Lecture 3: Emission and absorption

Senior Astrophysics

2018-03-09

Outline

Radiative transfer equation

- Last lecture we derived an expression for how radiation intensity changes with pure absorption:
$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu$$
- If the absorption coefficient is a **constant** (e.g. uniform density gas of ionised hydrogen), then
$$I_\nu(\Delta s) = I_0 e^{-\alpha_\nu \Delta s}$$
- i.e. specific intensity after distance Δs
$=$ initial intensity $\times$ radiation exponentially absorbed with distance

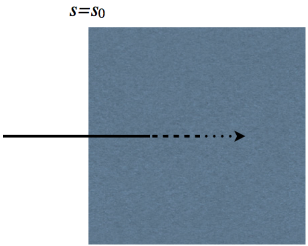
Absorption: Macroscopic version

- Imagine radiation travelling into a cloud of absorbing gas. The exponential term defines a **scale** over which radiation is attenuated.

- When

$$-\alpha_\nu \Delta s = 1$$

the intensity will be reduced to $1/e$ of its original value.



Absorption: Macroscopic version

- We define the **optical depth** τ as

$$\tau_\nu(s) = \alpha_\nu \Delta s \qquad [\text{more generally: } \tau_\nu(s) = \int_{s_0}^{s} \alpha_\nu(s')ds']$$

- A medium is **optically thick** at a frequency ν if the optical depth for a typical path through the medium satisfies

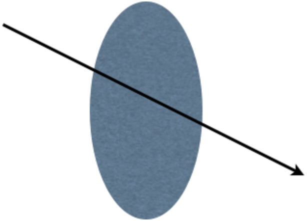
$$\tau_\nu \gg 1$$

- Medium is **optically thin** if instead

$$\tau_\nu \ll 1$$

- We take $\tau_\nu = 1$ to be "just optically thick".

- An optically thin medium is one which a typical photon of frequency ν can pass through without being absorbed.



Smog

Smog

www.smh.com.au/environment/fresh-out-of-fresh-air-report-20090324-98wq.html

Interstellar clouds

apod.nasa.gov/apod/ap091001.html

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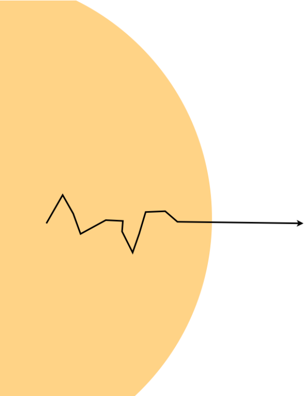
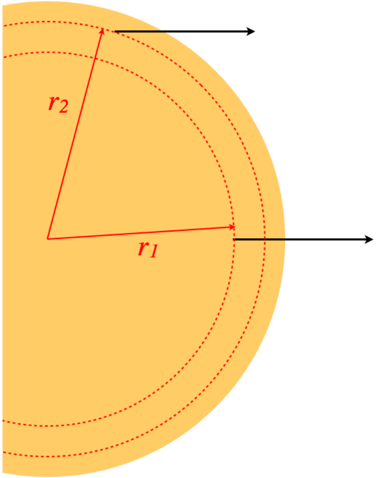
Optical depth

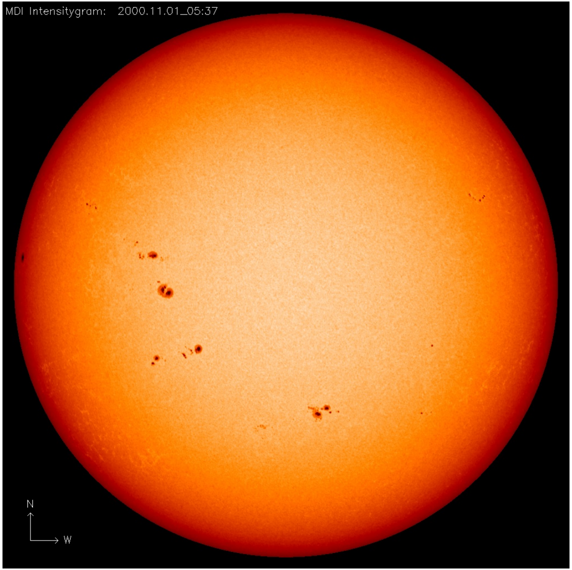


- For light arising inside a gas (e.g. light from inside a star), the optical depth is the number of mean free paths from the original position to the surface of the gas, measured along the ray's path.
- We typically see no deeper into an atmosphere at a given wavelength than $\tau_\lambda \simeq 1$.

Limb darkening



This explains the phenomenon of **limb darkening**. Near the edge of the Sun's disk, we do not see as deeply into the solar atmosphere.
Since $T(r_2) < T(r_1)$, we see a lower temperature, and hence the limb of the Sun appears darker than its centre.



apod.nasa.gov/apod/ap091001.html

Radiative transfer

- We can write the radiative transfer equation with both absorption and emission:
$$\frac{dI_\nu}{ds} = -\alpha I_\nu + j_\nu$$

Source function

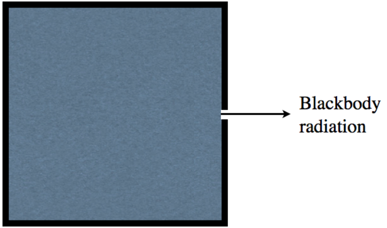
- Rewrite this using the optical depth as a measure of 'distance' rather than s:
$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$
- Divide by the absorption coefficient:
$$\frac{dI_\nu}{\alpha_\nu ds} = -I_\nu + \frac{j_\nu}{\alpha_\nu}$$
or
$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$
where $S_\nu = j_\nu/\alpha_\nu$ is the **source function**. This is sometimes a more convenient way to write the equation.

Source of radiation

- Now we can describe what happens to radiation as it propagates through a medium, which might remove the radiation (absorption) or add to it (emission).
- Where does the radiation come from in the first place?

Blackbody radiation

- The most important type of radiation is **blackbody radiation**: radiation which is in thermal equilibrium with matter at some temperature T



- Emission from many objects is (at least roughly) of this form. In particular, interiors of stars are like this.

Blackbody radiation

- A blackbody of temperature T emits a **continuous spectrum** with some energy at all wavelengths. The frequency dependence of blackbody radiation is given by the **Planck function**

$$B_\nu(T) = \frac{2h\nu^3/c^2}{e^{h\nu/kT} - 1}$$

or

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}$$

where $h = 6.626 \times 10^{-34}$ J s is **Planck's constant**

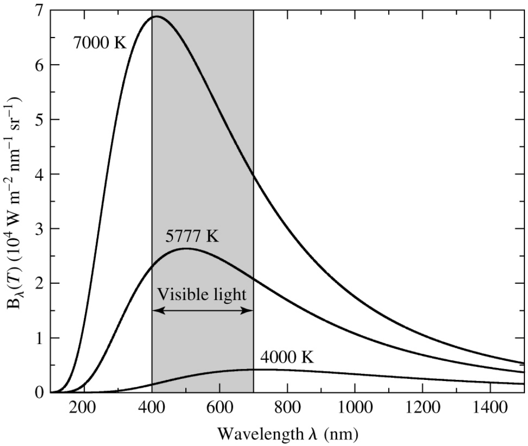
Blackbody radiation

- B_ν has the same units as specific intensity:
  W m^{-2} Hz^{-1} sr^{-1}
  i.e. amount of energy per unit area, per unit time, per unit frequency, per unit solid angle.
- B_λ has units W m^{-2} nm^{-1} sr^{-1}

Blackbody radiation

Plot $B_\lambda(T)$. Properties:

- continuous spectrum
- increasing T increases B_λ at **all** wavelengths
- higher T shifts peak to shorter wavelength / higher frequency



CO fig. 3.8

Blackbody radiation

- Note that for a source of blackbody radiation,

$$I_\nu = B_\nu$$

the specific intensity of radiation emitted is given by the Planck function.

Blackbody radiation

- Differentiate $B_\lambda(T)$ with respect to wavelength and set resulting expression to zero to find where Planck function peaks
- Find
$$\lambda_{\max} = \frac{2.88 \times 10^{-3}}{T} \qquad \text{(m)}$$
i.e. hotter T, smaller wavelength: **Wien's displacement law**

Stefan-Boltzmann equation

- The luminosity of a blackbody of area A and temperature T is given by the Stefan-Boltzmann equation

$$L = A\sigma T^4$$

where $\sigma = 5.670 \times 10^{-8}$ W m^{-2} K^{-4}

- So for a spherical star of radius R, $A = 4\pi R^2$ and the luminosity is

$$L = 4\pi R^2 \sigma T^4$$

Effective temperature

- Since stars are not **perfect** blackbodies, we use this equation to define the **effective temperature** T_e of a star's surface

$$F_{\text{surf}} = \sigma T_e^4$$

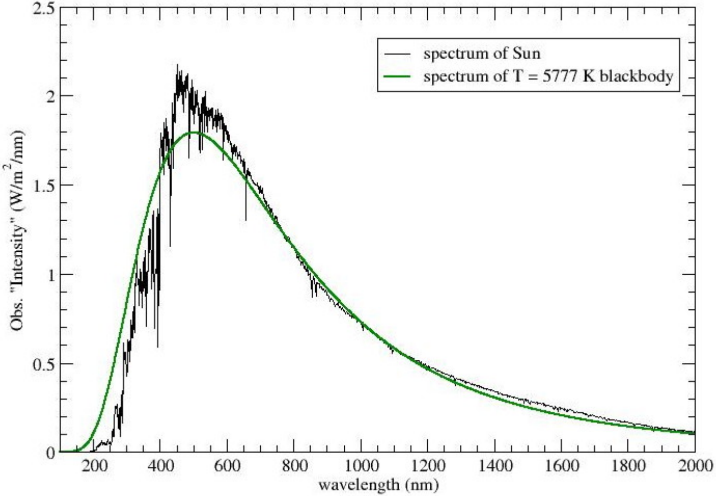
  Effective temperature is the temperature of a blackbody that emits the same flux

- e.g. for the Sun

$$L_\odot = 4\pi R_\odot^2 \sigma T_e^4$$

- Find $T_e = 5770$ K
- Note that T_e is perfectly well-defined even if the spectrum is nothing like a blackbody.

Effective temperature



The Sun's spectrum is a good match to the spectrum of a blackbody of temperature T = 5777 K (in green).
(http://homepages.wmich.edu/˜korista/phys325.html)

Which objects have blackbody spectra?

- We get a blackbody spectrum wherever we have matter in thermal equilibrium with radiation.
- Go back to our radiative transfer equation again:

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

where $S_\nu = j_\nu/\alpha_\nu$ is the source function. Since the gas is in thermal equilibrium with the radiation, then we take this source function to be the Planck function, and assume T is constant:

$$\frac{dI_\nu}{d\tau_\nu} = I_\nu + B_\nu(T)$$

- We can integrate this equation (hint: multiply both sides by e^{τ_ν}), with solution

$$I_\nu(\tau_\nu) = B_\nu + e^{-\tau_\nu}\left[I_0 - B_\nu\right]$$

where I_0 is the value of I_ν at $\tau_\nu = 0$.

- The second term approaches 0 as τ_ν becomes large, so at high optical depth $I_\nu = B_\nu$, e.g. in the centre of a star.
- Alternately:

$$I_\nu(\tau_\nu) = I_0 e^{-\tau_\nu} + B_\nu(1 - e^{-\tau_\nu})$$

In general:

- in thermal equilibrium, the source function is the Planck function, $S_\nu = B_\nu$
- However, even in thermal equilibrium, the intensity of the radiation I_ν will not necessarily be equal to B_ν **unless** the optical depth is large, $\tau_\nu \gg 1$.
- Saying $I_\nu = B_\nu$ is a statement that the radiation field is described by the Planck function
- Saying $S_\nu = B_\nu$ describes the physical source of the radiation, j_ν / k_ν, as one that produces blackbody radiation

Atomic processes

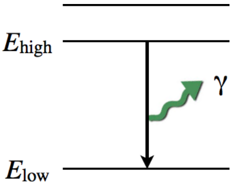
So now, we're going to look at the star's atmosphere
We need to understand the different ways that matter can interact with radiation.

Atomic processes: review

- Radiation can be emitted or absorbed when electrons make transitions between different states. There are three main categories:
  - Bound-bound (excitations and de-excitations): electron moves between two bound states (orbitals) in an atom or ion, and a photon is emitted or absorbed
  - Bound-free:
    - bound → unbound: **ionisation**
    - unbound → bound: **recombination**
  - Free-free: free electron gains energy by absorbing a photon in the vicinity of an ion, or loses energy by emitting a photon: **bremsstrahlung** (‘braking radiation’)

Bound-bound transitions

- Transitions between two atomic energy levels



- Energy of the emitted/absorbed photon is the difference between the energies of the two levels

$$h\nu = |E_{\text{high}} - E_{\text{low}}|$$

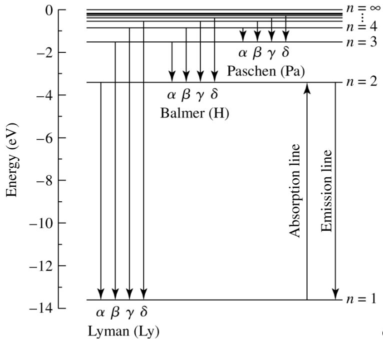
Hydrogen energy levels

- Energy levels are labelled by n the **principal quantum number**
- Lowest level ($n = 1$) is the ground state.
- States with larger n have energy

$$E_n = -\frac{R}{n^2}$$

  where $R = 13.6$ eV is a constant
- The n-th energy level has $2n^2$ degenerate quantum states (same E)

Hydrogen spectrum



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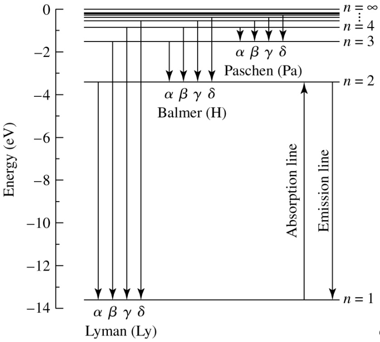
- Special terminology: transitions involving $n = 1, 2, 3, 4$ are part of the **Lyman**, **Balmer**, **Paschen**, **Brackett** series

Hydrogen spectrum



CO fig. 5.7

- Different transitions are labelled with Greek letters, so Lyα arises from the $n = 2$ to $n = 1$ transition; Balmer α (written Hα) arises from $n = 3$ to $n = 2$, Hβ is $n = 4$ to $n = 2$, etc.

Next lecture

Sources of radiation

- Atomic processes
- Absorption and emission line spectra
- Optically thick and thin sources