

# Conservation of mechanical energy

Pre-reading: KJF §10.6

## Conservation of Mechanical Energy

Under the influence of **conservative** forces only  
(i.e. no friction or drag etc.)

$$M.E. = K + U = \text{constant}$$

Note that  $U$  and  $K$  can include such things as elastic potential energy, rotational kinetic energy, etc.

*Example:* simple pendulum or slippery dip  
(if friction & air resistance are negligible).

## Simple Pendulum

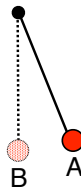
The system is (pendulum + earth).

$F_T$  (tension in string) is always perpendicular to motion so does no work.

Weight (gravity) does all the work.

Because gravitational force is conservative, if drag & friction negligible then,

$$M.E. = K + U = \text{constant, i.e. } (K + U)_A = (K + U)_B$$



## Example: Tarzan



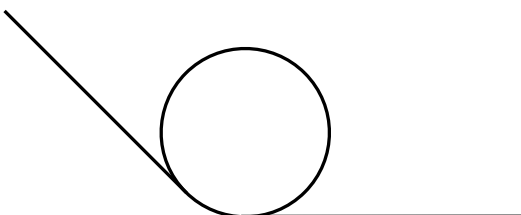
Tarzan who weighs 688N swings from a cliff at the end of a convenient vine that is 18m long. From the top of the cliff to the bottom of the swing he descends by 3.2m.

- What is his speed at the bottom of the swing? Neglect air resistance.
- The vine will break if the force on the vine exceeds 950N. Does it break at the bottom of the swing?

[7.9 m.s<sup>-1</sup>, no]

## Loop the loop

What height does the ball have to start at to make it through the loop?



## Non-Conservative Forces

These are **dissipative** forces such as drag and friction.

Mechanical energy is not conserved when non-conservative forces are acting because friction (and other dissipative forces) convert work or ME directly into thermal energy.

*Thermal energy is just the sum of all the kinetic & potential energies of the molecules of a body.*

Include this in our expression for conservation of energy:

$$K_i + U_i + W = K_f + U_f + \Delta E_{th}$$

## Work and Friction (1)

### Example 1:

Block on horizontal surface slides to rest due to kinetic friction. Work done **by** friction is

$$\Delta ME = \Delta K = -F_k d$$

### Example 2:

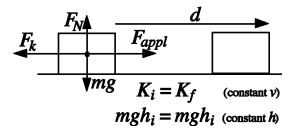
Block sliding along a horizontal surface at constant velocity. If work is done **AGAINST** friction by an applied force  $F_{app}$  and  $\Delta K$  &  $\Delta U = 0$  then;

The amount of thermal energy produced must be exactly equal to the amount of work done, in other words...

7

## Work and Friction (2)

$$W = F s \cos\theta$$



Force working **AGAINST** friction is  $F_{appl} = -F_k$  (why?) but  $F_k = \mu_k F_N$ ,  $\cos\theta = 1$ , and  $s = d$ , so the amount of thermal energy produced is

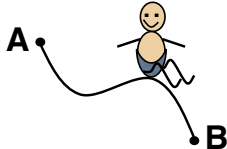
$$\Delta E_{th} = F_k d = \mu_k F_N d$$

Clearly, here work is **not** reversible. (Why not?)

Work done **BY** friction - same magnitude, opposite sign

8

### Example: child on a slide with friction



Since friction (a non-conservative force) is acting,

- $ME_A$  does not equal  $ME_B$ ,
- $ME_B$  will be less, and
- $ME_A - ME_B =$  thermal energy produced.

Also note:

- Work done by gravity (or weight) is always  $= mgh$
- When sliding, work done by normal force  $= 0$  because  $\cos 90^\circ = 0$

9

A 20 kg child, starting from rest, slides down a 3m high frictionless slide.

How fast is he going at the bottom?



[7.7 ms<sup>-1</sup>]

Now he slides down a slide with friction, and his speed at the bottom is 6.0 ms<sup>-1</sup>. How much thermal energy has been produced by friction?

[228 J]

10

## Problem: Skier

A 60 kg skier leaves the end of a ski jump ramp with a velocity of 24 ms<sup>-1</sup> directed 25° above the horizontal. Suppose that as a result of air resistance the skier returns to the ground with a speed of 22 ms<sup>-1</sup> and lands at a point down the hill that is 14m below the ramp.

How much energy is dissipated by air resistance during the jump?



[11 kJ]

11

## Problem

During a rockslide, a 520kg rock slides from rest down a hillside 500m long and 300m high. Coefficient of kinetic friction between the rock and the hill surface is 0.25.

- If the gravitational potential energy of the rock-Earth system is set to zero at the bottom of the hill, what is the value of  $U$  just before the slide?
- How much work is done by frictional forces during the slide?
- What is the kinetic energy of the rock as it reaches the bottom of the hill?
- What is its speed then?

[1.53MJ, -0.510MJ, 1.02MJ, 63ms<sup>-1</sup>]

12

## NEXT LECTURE

Energy ILD