Lecture 9

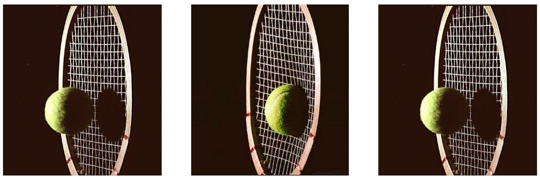
Momentum, impulse and energy

Pre-reading: KJF §9.1 and 9.2

MOMENTUM AND IMPULSE

KJF chapter 9

before *after*



COLLISION
complex interaction

Linear Momentum of a Body

We define the momentum of an object as:

$$\underline{p} = m\,\underline{v}$$

where m = mass and $\underline{v}$ = velocity.

$\underline{p}$ is a vector and is in the same direction as v.

(Don't confuse p with power or pressure.)

Units: kg.m.s^{-1}

Momentum and Newton's 2nd Law

If $\underline{F}_{\text{net}}$ and m are constant, then

$$\underline{F}_{\text{net}} = m\underline{a} = m\,\Delta\underline{v}/\Delta t$$

$$= \Delta m\underline{v}/\Delta t = \Delta\underline{p}/\Delta t$$

Newton originally expressed his second law in terms of momentum.

If m or $\underline{F}$ are NOT constant then: $\underline{F}_{\text{net}} = \mathrm{d}\underline{p}/\mathrm{d}t$

Momentum of a system of particles

Total momentum $\underline{p}$ is the vector sum of individual momenta:

$$\underline{p} = \sum \underline{p}_i = \sum m_i\,\underline{v}_i \qquad \textit{(Vector sum!)}$$

Now consider the particles as one system:

N2L becomes:

$$\sum \underline{F}_{\text{ext}} = \mathrm{d}\underline{p}/\mathrm{d}t$$

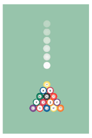
where $\sum \underline{F}_{ext}$ = net external force
i.e. not forces between particles in the system.

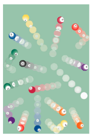
Conservation of Momentum

So if $\sum \underline{F}_{ext} = 0$ for a system

then $d\underline{p}/dt = 0$ *total momentum is constant*

When the particles interact (e.g. billiard ball collision, explosion etc.), if net external force is zero then total momentum *before* interaction equals total momentum *after* i.e. **momentum is conserved**

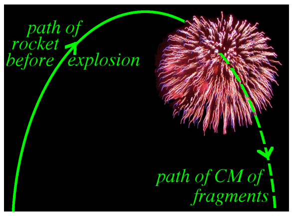




$$\underline{p}_{initial} = \underline{p}_{final}$$

$$(\sum m_i \underline{v}_i)_{initial} = (\sum m_i \underline{v}_i)_{final}$$

$$= m_{tot}\, \underline{v}_{cm}$$



Example

Calculate the recoil speed of a pistol (mass 0.90 kg) given that the bullet has mass 8.0 g and emerges from the pistol with a speed of 352 ms^{-1}. Assume the momentum of the exhaust gas is negligible.

[3.1 ms^{-1}]

Impulse

The impulse $\underline{J}$ of a force is defined as the change in momentum $\Delta\underline{p}$ caused by that force.

From Newton's Second Law, if $\underline{F}$ is constant

$$\underline{F} = \Delta\underline{p}/\Delta t$$

Then $\underline{F}\,\Delta t = \Delta\underline{p} = \underline{J}$

Example: 1.0 kg object falls under gravity. Calculate the impulse the object experiences due to its weight after falling for 10 s.

[98 kg m s^{-1} downwards]

Impulse (2)

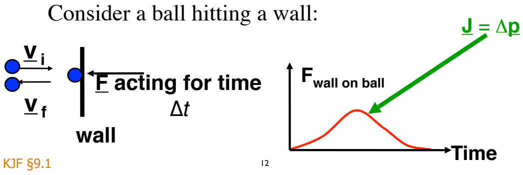
However, if $\underline{F}$ is not constant

$$\underline{J} = \underline{F}_{av}\,\Delta t \quad \text{or} \quad \underline{J} = \int \underline{F}(t)\,dt$$

i.e. impulse = area under $\underline{F}$ vs t curve

Remember that if $\underline{F}$ is constant $\underline{F}_{av} = \underline{F}$

Consider a ball hitting a wall:



Impulse (3)

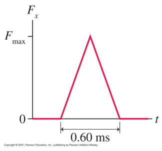
$$\underline{J} = \Delta \underline{p} = \underline{F}_{av}\ \Delta t$$

You can minimise the average force $\underline{F}_{av}$ during an impact, by increasing the impact time Δt

e.g. seat belts, driver's air bag, wicket-keeper's glove, thick landing mat for high jumper.

Example: Hitting a cricket ball

A 150g cricket ball is bowled with a speed of 20 ms^{-1}. The batsman hits it straight back to the bowler at 40 ms^{-1}, and the impulsive force of bat on ball has the shape as shown.



(a) What is the maximum force the bat exerts on the ball?

(b) What is the average force the bat exerts on the ball?

[30 kN, 15 kN]

ENERGY

KJF chapter 10

What is Energy?

Energy is needed to do useful work.

Energy can move things, heat things up, cool them down, join things, break things, cut things, make noise, make light, and power our electronics, etc.

Energy is a scalar *(no direction, not a vector — easy maths!)*

S.I. unit of energy is the joule, J

Examples of energy?

- Energy of motion - "kinetic energy"
- Stored energy - "potential energy": gravitational, elastic, chemical
- Energy in hot objects - "thermal energy"

KJF §10.1–10.2

Kinetic Energy

Simplest form of energy is energy of motion – kinetic energy

$$K = \tfrac{1}{2} mv^2$$

where m is mass (kg) and v is magnitude of velocity (ms^{-1}).

Unit definition: 1 J = 1 joule = 1 kg.$(m.s^{-1})^2$ = 1 kg.$m^2.s^{-2}$

Example: A 1.0 kg mass moves @ 2.0 ms^{-1}. Find K.E.

[$K = \tfrac{1}{2} \times 1.0\ kg \times 4.0\ m^2.s^{-2} = 2.0\ J$]

KJF §10.5

Gravitational Potential Energy

Stored energy due to height in a gravitational field:

G.P.E. or $U = mgh$

where m is mass (kg) and h is height above the origin level (m).

The origin position ($h = 0$) can be freely chosen
U is always *relative* to some reference level or position.

Example: A 1.0 kg mass is held 10 m above the ground. Find its G.P.E. relative to the ground.

[$U = 1.0 \text{ kg} \times 9.8 \text{ ms}^{-2} \times 10 \text{ m} = 98 \text{ J}$]

Mechanical Energy

Kinetic energy and potential energy added together are called *Mechanical Energy*.

Potential energy is stored energy resulting from any force which depends only on position (e.g. gravity, force in a spring, electrostatic attraction).

Gravitational potential energy is only one example of this.

Law of Conservation of Energy

Energy cannot be created or destroyed
(i.e. it is "conserved")

It can only be changed from one form to another

OR

In an isolated system — one where there is no energy transfer into or out of the system — the total energy E_{tot} is conserved.

NEXT LECTURE

Work, power and potential energy

Read: KJF §9.1, 9.2