Lecture 6: The main sequence

Senior Astrophysics

2018-03-20

Outline

Dynamic equilibrium

- Stars exist in a state of equilibrium between gravity, which seeks to collapse the star, and pressure, which supports the star.
- A star is

  "a gravitationally confined thermonuclear reactor whose composition evolves as energy is lost to radiation and neutrinos"
  (Woosley, Heger and Weaver 2002).

Dynamic equilibrium

- Stars are **stable**, because cooling leads to contraction and heating, while overheating leads to cooling and expansion.
- However, since gas pressure depends on temperature, stars must remain hot, and hence they must radiate. In order to replenish the energy lost to radiation, stars must either contract or obtain energy from nuclear reactions. Since nuclear reactions change the composition, stars must **evolve**.
- In today's lecture, we will look at the nuclear reactions which power stars.

Nuclear reactions

- General description of a nuclear reaction

$$I(A_i, Z_i) + J(A_j, Z_j) \leftrightarrow K(A_k, Z_k) + L(A_l, Z_l)$$

where

$$A = \text{nuclear number (nuclear mass)} = n + p \quad (1)$$
$$Z = \text{nuclear charge} = p \quad (2)$$
$$N = \#\ \text{neutrons} = A - Z \quad (3)$$

Reactions must conserve

- baryon number
- lepton number
- charge

Binding energy

- The total mass of a nucleus is *less* than the mass of its constituents.
- The difference in mass (the *mass defect*) is called the **binding energy**. This is the energy released in the reaction.

Binding energy

- If nuclei j with mass $\sum M_j$ fuse to form nucleus y with mass M_y, with $M_y < \sum M_j$, then the mass defect $\Delta M = \sum M_j - M_y$ is available for the star's energy balance:
$$E = \Delta M\, c^2$$
- Stars provide their energy by conversion of **rest-mass** into **kinetic energy**

Binding energy

E.g. hydrogen burning: 4 H $\rightarrow$ He

$$m_{\rm H} = 1.0081u, m_{\rm He} = 4.0039u$$

so

$$E = (4 \times 1.0081 - 4.0039)c^2 = 2.85 \times 10^{-2} c^2$$
$$= 26.7 \text{ MeV per He nucleus}$$

Fraction of rest-mass energy liberated:

$$\varepsilon = 2.85 \times 10^{-2}/(4 \times 1.0081) = 0.007$$

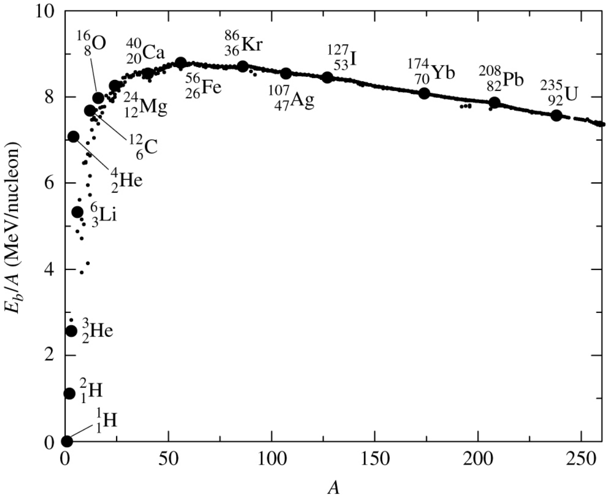
The transformation of H into He liberates 0.7% of the rest-mass of the system in the form of energy

Binding energy

- The mass defect is linked to the **binding energy** of the nucleus: the energy required to separate the nucleons against their mutual attraction by the strong (short-range) nuclear forces

$$Q(Z, N) = [Zm_{\mathrm{p}} + Nm_{\mathrm{n}} - m(Z, N)]c^2$$

  for a nucleus with Z protons and N neutrons.
- We can also define the average binding energy per nucleon $= Q(Z, N)/A$



Binding energy per nucleon as a function of mass number A. Several nuclei — particularly ^{4}He, but also ^{12}C and ^{16}O — lie well above the general trend, indicating unusual stability. The most stable nucleus is ^{56}Fe, at the peak of the curve. (From Carroll & Ostlie, Fig. 10.9)

Binding energy

General characteristics:

- $Q(Z, A)/A$ increases up to $A = 56$ (Fe), then slow decline
- energy gained by **fusion** of light elements to heavier (up to iron)
- or **fission** of heavy nuclei into lighter ones (above iron)
- **steep** rise from H $\rightarrow$ ^{2}H $\rightarrow$ ^{3}H $\rightarrow$ ^{4}He $\Rightarrow$ most energy released in first steps

Binding energy

e.g.

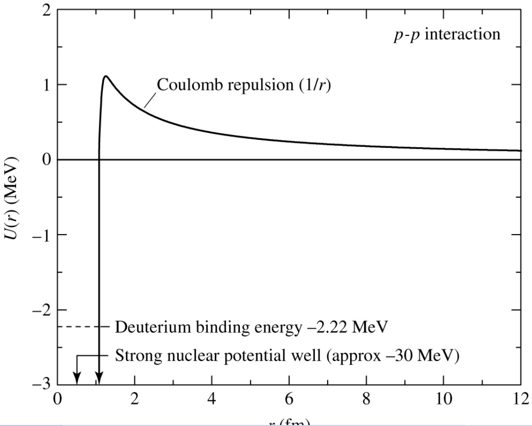
$$\begin{aligned} 4H &\rightarrow {}^{4}\text{He} \qquad 6.3 \times 10^{14}\ \text{J kg}^{-1} = 0.007c^2 \quad (\varepsilon = 0.007) \\ 56H &\rightarrow {}^{56}\text{Fe} \qquad 7.6 \times 10^{14}\ \text{J kg}^{-1} = 0.0084c^2 \quad (\varepsilon = 0.0084) \end{aligned}$$

i.e. hydrogen burning already releases 85% of the available nuclear binding energy.

No energy can be gained from fusion beyond Fe, because formation of heavier elements requires an *input* of energy rather than causing energy to be released.

Nuclear fusion

In order to fuse, two positively charged nuclei must overcome the Coulomb barrier (long range force $\propto 1/r^2$) to reach separations where the strong force dominates ($d \sim 10^{-15}$ m = typical size of nucleus)



Quantum tunnelling

- Height of Coulomb barrier is

$$U_{\text{classical}} = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 r}$$

  with corresponding temperature $T \sim 10^{10}$ K ($\sim$ MeV)
- Temperature at the centre of the Sun

$$T_c \approx 1.5 \times 10^7 \text{ K} \sim 10^3 \text{ eV}$$

so the KE of particles is extremely low compared to the Coulomb barrier.

- High-energy tail of the Maxwell-Boltzmann distribution $\sim e^{-E/kT} = e^{-1000} \approx 10^{-434}$.
- With $\sim 10^{57}$ nucleons in the Sun, expect **no nuclear reactions**!

Quantum tunnelling

- But according to QM, there is a finite probability that a particle will tunnel through the barrier, with

$$P_{\text{Gamow}} = e^{-\frac{\pi Z_1 Z_2 e^2}{\varepsilon_0 h v}}$$

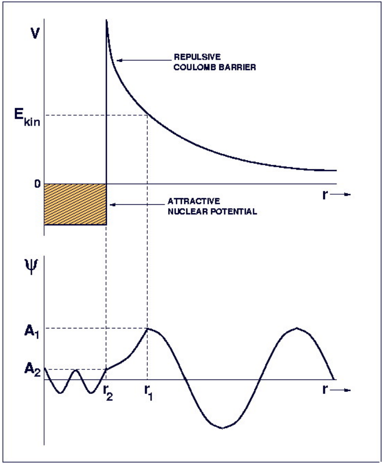
- This increases with increasing particle speed v, but the number of particles with speed v depends on T via the Maxwell distribution, so

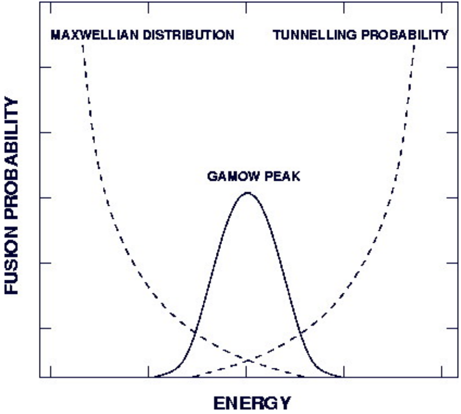
$$P_{\text{fusion}} \propto e^{-\frac{\pi Z_1 Z_2 e^2}{\varepsilon_0 h v}} \times e^{-\frac{m v^2}{2kT}}$$

Quantum tunnelling

- Fusion is most likely to occur in a particular energy window: the *Gamow peak*.
- The higher the charge of the interacting nuclei, the higher the E_{kin} and T before reactions occur, so light elements fuse at lower T than heavy elements
- This explains why the different nuclear burning phases in stars (H, He, C, etc...) are well separated (cannot proceed at the same T)

Quantum tunnelling





from http://www.vikdhillon.staff.shef.ac.uk/teaching/phy213/phy213_fusion2.html

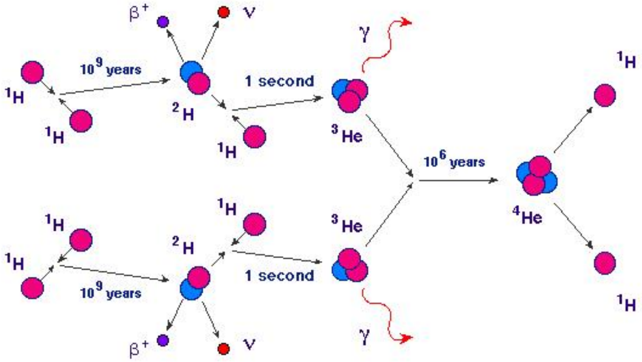
Hydrogen burning

- The most important fusion reaction in stars involves converting H to He ("H burning"): this dominates $\sim 90\%$ of a star's life.
- Overall reaction involves fusing 4 H nuclei (protons) $\rightarrow$ ^{4}He, but this can't be done in a single step.
- The reaction proceeds in a series of steps, each involving two particles. The whole process is

$$4\,^1\mathrm{H} \rightarrow \,^4\mathrm{He} + 2e^+ + 2\nu_e + 2\gamma \qquad \textbf{(PP chain)}$$

- In steps:

$$p + p \rightarrow {}^2\mathrm{H} + e^+ + \nu_e \qquad \text{weak force} \qquad (1)$$
$${}^2\mathrm{H} + p \rightarrow {}^3\mathrm{He} + \gamma \qquad (2)$$
$${}^3\mathrm{He} + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + 2p \qquad (3)$$



Hydrogen burning

- Reaction ① is a weak interaction $\longrightarrow$ bottleneck of the reaction chain
- Typical reaction times for $T = 3 \times 10^7$ K are
  1. 14×10^9 yr
  2. 6 s
  3. 10^6 yr

  $\Rightarrow$ Deuterium is burned up very rapidly.
- If ^{4}He is sufficiently abundant, two further chains can occur: PPII and PPIII. Total energy released is the same, but the energy carried away by the neutrino is different. The relative importance of the different branches depends on density, temperature and composition.

Hydrogen burning

- PP II:

$$^{3}\text{He} + {}^{4}\text{He} \rightarrow {}^{7}\text{Be} + \gamma$$
$$^{7}\text{Be} + e^{-} \rightarrow {}^{7}\text{Li} + \nu_e$$
$$^{7}\text{Li} + p \rightarrow {}^{4}\text{He} + {}^{4}\text{He}$$

- PP III:

$$^{7}\text{Be} + p \rightarrow {}^{8}\text{B} + \gamma$$
$$^{8}\text{B} \rightarrow {}^{8}\text{Be} + e^{+} + \nu_e$$
$$^{8}\text{Be} \rightarrow {}^{4}\text{He} + {}^{4}\text{He}$$

Neutrinos

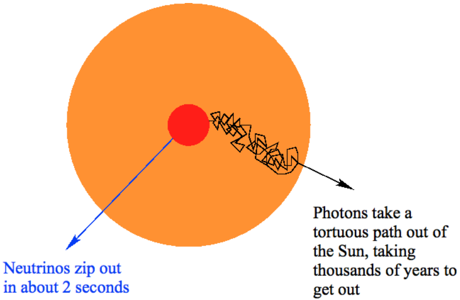
- Energy release

$$Q_{\rm pp} = [4\Delta M(^1{\rm H}) - \Delta M(^4{\rm He})]c^2 = 26.7\ {\rm MeV}$$
$$\sim 6 \times 10^{14}\ {\rm J\,kg^{-1}}$$

- Since two protons are turned into two neutrons, two neutrinos are also emitted, which carry energy away.
- These neutrinos directly confirm the occurrence of nuclear reactions in the Sun.
- Mean neutrino energy is ~ 0.26 MeV.

Neutrinos

- Neutrinos have a very tiny cross-section for interacting with normal matter ($\sigma \sim 10^{-48}\,\mathrm{m}^2$), so they escape from the core of the Sun almost directly. However, it is possible to detect them at the Earth. The number detected should be proportional to the number of fusions taking place.



Neutrinos

- Raymond Davis built the first neutrino detector in the 1960s: 400,000 litres of cleaning fluid (C_2Cl_4) one mile underground
- Very rarely, a neutrino would encounter a chlorine atom, and would be transformed into an atom of radioactive argon: the expected rate was a few dozen captures per month ($\sigma_{\rm Cl} \sim 10^{-46}\,{\rm m}^2$).

Neutrinos

- The experiment was a brilliant success: despite the difficulty, solar neutrinos were in fact detected. However, the number was much lower than expected: between a third and a quarter of the expected rate.
- In his Nobel Prize lecture (2002), Raymond Davis says

  "The solar neutrino problem lasted from 1967–2001. Over this period neither the measured flux nor the predicted flux changed significantly. I never found anything wrong with my experiment. John Bahcall never found anything wrong with the standard solar model."

Neutrinos

- In 1998 a Super-Kamiokande team announced they had detected evidence of neutrinos oscillating between different varieties, only one of which – the electron neutrino – can be detected.
- This not only explains the results of the solar neutrino experiment, but also implies that neutrinos have mass.

CNO cycle

- Stars like the Sun contain trace amounts of C, N and O at birth:

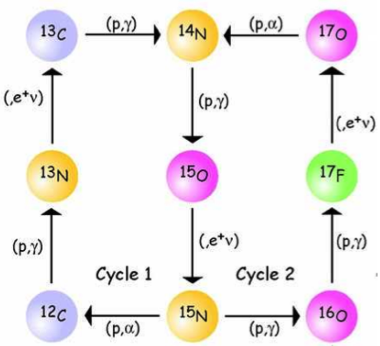
$$X_{^{12}C} = 3.0 \times 10^{-3}$$
$$X_{^{14}N} = 1.1 \times 10^{-3}$$
$$X_{^{16}O} = 9.6 \times 10^{-3}$$

  (cosmic abundances in mass fraction)

- If C, N and O are already present, then at high temperatures they can be used as catalysts for the fusion of H. CNO are not destroyed by the reaction, so the net result is still $4\text{H} \rightarrow {}^{4}\text{He}$.

CNO cycle

- In each cycle, 4H $\longrightarrow$ 1 He
- Total abundance of CNO nuclei is conserved
- Cycle 2 is much less likely (0.04%)
- Slowest reaction is $^{14}\mathrm{N} + p$, so abundances of $^{12}\mathrm{C}$ and $^{16}\mathrm{O}$ decrease and abundance of $^{14}\mathrm{N}$ increases if CNO cycle has time to reach an equilibrium: all initial CNO nuclei are found as $^{14}\mathrm{N}$, waiting to be transformed to $^{15}\mathrm{O}$.



CNO cycle

- CNO energy generation rate is a *strong* function of temperature

$$\varepsilon_{\mathrm{PP}} \propto \rho T^4$$
$$\varepsilon_{\mathrm{CNO}} \propto \rho T^{20}$$

  - in low mass stars (=low T) PP dominates
  - in high mass stars (=high T) CNO dominates

Helium burning

- Next possible reaction is ^{4}He burning (^{3}He and ^{2}H (deuterium) are used up during H burning).
- Obvious reaction would be $^4\mathrm{He} + {}^4\mathrm{He}$, but there is no stable nucleus with $A = 8$

$$\text{e.g.} \qquad {}^4\mathrm{He} + {}^4\mathrm{He} \rightleftharpoons {}^8\mathrm{Be} \qquad \tau \sim 2.6 \times 10^{-16}\,\mathrm{s}$$

- However, **if** a third ^{4}He nucleus can collide before the ^{8}Be decays, can form ^{12}C via the *triple-α process*

$$ {}^4\mathrm{He} + {}^4\mathrm{He} \rightarrow {}^8\mathrm{Be}$$
$$ {}^8\mathrm{Be} + {}^4\mathrm{He} \rightarrow {}^{12}\mathrm{C} + \gamma$$

Helium burning

Fred Hoyle suggested that the very small probability of this reaction occurring would be greatly enhanced if the C nucleus had an excited state with an energy level close to the combined energies of the ^{8}Be and ^{4}He nuclei: the first (and only!) successful prediction of the anthropic principle.

Triple-α reaction

- Energy release

$$Q_{3\alpha} = [3\Delta M(^4\text{He}) - \Delta M(^{12}\text{C})]c^2 = 7.275\ \text{MeV}$$
$$\sim 5.8 \times 10^{13}\ \text{J}\,\text{kg}^{-1}$$
$$\sim 10\%\ \text{of H rate}$$

- Temperature dependence is enormous

$$\varepsilon_{3\alpha} \propto \rho T^{40}$$

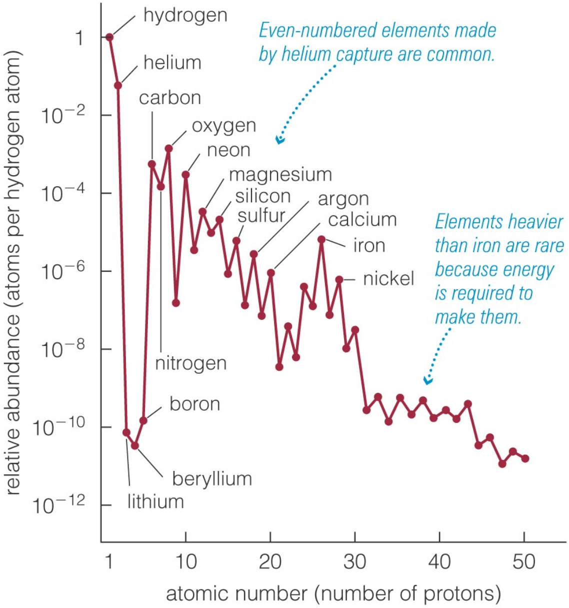
i.e. 10% increase in $T \Rightarrow 50\times$ increase in ε

Further reactions

- Once ^{12}C has formed, O and Ne also form easily

$$^{12}\mathrm{C} + {}^{4}\mathrm{He} \rightarrow {}^{16}\mathrm{O} + \gamma$$
$$^{16}\mathrm{O} + {}^{4}\mathrm{He} \rightarrow {}^{20}\mathrm{Ne} + \gamma \qquad \text{etc.}$$

- In general, as T increases, nuclei with higher and higher Z can form by adding ^{4}He (alpha particles).
- Result: the cosmic abundance distribution shows that even-numbered nuclei are $\sim$ 10 times more abundant than the odd.



hydrogen
helium
carbon
oxygen
neon
magnesium
silicon
sulfur
argon
calcium
iron
nickel
nitrogen
boron
beryllium
lithium
Even-numbered elements made by helium capture are common.
Elements heavier than iron are rare because energy is required to make them.
relative abundance (atoms per hydrogen atom)
1
10−2
10−4
10−6
10−8
10−10
10−12
1
10
20
30
40
50
atomic number (number of protons)

Next lecture

- Stellar scaling relations
- Evolution of a $1M_{\odot}$ star