction



Since friction (a non-conservative force) is acting,

- ME_A does not equal ME_B ,
- ME_B will be less, and
- $ME_A - ME_B = \text{thermal energy produced.}$

Also note:

- Work done by gravity (or weight) is always $= mgh$
- When sliding, work done by normal force $= 0$ because $\cos 90^\circ = 0$

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A 20 kg child, starting from rest, slides down a 3m high frictionless slide.

How fast is he going at the bottom?



[7.7 ms⁻¹]

Now he slides down a slide with friction, and his speed at the bottom is 6.0 ms⁻¹. How much thermal energy has been produced by friction?

[228 J]

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Problem

During a rockslide, a 520kg rock slides from rest down a hillside 500m long and 300m high. Coefficient of kinetic friction between the rock and the hill surface is 0.25.

- If the gravitational potential energy of the rock-Earth system is set to zero at the bottom of the hill, what is the value of U just before the slide?
- How much work is done by frictional forces during the slide?
- What is the kinetic energy of the rock as it reaches the bottom of the hill?
- What is its speed then?

[1.53MJ, -0.510MJ, 1.02MJ, 63ms⁻¹]

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Problem: Skier

A 60 kg skier leaves the end of a ski jump ramp with a velocity of 24 ms⁻¹ directed 25° above the horizontal. Suppose that as a result of air resistance the skier returns to the ground with a speed of 22 ms⁻¹ and lands at a point down the hill that is 14m below the ramp.

How much energy is dissipated by air resistance during the jump?



[11 kJ]

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