Lecture 6

Circular Motion

Pre-reading: KJF §6.1 and 6.2

Circular motion

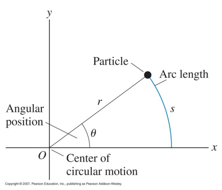
KJF §6.1–6.4

Angular position

If an object moves in a circle of radius r, then after travelling a distance s it has moved an angular displacement θ:

$$\theta = \frac{s}{r}$$

θ is measured in radians
(2π radians = 360°)

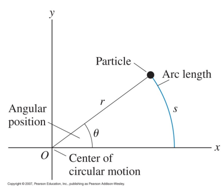


KJF §3.8

Tangential velocity

If motion is *uniform* and object takes time t to execute motion, then it has tangential velocity of magnitude v given by

$$v = \frac{s}{t}$$



Period of motion T = time to complete one revolution (units: s)

Frequency f = number of revolutions per second (units: s^{-1} or Hz)

$$f = \frac{1}{T}$$

Angular velocity

Define an angular velocity ω

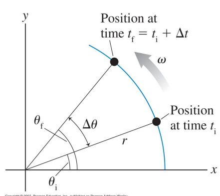
$$\omega = \frac{\text{angular displacement}}{\text{time interval}} = \frac{\theta}{t}$$

Uniform circular motion is when ω is constant.

Combining last 3 equations:

$$v = r\omega$$

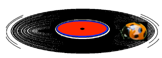
period $$T = \frac{2\pi}{\omega}$$



KJF §6.1

Question

You place a beetle on a uniformly rotating record

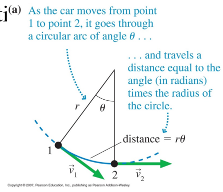


Is the beetle's *tangential* velocity different or the same at different radial positions?

Is the beetle's *angular* velocity different or the same at the different radial positions?

Remember; all points on a rigid rotating object will experience the ***same*** *angular velocity*

Consider an object is moving in uniform circular motion – tangenti speed is constant.



Is the object accelerating?

Velocity is a *vector*
∴ changing direction

⇒ acceleration

⇒ net force

The change in velocity

$\Delta \underline{v} = \underline{v}_2 - \underline{v}_1$

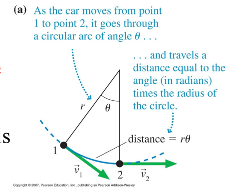
and $\Delta \underline{v}$ points towards the centre of the circle

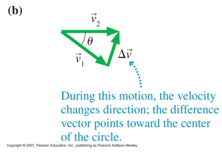
Angle between velocity vectors is θ so

$\Delta v = v\theta$

and so

$$a = \frac{\Delta v}{\Delta t} = \frac{v\theta}{r\theta/v} = \frac{v^2}{r}$$





KJF §3.8

Centripetal acceleration

Acceleration points towards centre
– centripetal acceleration a_c

$$a_c = \frac{v^2}{r} = \omega^2 r$$

Since the object is accelerating, there must be a force to keep it moving in a circle

$$F_c = \frac{mv^2}{r} = m\omega^2 r$$

This centripetal force may be provided by friction, tension in a string, gravity etc. or combinations.
Examples?

KJF §6.2

Note that centripetal force is the *name* given to the resultant force: it is **not** a separate force in the free-body diagram.

The centripetal acceleration has to be provided by some other force (tension, friction, normal force) in order for circular motion to occur.

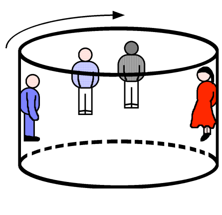
Solving CM problems

- Draw a free-body diagram
- If the object is moving in a circle, there must be a **net force** pointing towards the centre of the circle.
- The magnitude of this net force is given by

$$F_c = \frac{mv^2}{r} = m\omega^2 r$$

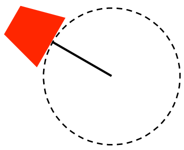
Problem 1

You enter the carnival ride called "The Rotor". The circular room is spinning and you and other riders are stuck to the circular wall.



- Draw a free-body diagram of the woman in red
- Is she in equilibrium? Explain
- What force is providing the centripetal force?

Whirling bucket



A bucket of water is whirled around in a vertical circle with radius 1m.

What is the minimum speed that it can be whirled so the water remains in the bucket?

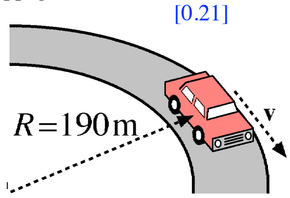
[3 ms^{-1}, or rotation period 2s]

Car around a corner

A car of mass 1.6 t travels at a constant speed of 72 km/h around a horizontal curved road with radius of curvature 190 m. (Draw a free-body diagram)

What is the minimum value of μ_s between the road and the tyres that will prevent slippage?

[0.21]

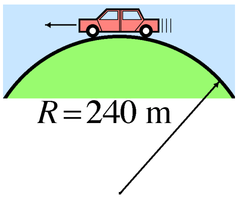


Car over a hill

A car is driving at constant speed over a hill, which is a circular dome of radius 240 m.

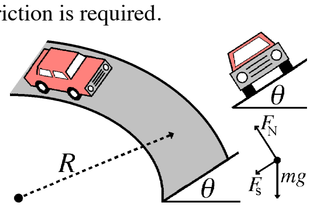
Above what speed will the car leave the road at the top of the hill?

[175 km/h]



Banked road

On a curve, if the road surface is "banked" (tilted towards the curve centre) then the horizontal component of the normal force can provide some (or all) of the required centripetal force. Choose v & θ so that less or no static friction is required.



KJF example 6.6

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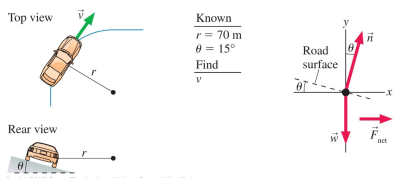
A curve of radius 70m is banked at a 15° angle. At what speed can a car take this curve without assistance from friction?

[14 ms^{-1} = 50 km h^{-1}]



KJF example 6.6

Next lecture

Centre of mass and Torque

Read: KJF §7.2, 7.3