

Lecture 10

Work, Power and Potential energy

WORK

KJF §10.4

What is Work?

“Doing work” means using a **force** to

- *transfer energy* from one object to another, or
- *convert energy* from one form to another.

Work (W) is equal to the **amount of energy** transferred or converted by the force.

Work is a scalar. S.I. unit is also the joule (J).

If force is constant then

$$W = Fs \cos \theta$$

where F is applied force, s is object's displacement while the force is applied and θ is angle between applied force and displacement.

Work and Kinetic Energy (1)

Consider a single constant force F acting on a body causing a change in kinetic energy only. Suppose it moves in the x -direction and the force acts at angle θ to that direction.

If body moves displacement $d = x_f - x_i$. From the motion equations;

$$v_f^2 = v_i^2 + 2a_x d \quad \text{where } d = x_f - x_i$$

$$\text{i.e. } 2a_x d = (v_f^2 - v_i^2)$$

$$\begin{aligned} \text{But the K.E. is changing; } \Delta K &= K_f - K_i \\ &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}m \times 2a_x d \end{aligned}$$

Work and Kinetic Energy (2)

Therefore $\Delta K = \frac{1}{2} m \times 2a_x d = ma_x d = F_x d$
 $= F \cos\theta d$

But our definition of work was “change in energy” and in this case the only change is in kinetic energy.

Therefore the work done is

$$W = \Delta K = Fd \cos\theta$$

Sign Convention for Work

- **Work can be negative!** (i.e. Energy will be transferred *from* the body). ("*signed scalar*")
- If the applied force is causing the object to **increase** in energy, then the work is **positive**, e.g. work done by legs walking upstairs.
- If the applied force is causing the object to **decrease** in energy, then work is **negative**, e.g. work done by legs walking downstairs.

Work-Energy Theorem

Work is a scalar, i.e. the work done by the individual forces can be added together "arithmetically":

$$W = W_1 + W_2 + \dots$$

Work-Energy Theorem: (*a special case of the law of conservation of energy*) The change in kinetic energy of a system equals the sum of work done by all the individual forces on the system:

$$\Delta K = \sum W$$

Strictly speaking this theorem applies to rigid ("non-squishy") objects

What is Power?

Power means the **rate** at which a force does work on an object.

Instantaneous power: $P = \frac{dW}{dt}$

$$\text{Average } P = \frac{\text{work done}}{\text{time taken}} = \frac{\Delta W}{\Delta t}$$

Also remember that $W = Fs \cos\theta$, so if F and θ are constant

$$P = \frac{\Delta W}{\Delta t} = F \frac{\Delta s}{\Delta t} \cos\theta = Fv \cos\theta = \underline{\mathbf{F}} \cdot \underline{\mathbf{v}}$$

Units

Units: watt (W) = joule.second⁻¹ (J.s⁻¹)

[1 horsepower (hp) = 746 W]

Note! Do not confuse the unit watt (W) with the algebraic symbol for work (W).

The 1st is the unit for power P, the 2nd represents an energy and has joules (J) as its unit.

	Algebraic Symbol	Unit Symbol
Energy, Work	$K, U, W...$	J
Power	P	W

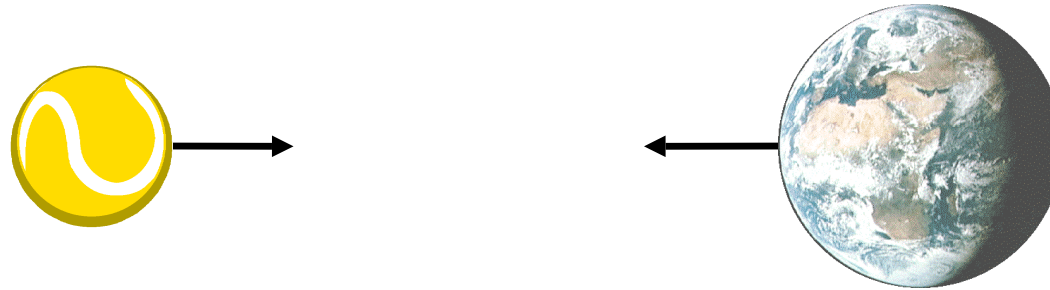
Problem

The loaded cab of an elevator has a mass of $3.0 \times 10^3 \text{ kg}$ and moves 210m up the shaft in 23s at constant speed.

At what average rate does the force from the cable do work on the cab?

$$[2.7 \times 10^5 \text{ W}]$$

Gravitational Potential Energy



Two bodies are attracted because of gravity caused by their mass, so we have to do work W to push them apart.

When we do this, energy is transferred to the system — called **gravitational potential energy**, U . Near earth's surface $U = mgh$

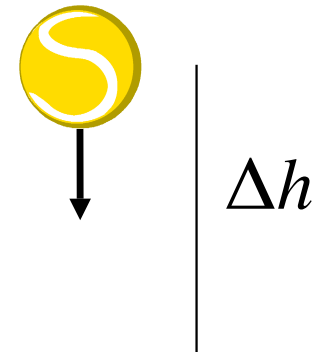
U is a property of the **whole system** *e.g.* system is earth + ball, but usually we simplify and say "the ball has potential energy"

Work done **by** weight (gravity) on the ball as it falls a distance h towards earth is

$$W = Fs \cos \theta = mg \times \Delta h \times \cos 0^\circ = mg\Delta h$$

i.e. work done by weight (gravity)
= minus the change in U

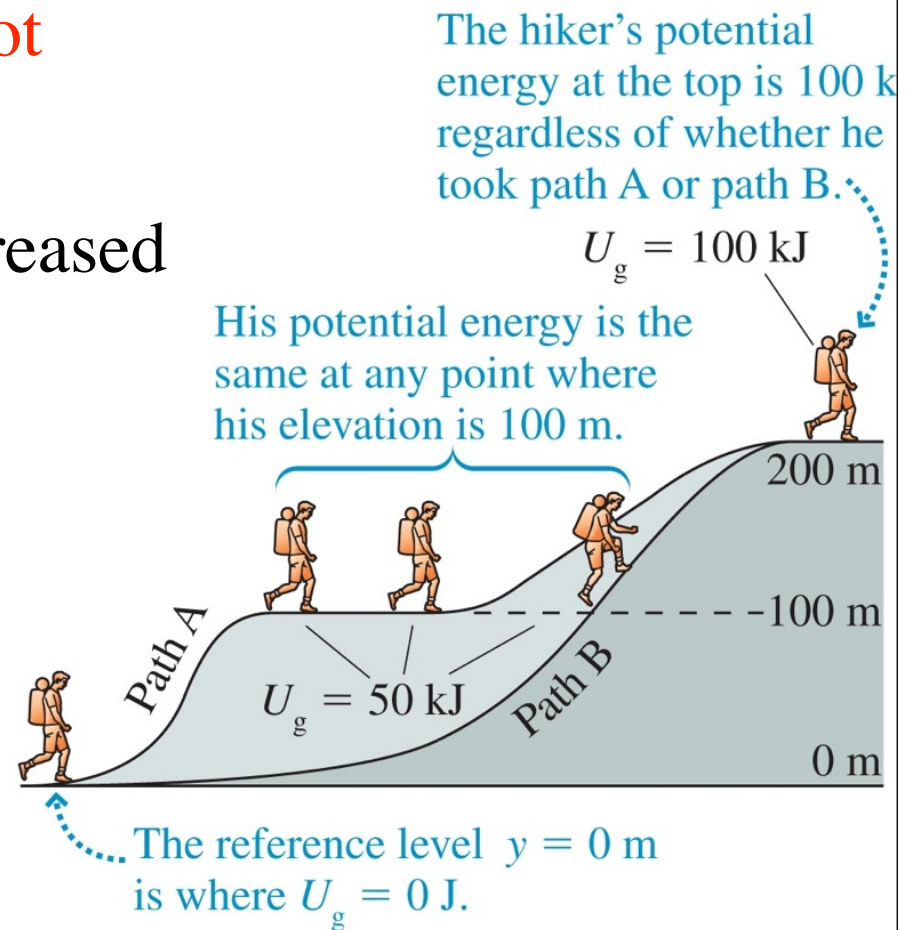
Only the **change** in height is significant
 \Rightarrow we are free to choose any reference
level we like where $U=0$



PE is independent of path

Because gravitational potential energy depends only on the height of the object above the reference level, the potential energy does **not** depend on the path taken.

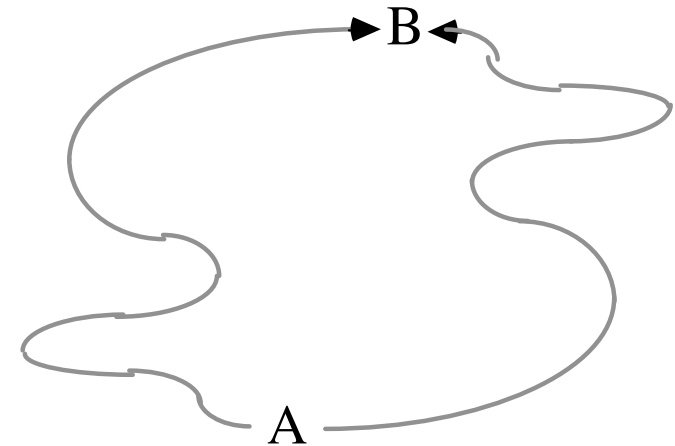
In both cases the G.P.E increased by the same amount: mgh





Conservative Forces

If work done by a force in moving an object from A to B does NOT depend on the path taken, we call it a **conservative force** (e.g. gravity, ideal springs).



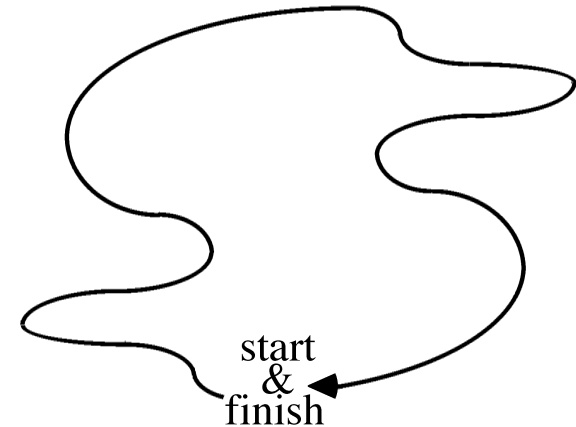
An object moving under the influence of a conservative force always conserves mechanical energy:

$$ME = K_i + U_i = K_f + U_f$$

EVERY conservative force has “potential energy” associated with it (e.g. gravity → gravitational P.E., spring force → elastic P.E.).

Forces like friction which dissipate energy instead of storing it are **non-conservative forces**.

If an object moves in a closed loop under the influence of a conservative force, the total work done is **ZERO**.



Work done in moving an object from $A \rightarrow B$ under the influence of a conservative force is the exact negative of work done in going from $B \rightarrow A$, i.e. **reversible**

$$W_{AB} = -W_{BA}$$

e.g. Work done by gravity going upstairs and back downstairs again.