Lecture 6

Circular Motion

Pre-reading: KJF §6.1 and 6.2

Please take a clicker

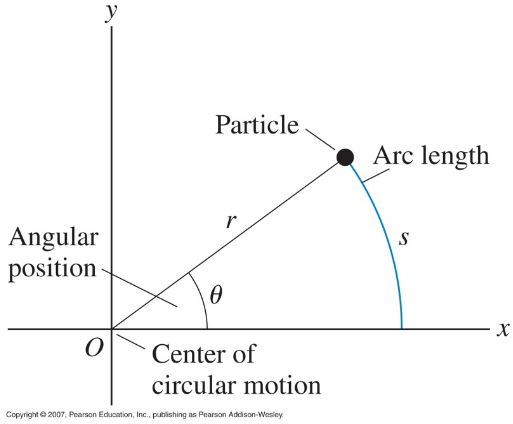
CIRCULAR MOTION

KJF §6.1–6.4

Angular position

If an object moves in a circle of radius r, then after travelling a distance s it has moved an angular displacement θ:

$$\theta = \frac{s}{r}$$



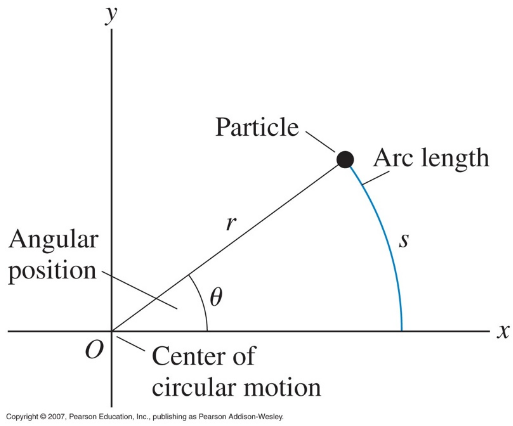
θ is measured in radians (2π radians = 360°)

KJF §3.8

Tangential velocity

If motion is *uniform* and object takes time t to execute motion, then it has tangential velocity of magnitude v given by

$$v = \frac{s}{t}$$



Period of motion T = time to complete one revolution (units: s)

Frequency f = number of revolutions per second (units: s^{-1} or Hz)

$$f = \frac{1}{T}$$

Angular velocity

Define an angular velocity ω

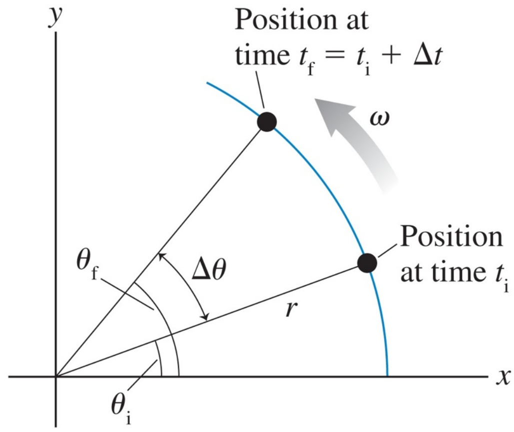
$$\omega = \frac{\text{angular displacement}}{\text{time interval}} = \frac{\theta}{t}$$

Uniform circular motion is when ω is constant.

Combining last 3 equations:

$$v = r\omega$$

period $T = \frac{2\pi}{\omega}$



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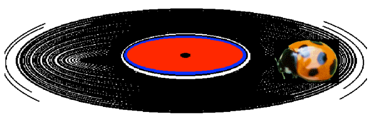
KJF §6.1

Question

You place a beetle on a uniformly rotating record



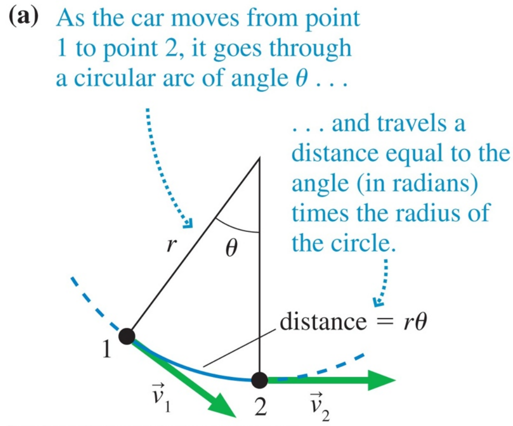
(a) Is the beetle's *tangential* velocity different or the same at different radial positions?

(b) Is the beetle's *angular* velocity different or the same at the different radial positions?

Remember; all points on a rigid rotating object will experience the ***same*** *angular velocity*

Consider an object is moving in uniform circular motion – tangential speed is constant.

Is the object accelerating?



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Velocity is a *vector*

$\therefore$ changing direction

$\Rightarrow$ acceleration

$\Rightarrow$ net force

The change in velocity

$\Delta \underline{v} = \underline{v}_2 - \underline{v}_1$

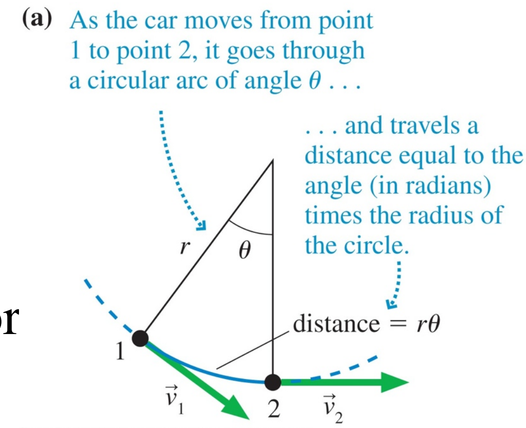
and $\Delta \underline{v}$ points towards the centre of the circle

Angle between velocity vector is θ so

$\Delta v = v\theta$

and so

$$a = \frac{\Delta v}{\Delta t} = \frac{v\theta}{r\theta / v} = \frac{v^2}{r}$$



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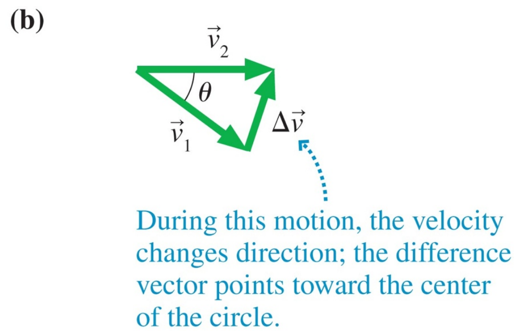

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KJF §3.8

Centripetal acceleration

Acceleration points towards centre
– centripetal acceleration a_c

$$a_c = \frac{v^2}{r} = \omega^2 r$$

Since the object is accelerating, there must be a force to keep it moving in a circle

$$F_c = \frac{mv^2}{r} = m\omega^2 r$$

This centripetal force may be provided by friction, tension in a string, gravity etc. or combinations. *Examples*?

KJF §6.2

Note that centripetal force is the *name* given to the resultant force: it is **not** a separate force in the free-body diagram.

The centripetal acceleration has to be provided by some other force (tension, friction, normal force) in order for circular motion to occur.

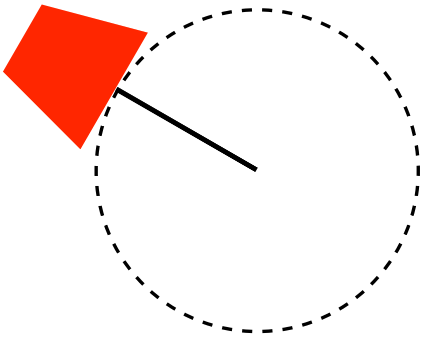
Solving CM problems

- Draw a free-body diagram
- If the object is moving in a circle, there must be a **net force** pointing towards the centre of the circle.
- The magnitude of this net force is given by

$$F_c = \frac{mv^2}{r} = m\omega^2 r$$

Problem 1

Whirling bucket



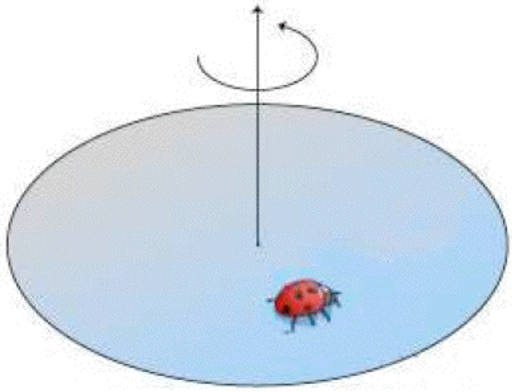
A bucket of water is whirled around in a vertical circle with radius 1m.

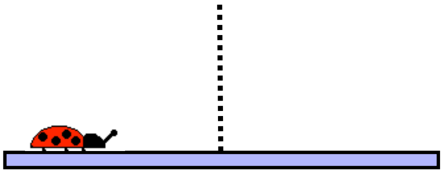
What is the minimum speed that it can be whirled so the water remains in the bucket?

[3 ms^{-1}, or rotation period 2s]

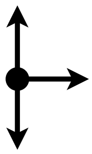
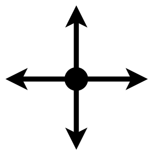
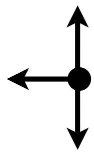
Clicker questions

A beetle is sitting on a rotating turntable. Looking at the turntable side on, so the centre is towards the right:





Which diagram correctly shows the forces acting on the beetle?

(a)  (b)  (c)  (d)  (e) other

There is a centripetal force acting on the beetle. What provides this force?

(a) the angular velocity of the turntable

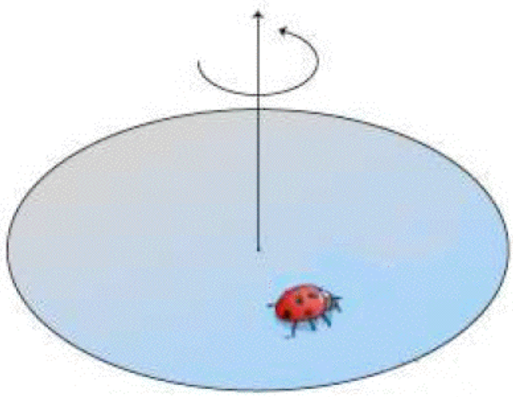
(b) gravity

(c) tangential velocity

(d) friction

(e) centripetal acceleration

(f) normal force



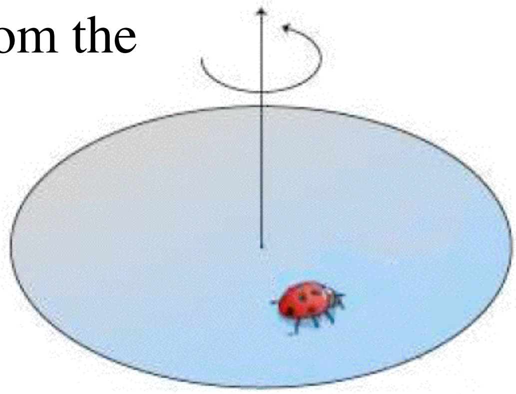
The turntable starts to spin faster. Which direction should beetle move so as not to slip?

(a) inwards

(b) outwards

(c) forward in the direction of motion

(d) backwards away from the direction of motion

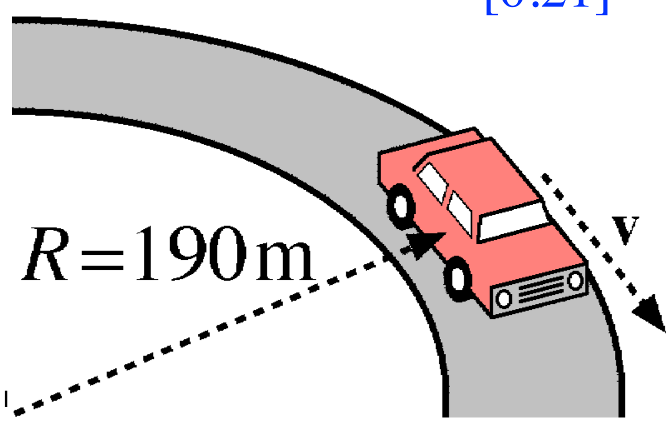


Car around a corner

A car of mass 1.6 t travels at a constant speed of 72 km/h around a horizontal curved road with radius of curvature 190 m. (Draw a free-body diagram)

What is the minimum value of μ_s between the road and the tyres that will prevent slippage?

[0.21]

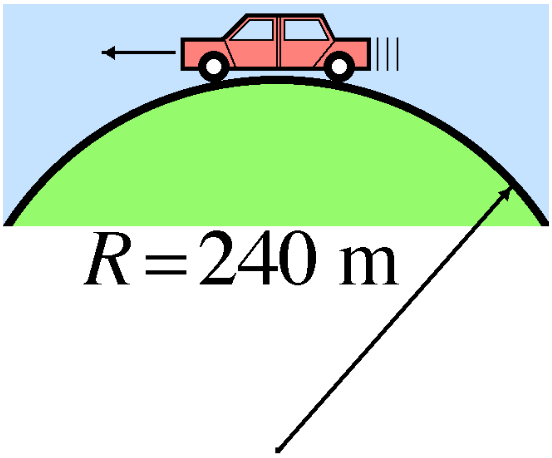


Car over a hill

A car is driving at constant speed over a hill, which is a circular dome of radius 240 m.

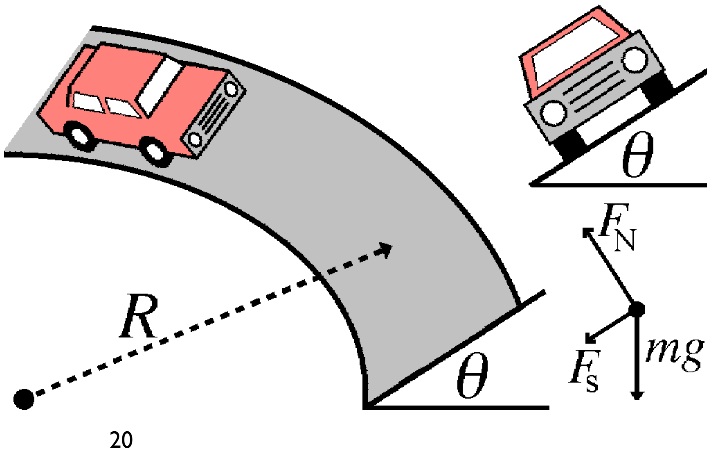
Above what speed will the car leave the road at the top of the hill?

[175 km/h]



Banked road

On a curve, if the road surface is "banked" (tilted towards the curve centre) then the horizontal component of the normal force can provide some (or all) of the required centripetal force. Choose v & θ so that less or no static friction is required.

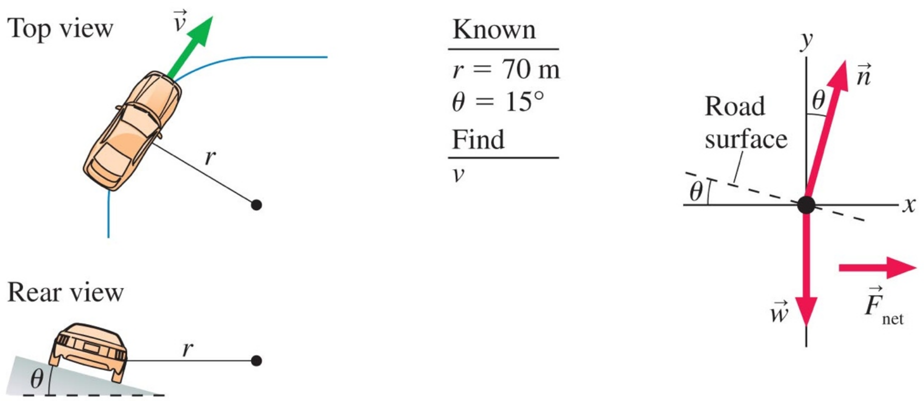


KJF example 6.6

KJF example 6.6

A curve of radius 70m is banked at a 15° angle. At what speed can a car take this curve without assistance from friction?

[14 ms^{-1} = 50 km h^{-1}]



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NEXT LECTURE

Centre of mass and Torque

Read: KJF §7.2, 7.3