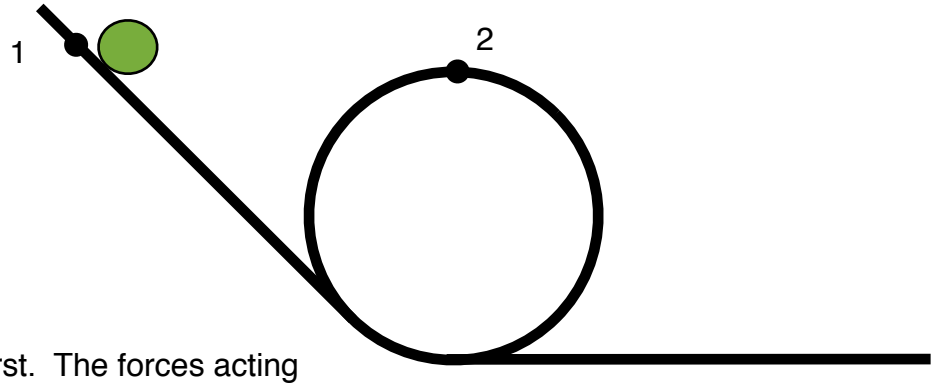
Loop-the-loop

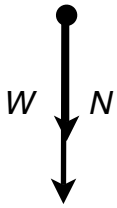
What height does the ball have to start at to make it through the loop?

Let point 1 be where the ball starts, at height x, and point 2 be at the top of the loop, at height $2r$ where r is the radius of the loop.



Solution: Look at point 2 first. The forces acting on the ball when it is at point 2 are the normal force and the weight force, both pointing down. So the net force is also pointing down.

The *minimum* value for the net force occurs when N=0, so the net force is just equal to the weight force. In that case, the weight force alone is providing the centripetal force, and we have



$$F_c = mv_2^2/r = W = mg$$

so

$$v_2^2 = gr$$

Now, when we release the ball at position 1, mechanical energy is conserved, so

$$ME_1 = U_1 + K_1 = ME_2 = U_2 + K_2$$

At point 1 (height x), we have

$$U_1 = mgx, \quad K_1 = 0$$

At point 2 (height $2r$), we have

$$U_1 = 2mgr, \quad K_1 = \tfrac{1}{2}mv_2^2 = \tfrac{1}{2}m \times gr \qquad \text{(from above)}$$

Hence conservation of mechanical energy gives us

$$mgx = 2mgr + \tfrac{1}{2}mgr = 2.5mgr$$

so

$$x = 2.5r$$

i.e. the ball needs to start at a height of **at least** diameter + $\tfrac{1}{2}r$ in order to make it through the loop.