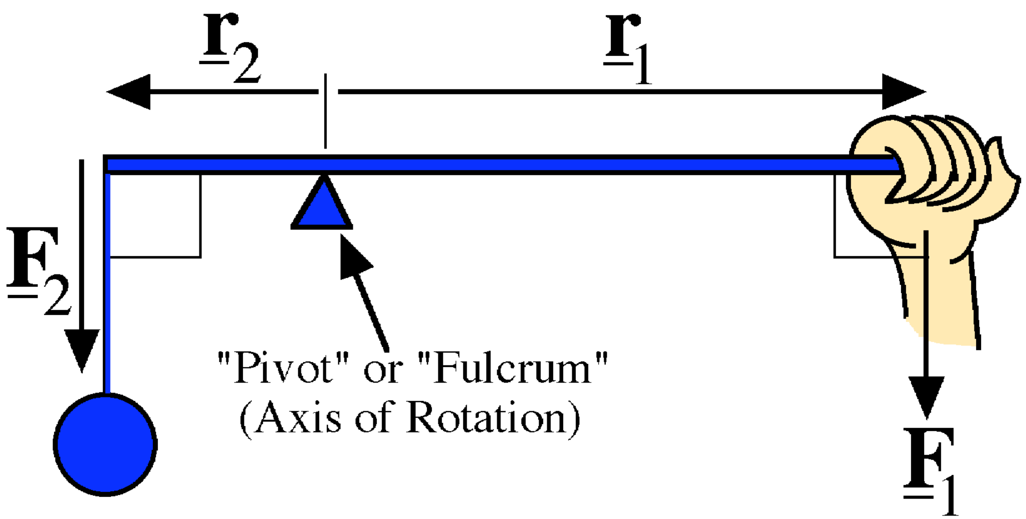
Lecture 8

Torque
and
Equilibrium

Pre-reading: KJF §8.1 and 8.2

Archimedes' Lever Rule



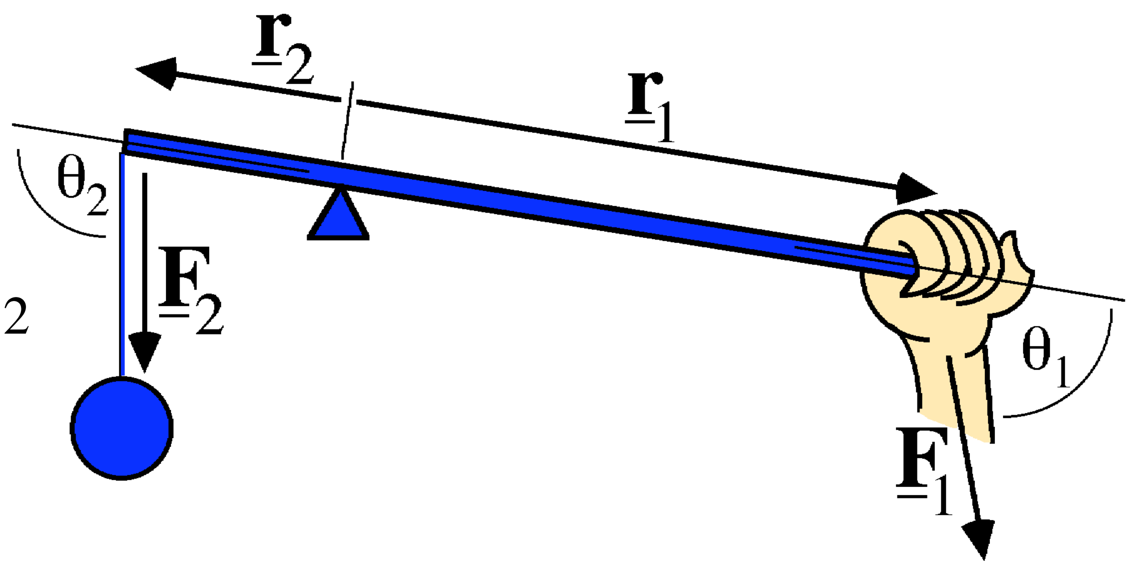
At equilibrium (and with forces 90° to lever):

$$r_1 F_1 = r_2 F_2$$

General Lever Rule

For general angles

$r_1 F_1 \sin\theta_1 = r_2 F_2 \sin\theta_2$



We call $rF \sin\theta = \tau$ torque

S.I. unit of torque: newton metre (Nm)

At equilibrium, the magnitude of torques exerted at each end of lever are equal

KJF §7.2

What is torque?

Crudely speaking, torque is "twisting or turning ability" of a force that can:

- change the angular velocity of an object (i.e. speed up or slow down rotation)
- cause a twisting or bending distortion of an object

A force with a "line of action" that does not cross the axis of rotation results in torque.

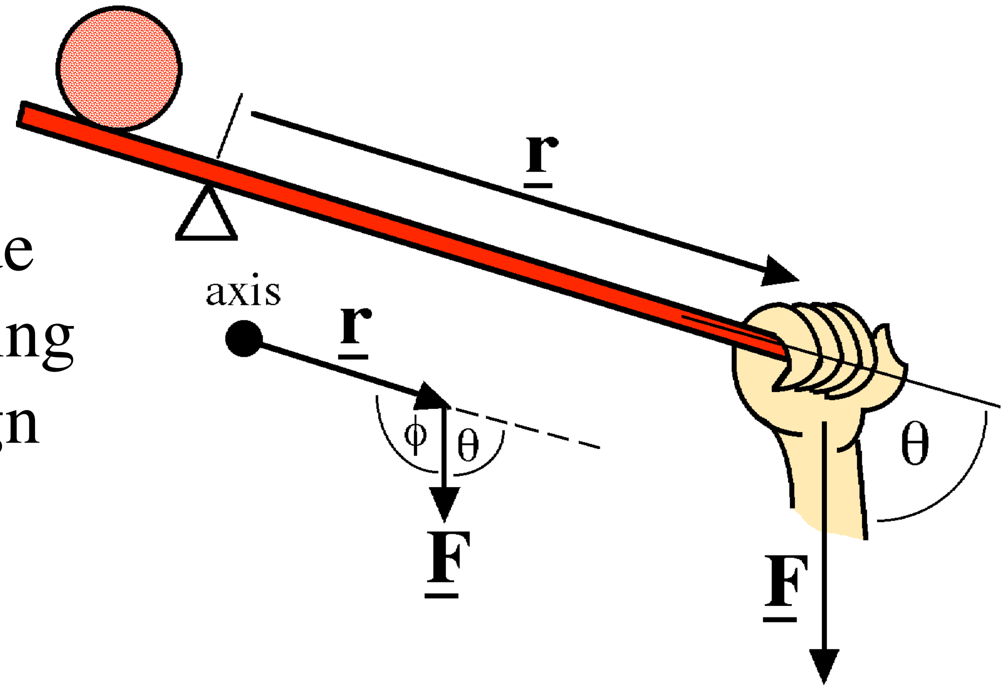
Note:

- torque is measured about a particular point.
  Usually this will be a hinge, pivot or axis

- torque has a sign.
  All forces that tend to rotate the object in the same direction produce torque with the same sign

Calculating torque (1)

Example: Calculate torque on lever exerted by hand:



Choose a sign convention (e.g. anti-clockwise +ve), then decide in which direction force is pulling or pushing lever. Write that sign in front of your answer.

Method 1:

If you're given r and θ, use formula for torque (magnitude)

$$\tau = r\,F\sin\theta$$

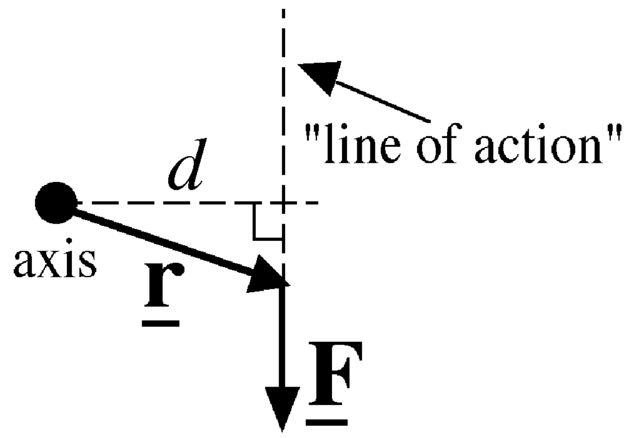
(Note: $\sin\theta = \sin\phi$, ∴ it doesn't matter which angle you use)

Calculating torque (2)

Method 2:

If you're given d the "perpendicular distance" from axis to the "line of action", then use formula

$$\tau = d\,F$$

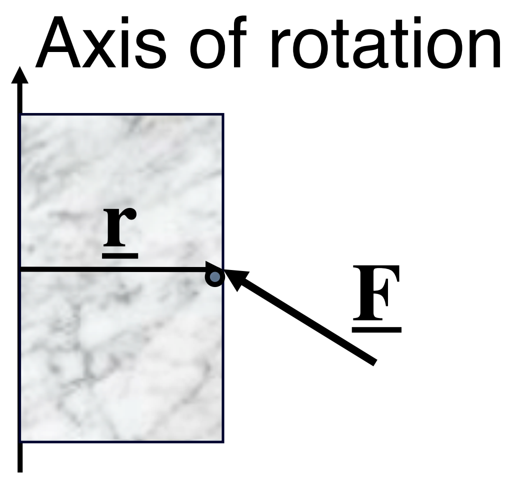


If the "line of action" crosses the axis (i.e. $d = 0$) then $\tau = 0$

Opening a door

- If r is perpendicular to F, then
  torque $\tau = r F$

- If r is not perpendicular to F, then
  torque $\tau = r F \sin\theta$
  where θ is the angle between r and F



- What happens if you push in the middle of the door; do you need more or less force? Why?

- What happens if you push along a line passing through axis of rotation? Explain.

Problem

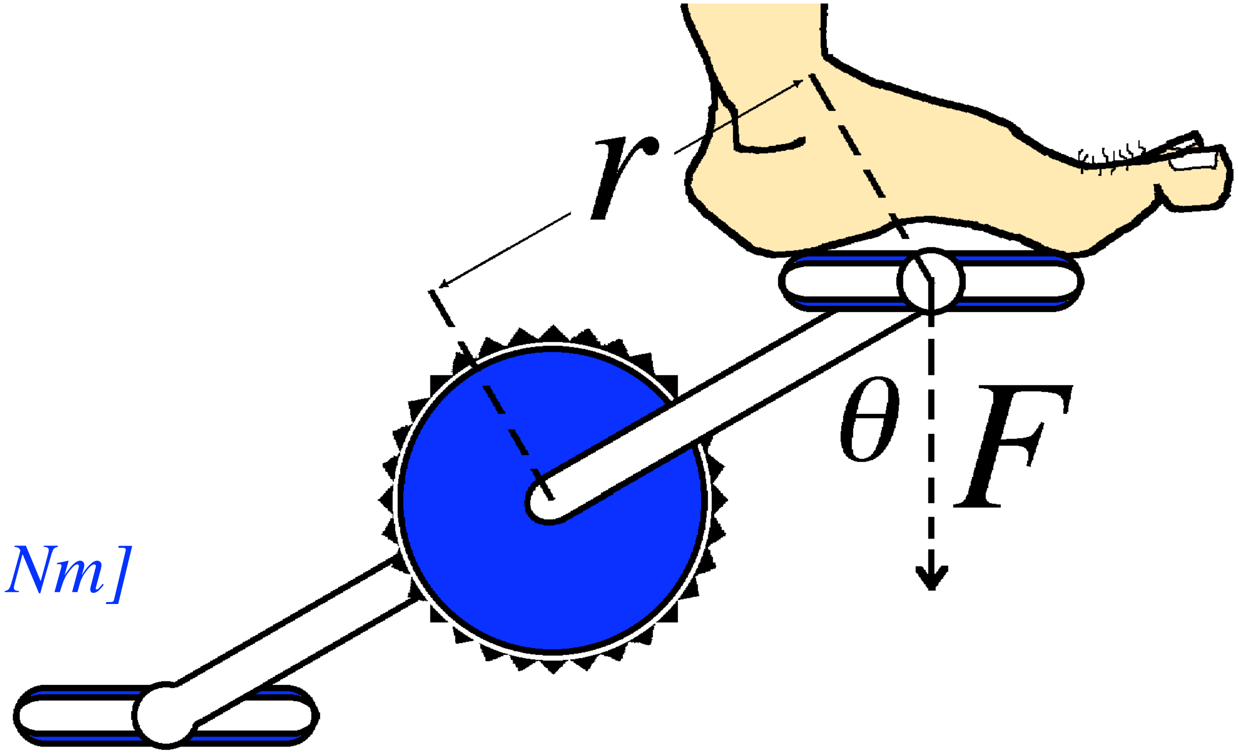
The length of a bicycle pedal arm is $r = 0.152$ m, and a downward force of $F = 111$ N is applied by the foot.

What is the magnitude of torque about the pivot point when the angle θ between the arm & vertical is;

(a) 30.0°?

(b) 90.0°?

(c) 180.0°?



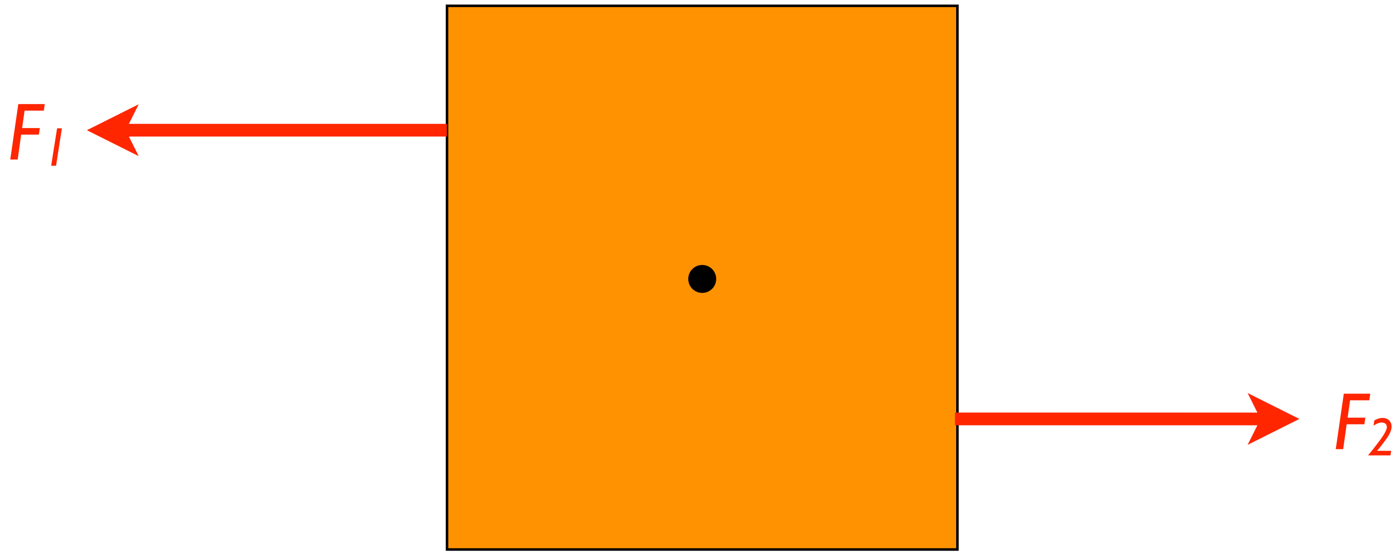
[8.44 Nm, 16.9 Nm, 0.00 Nm]

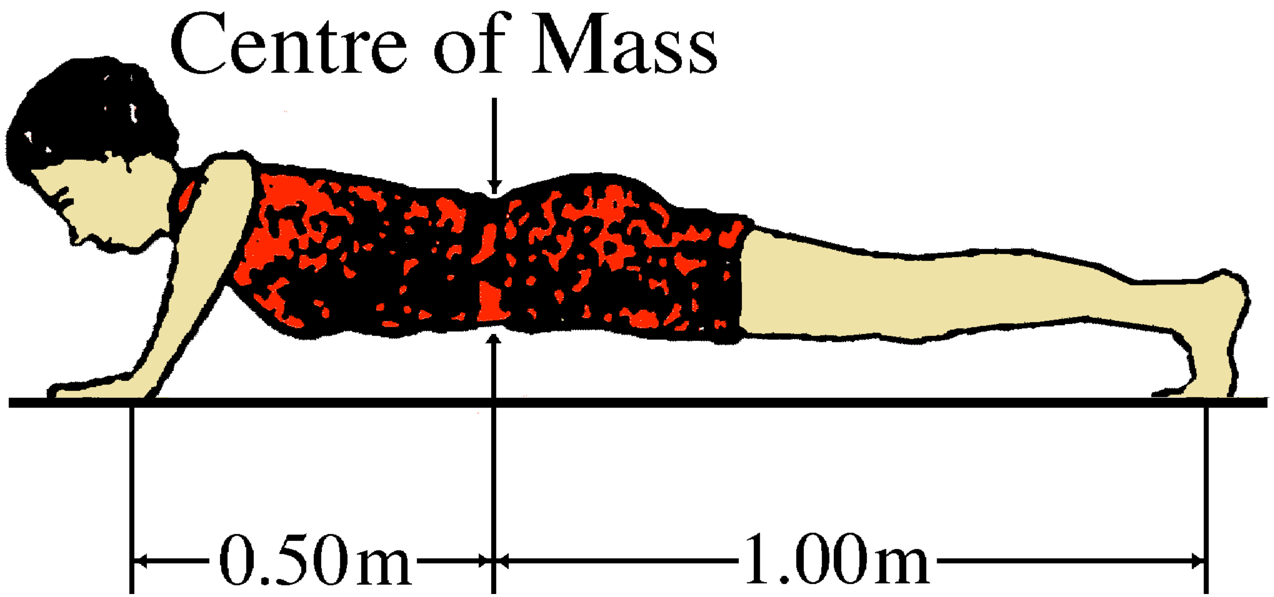
Adding up Torques

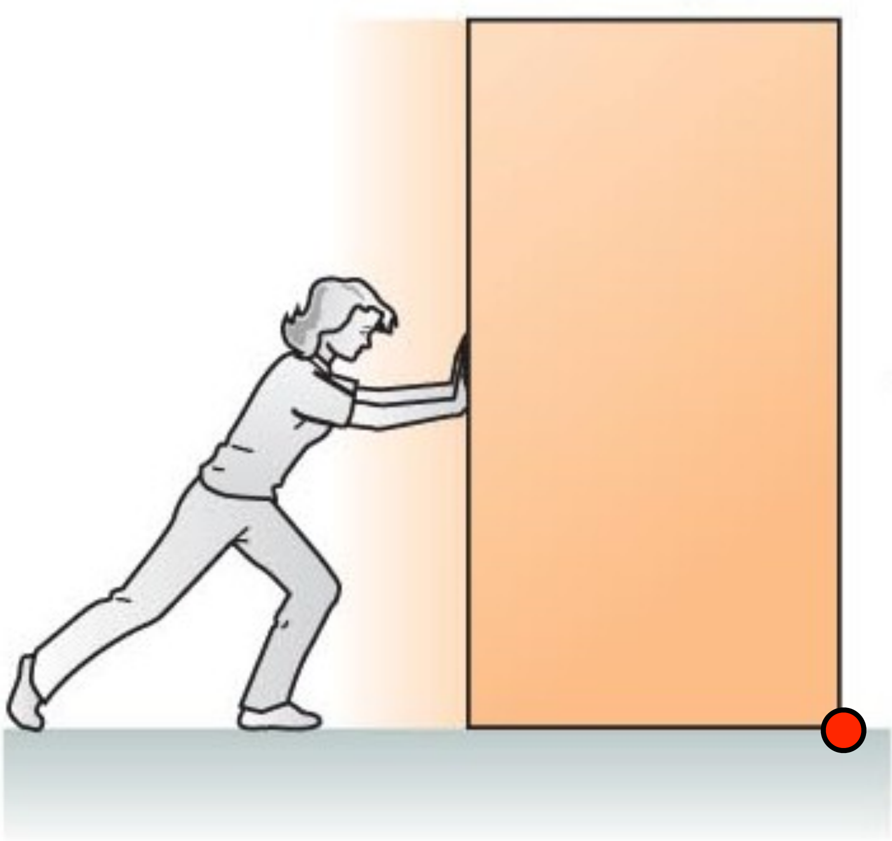
We will only consider torques acting in 2D (flat on page)

- Choose a sign convention (e.g. anti-clockwise is positive).
- Choose the rotation axis around which to calculate torque (unless it's already given).
- Draw the line of action for each force
- For each force, calculate the resulting torque (including sign).
- Add up all the torques.

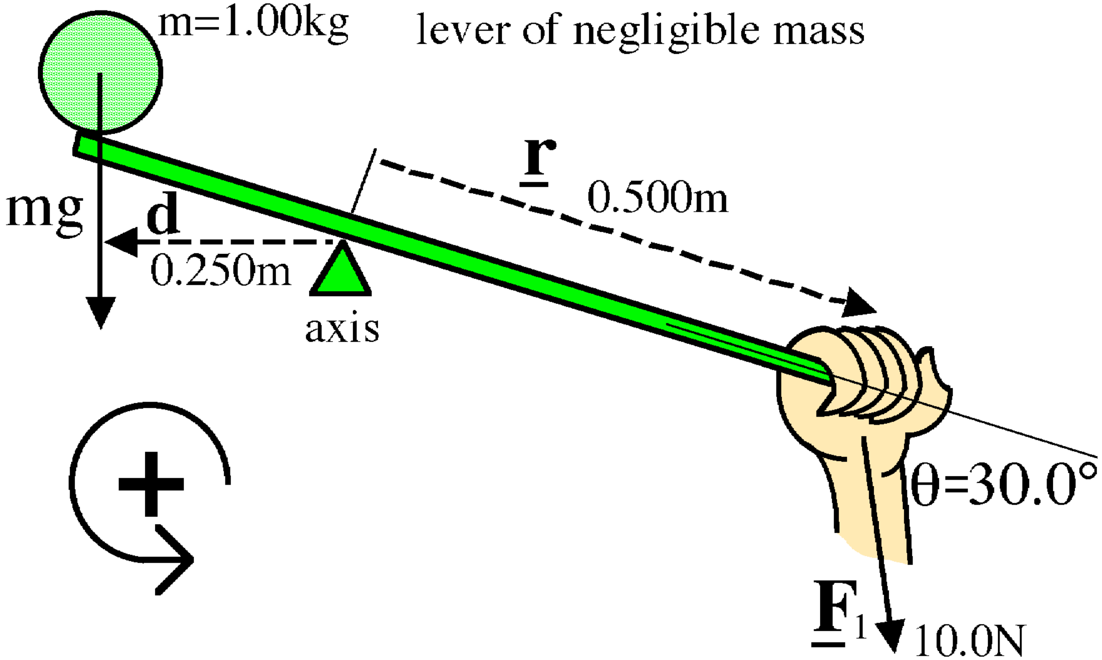
KJF §7.2, see p. 214


F_1
F_2


Centre of Mass
0.50m
1.00m



Adding up Torques: Example



torque 1; $\underline{\tau}_1 = -rF_1 \sin\theta = -0.5 \times 10 \times \sin 30 = -2.50$ Nm

torque 2; $\underline{\tau}_2 = +mgd = 1 \times 9.8 \times 0.25 = +2.45$ Nm

$\therefore$ net torque $= \sum\underline{\tau} = \underline{\tau}_1 + \underline{\tau}_2 = 2.45 + (-2.50) = -0.05$ Nm

(i.e. clockwise)

EQUILIBRIUM

KJF §8.1, 8.2

Conditions for Equilibrium

For an object to be in static equilibrium

- $\Sigma \underline{F} = 0$ no net force

  $\Rightarrow \Sigma F_x = 0, \Sigma F_y = 0$

- $\Sigma \underline{\tau} = 0$ no net torque

Because this is true for *all* pivot points, we are free to choose any point we like for calculating the torque

$\Rightarrow$ choose point where some torques disappear

KJF §8.1

Hints for Statics Problems

Usually you're given some forces on a static body & need to find unknown forces or torques.

- Draw a diagram!
- Decide on system
- Put in forces ON system only
- All forces in mechanics are either contact or gravity
- Define sign conventions

Solving Static Equilibrium Problems

- Decide on the "system"
- Choose a rotational axis and sign convention
  Best to choose one that causes some torques to disappear
  Remember nothing is rotating anyway so you're free to choose the axis.
- Calculate all horizontal components of forces acting on the system and write equation $\sum \underline{F}_h = 0$.
- Calculate all vertical components of forces acting on the system and write equation $\sum \underline{F}_v = 0$.
  Assume each object's weight force is acting at its centre of mass.
- Calculate all torques and write equation $\sum \underline{\tau} = 0$.
  Remember that all external forces are possible sources of torque.
- Solve the equations simultaneously.

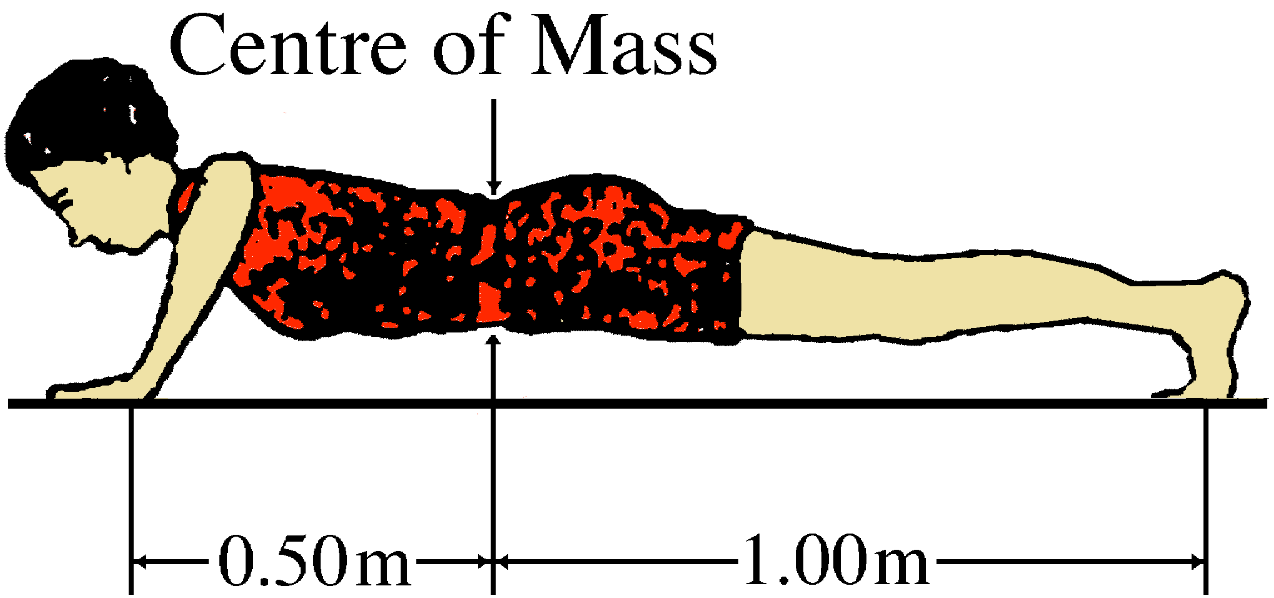
KJF §8.1

Example

A 65 kg woman is horizontal in a push-up position.

What are the vertical forces acting on her hands and her feet?

[hands 420 N, feet 210 N]



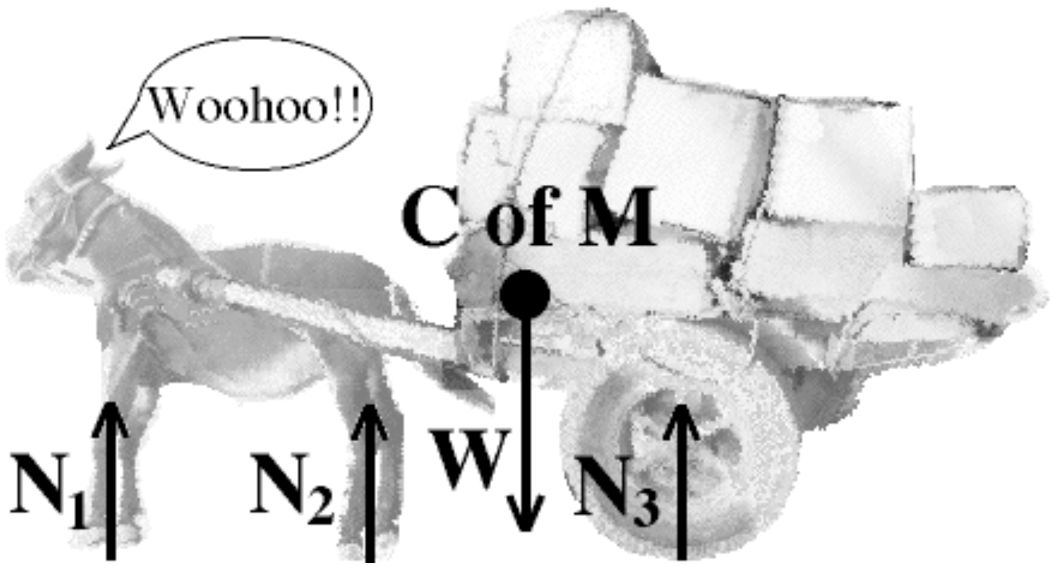
Example 2

The "system" is the ass, the cart and the cargo.

Here the cargo is loaded correctly.

Whatever rotation axis is chosen, there's always some normal forces opposing the torque due to the total system weight (treated as though it lies at the centre of mass)

No net torque
∴ equilibrium.



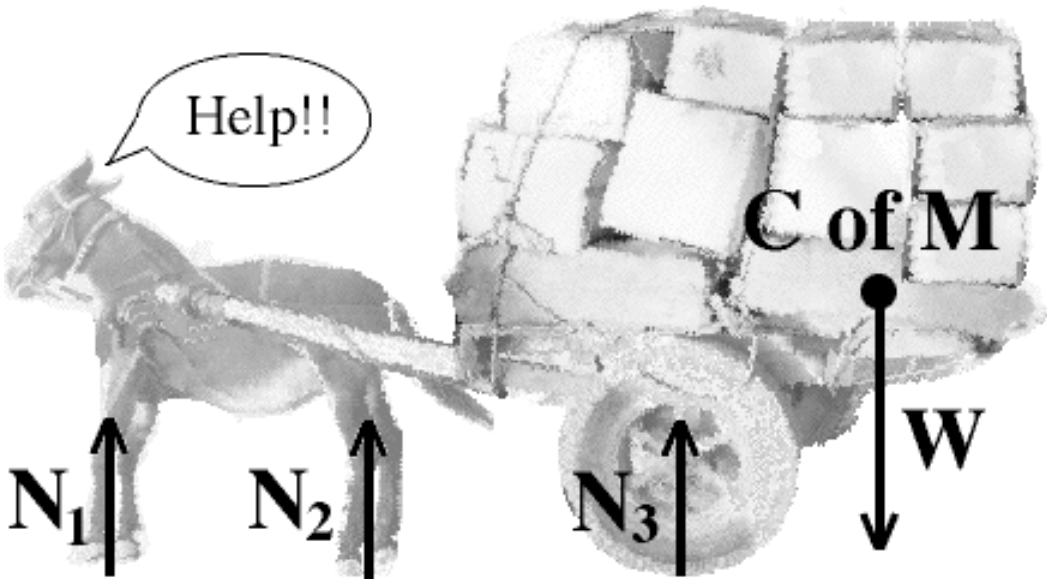
But...

Too much cargo is loaded at the back.

If the wheel is chosen as the rotation axis, all resulting torques are acting in the clockwise direction.

There is no torque opposing the torque due to the weight of the system, hence there is a net clockwise torque.

The system will rotate until the cart hits the ground.
The donkey will be lifted off the ground.



Sydney Morning Herald Jan '02

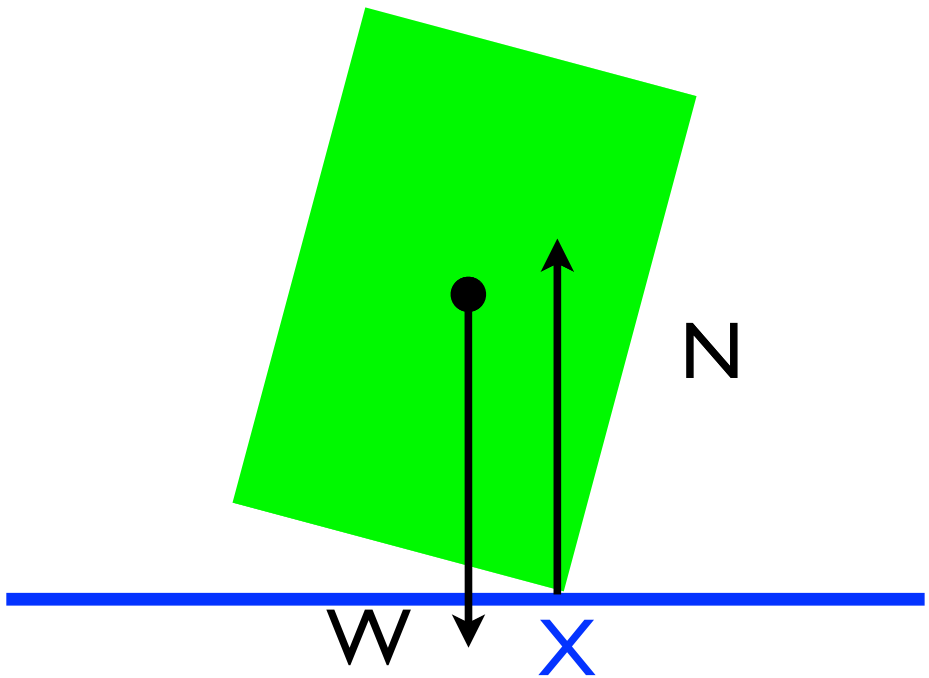
Types of Equilibrium

Neutral: with a small displacement, remains at new position.

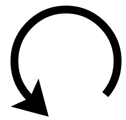
Stable: with a small displacement, returns to original position.

Unstable: with a small displacement, continues to move away from equilibrium position.

Stable

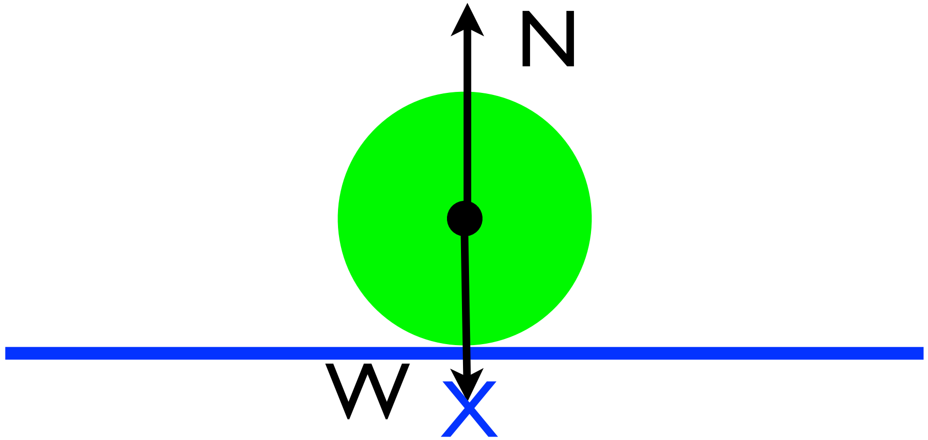


net torque
around X

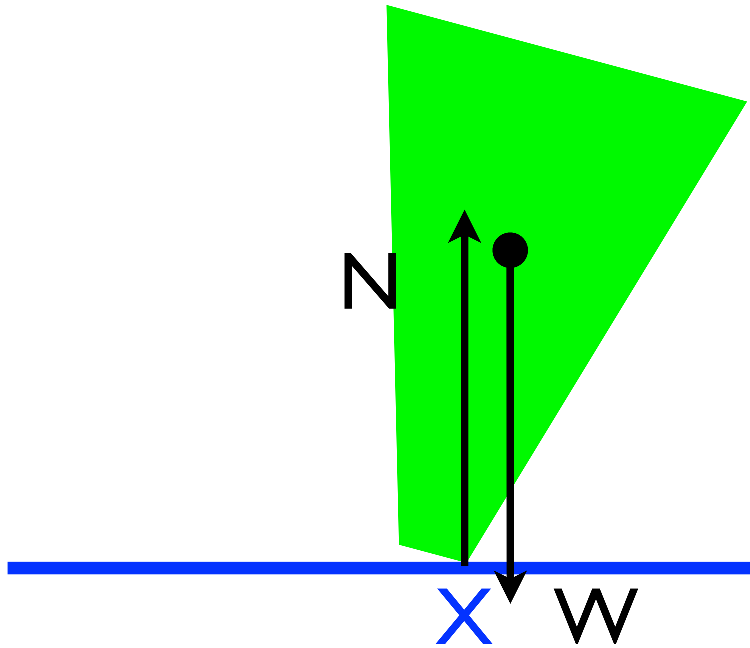


Neutral

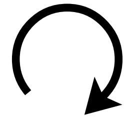
NO net torque
around X



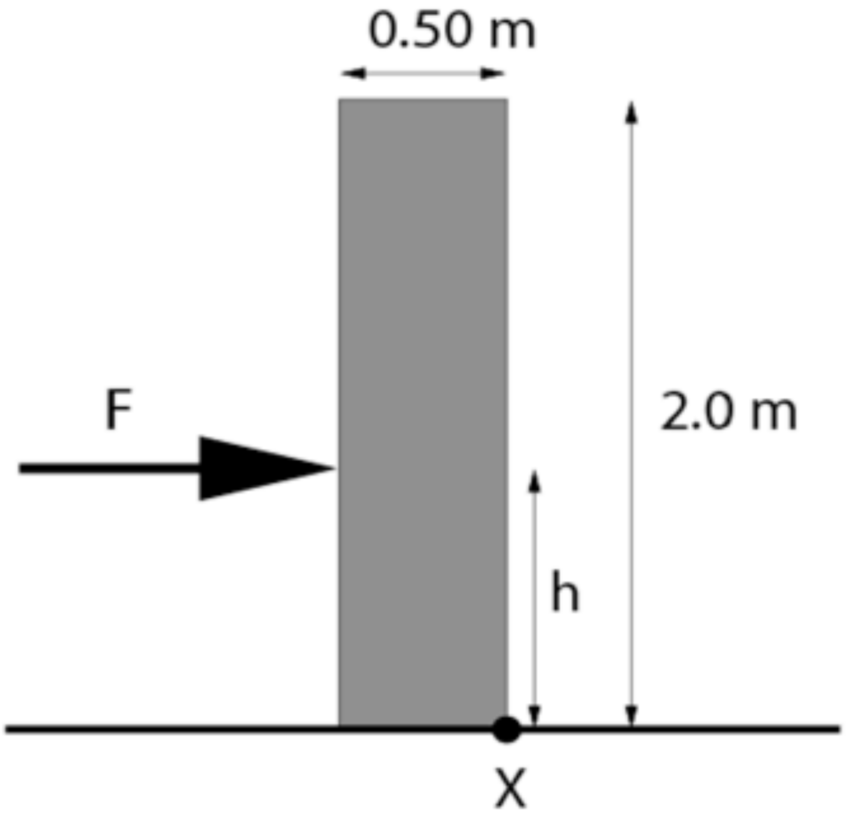
Unstable



net torque around X



Question 10



If the block is pushed too near to the top, it will tip over instead of sliding. It will try to rotate about the front corner of the block (labelled X in the diagram).

(d) What are the torques acting on the block around point X due to the force of magnitude F and the weight?

NEXT LECTURE

Momentum, impulse and energy

Read: KJF §9.1, 9.2