PHYS1002 Fundamentals Mechanics
Lecture 13 : Solution to Balls on String Problem

Problem

A ball of mass 700g is fastened to a cord 800mm long and fixed at the far end of a support, and is released when the cord is horizontal. At the bottom of its path, the ball strikes a stationary 350g ball suspended from the same support with a cord 800mm long. The two balls stick together after the collision.

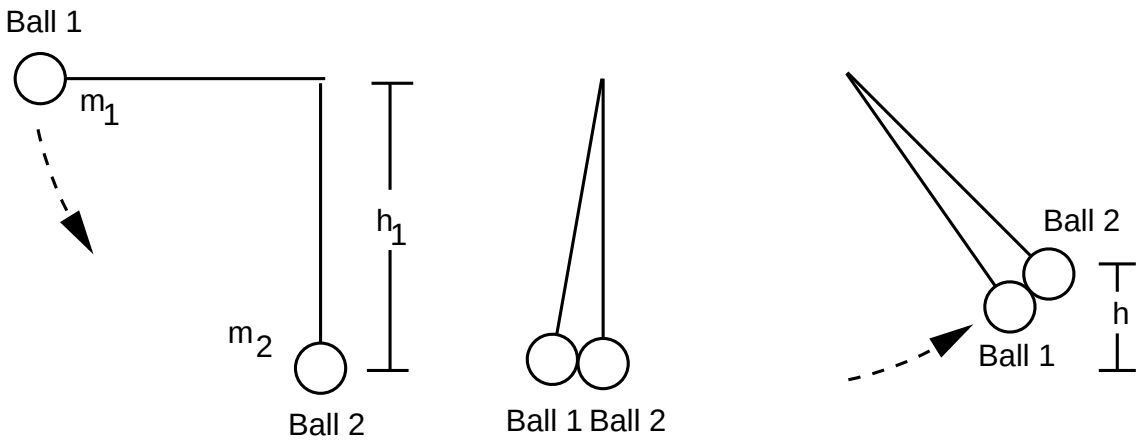
(a) Calculate the speed of the falling ball just before it hits the stationary ball.

(b) Calculate the speed of the two balls immediately after the collsion.

(c) Calculate the maximum vertical height the balls will reach after the collision.

Your answers must clearly show the physical principles you used in obtaining your numerical answers.

Solution



(a) The mechanical energy of ball 1 must be the same at the top of its swing and at the bottom where it is about to strike ball 2. We choose to set the potential energy of ball 1 to be zero when it is at the position of ball 2. Ball 1 has zero kinetic energy at the top because it is at rest initially.

$$ME_1(top) = KE_1(top) + PE_1(top) = 0 + m_1 g h_1 = m_1 g h_1$$

$$ME_1(bot) = KE_1(bot) + PE_1(bot) = \frac{1}{2} m_1 v_1^2 + 0 = \frac{1}{2} m_1 v_1^2$$

Hence

$$\begin{aligned}\frac{1}{2}m_1v_1^2 &= m_1gh_1\\ \frac{1}{2}v_1^2 &= gh_1\\ v_1^2 &= 2gh_1\\ v_1 &= \sqrt{2gh_1} = \sqrt{2\times 9.81\text{m s}^{-2}\ \times 0.800\text{m}\ } = 3.96\text{m s}^{-1}\end{aligned}$$

Note that the result does not depend on the mass of the ball.

(b) We assume that ball 1 is travelling horizontally to the right when it strikes ball 2 and define this as the positive direction. Ball 1 will effectively be travelling horizontally if the balls are not too large.

The pair of balls will also be travelling horizontally to the right immediately after they collide and stick together. If v is their common final speed at that point, conservation of linear momentum gives

$$p_{tot(before)} = p_{tot(after)}$$

i.e.

$$m_1v_1 + m_2v_2 = (m_1 + m_2)\, v$$

Since $v_2 = 0$ we get

$$m_1v_1 = (m_1 + m_2)\, v$$

Solve this equation for v to get

$$v = \frac{m_1}{(m_1+m_2)}v_1 = \frac{0.700\text{ kg}}{(0.700\text{ kg}\ + 0.350\text{ kg}\ )} \times 3.96\text{ m s}^{-1}\ = 2.64\text{ m s}^{-1}$$

(c) Note that we can use the same arguments as used in part(a) to solve this. By conservation of mechanical energy, and noting that the balls come to rest at height h (no kinetic energy) and start at the bottom with zero potential energy, we must have

$$\frac{1}{2}(m_1 + m_2)\, v^2 = (m_1 + m_2)\, gh$$

We solve this for h

$$\begin{aligned}h &= \frac{1}{2}\frac{(m_1+m_2)}{(m_1+m_2)}\frac{v^2}{g}\\ &= \frac{v^2}{2g} = \frac{(2.64\text{ m s}^{-1}\ )^2}{2\times 9.81\text{ m s}^{-2}}\\ &= 0.355\text{ m}\end{aligned}$$