Lecture 7: Stellar evolution I: Low-mass stars

Senior Astrophysics

2018-03-21

Outline

Scaling relations

- Estimate relations between stellar quantities: Start from the equation of hydrostatic equilibrium

$$\frac{dP}{dr} = -\frac{Gm(r)}{r^2}\rho(r)$$

- For “typical” values of the pressure, we can write

$$\frac{P}{R} \propto \frac{M}{R^2}\rho$$

and since $\rho \propto M/R^3$, then

$$P \propto \frac{M^2}{R^4}$$

Scaling relations

- From the equation of state $P \propto \rho T$, this implies

$$P \propto \frac{M}{R^3} T$$

and hence for these to hold simultaneously, we must have

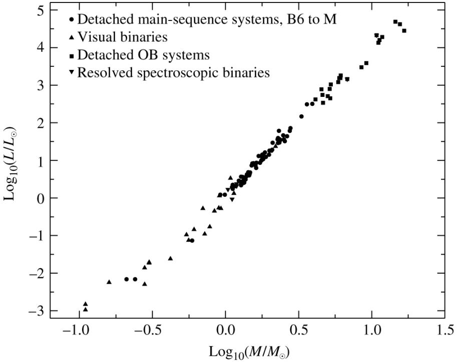
$$T \propto \frac{M}{R} \qquad ①$$

- Similar handwaving from the radiative transfer equation gives us

$$L \propto M^3 \qquad ②$$

Scaling relations

- This is the **mass-luminosity relation**; more massive stars are *much* more luminous.



Observed mass-luminosity relation from binary stars (Data from Popper 1980)

Scaling relations

- The Stefan-Boltzmann equation relates the luminosity of a star to its effective temperature

  $$L = 4\pi R^2 \sigma T_{\mathrm{eff}}^4$$

  If we assume that hotter stars have hotter internal temperatures, so $T \propto T_{\mathrm{eff}}$, then this implies

  $$L \propto R^2 T^4$$

  From the previous slides, we have $L \propto M^3$ and $RT \propto M$, so

  $$M^3 \propto M^2 T^2, \text{ or}$$
  $$M \propto T^2$$

Scaling relations

- Hence from 2, and using $T \propto T_{\rm eff}$ again, we have

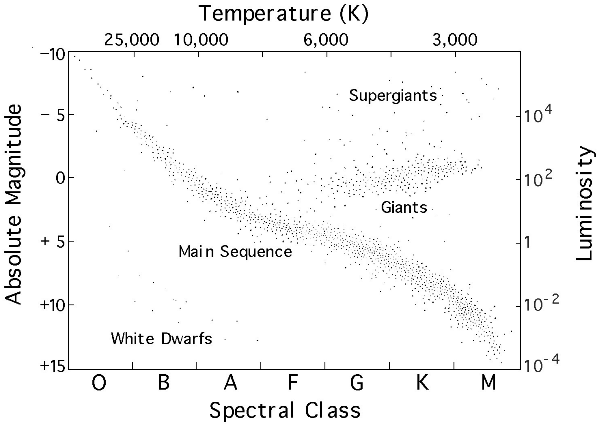
$$L \propto T_{\rm eff}^6$$

- i.e. there should be a relation between the luminosity and surface temperature of a star.

The HR diagram

- Plot L vs T for stars of known distances $\Rightarrow$ **Hertzsprung-Russell diagram** (or HR diagram).
- 90% of stars fall on the **main sequence**, a narrow strip running from cool/faint to hot/bright.
- The main sequence is a **mass** sequence: as we ascend the main sequence from cool and faint to hot and bright, the mass increases smoothly.

The HR diagram



http://heasarc.gsfc.nasa.gov/docs/RXTE_Live/class.html

Stellar lifetimes

- Since the amount of fuel that a star has is $\propto M$, and the rate at which it is consumed is $\propto L$, then the lifetime of a star τ should be given by

$$\tau \propto \frac{M}{L}$$

and so, using the mass-luminosity relation,

$$\tau \propto M^{-2}$$

- More accurate calculations give

$$\tau \propto M^{-2.5}$$

so massive stars have *much* shorter lifetimes than low-mass stars.

Stellar models

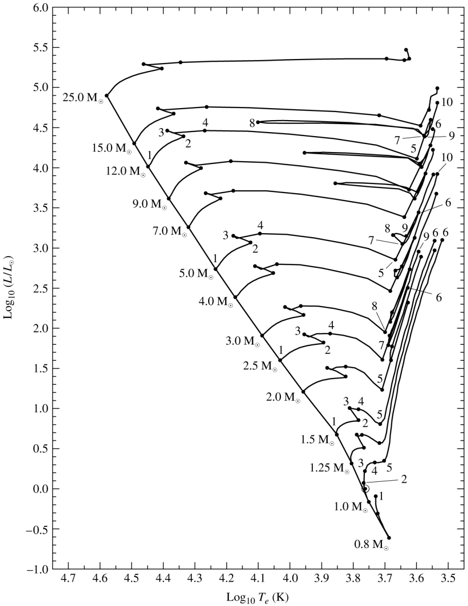
- We have now learned enough about the internal processes in stars to see how stars evolve.
- The details, however, are extremely complicated, and require large computer codes. The results of these computations are not always anticipated or intuitively expected from fundamental principles: the equations are non-linear and the solutions complex.

Stellar models

- We have discussed how we build a static model for a star using the stellar structure equations in lecture 5.
- In order to model a star's evolution we also need to track the composition of the star. which is altered by nuclear reactions and also by material being mixed throughout the star by convection (or other processes). This gives us an additional **time-dependent** equation.
- To solve these equations, we take a **time-step** from a model at time t to a model at time $t + \delta t$. The solution is iterated until it converges to a given accuracy. This produces a new model at the later time.

Stellar models

- The procedure:
  - At time t: static model describing $\rho(m,t)$, $T(m,t)$, $P(m,t)$, $X_i(m,t)$ ...
  - Want: $\rho(m,t+\delta t)$, $T(m,t+\delta t)$, $P(m,t+\delta t)$, $X_i(m,t+\delta t)$.
- Divide the star into shells and apply the structure equations.
  - for N shells $\Rightarrow 4N + N_{\text{isotopes}}$ equations (one for each species) to be solved.
  - Typically $N \geq 200$ shells $\Rightarrow$ need to solve at least 2000 coupled equations at each time step.
- A model **sequence** is a series of static stellar models for many different consecutive times $t, t+\delta t, \ldots$
- We will now look at the output of some sets of "standard" stellar evolution models. You will explore some of these models in the computer lab on Friday.



Log10 (L/L☉)
Log10 Te (K)
25.0 M☉
15.0 M☉
12.0 M☉
9.0 M☉
7.0 M☉
5.0 M☉
4.0 M☉
3.0 M☉
2.5 M☉
2.0 M☉
1.5 M☉
1.25 M☉
1.0 M☉
0.8 M☉

Mass ranges

- Stars of all masses begin on the main sequence, but subsequent evolution differs enormously.
- Identify four regions of the HR diagram:
  - **red dwarfs**: $M < 0.7M_\odot$. Main sequence lifetime exceeds age of Universe
  - **low-mass**: $0.7M_\odot < M < 2M_\odot$. End lives as WD and possibly PN
  - **intermediate-mass**: $2M_\odot < M <$ 8–10$M_\odot$. Similar to low-stars but at higher L; end as higher mass WD and PN
  - **massive**: $M >$ 8–10$M_\odot$. Distinctly different evolutionary paths; end as supernovae, leaving neutron stars or black holes
- Boundaries uncertain, mass ranges approximate.

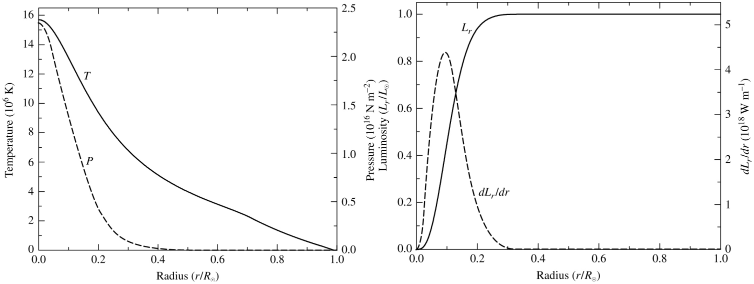
Mass ranges

Upper and lower limits to the mass of stars:

- **minimum mass** for a star $M_{\text{min}} = 0.08 M_\odot$
  Stars with masses below this value do not produce high enough temperatures to begin fusing H to He in the core $\rightarrow$ brown dwarf
- **maximum mass** for a star $M_{\text{max}} \sim 100\text{–}200 M_\odot$??? Stars with masses above this value produce too much radiation $\rightarrow$ unstable (see: Eddington limit, discussed later)

Evolution of a $1M_{\odot}$ star

- Look at evolution of a $1M_{\odot}$ star in detail.
- We can use the equations of stellar structure to calculate the structure of the star on the main sequence. The temperature rises steeply towards the centre of the star, which means that the energy generation is confined to the core.

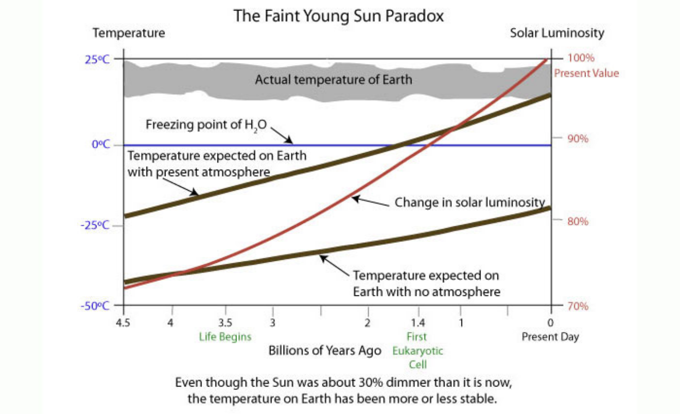


Changes on main sequence

- When H is converted to He (either chain), mean molecular weight of the gas increases and the number of particles decreases
  $\rightarrow P_c$ must decrease
  $\rightarrow$ star is not in equilibrium $\rightarrow$ collapse
- Thus T, ρ increase, so the core gets (slightly) hotter as evolution proceeds.
- Since energy generation rate depends on T, L also increases.
- As a result, the main-sequence is a band, not a line.

Changes on main sequence

- Sun's luminosity has increased by $\sim 30\%$ since birth ("faint young Sun paradox").



http://zebu.uoregon.edu/~imamura/122/lecture-1/lecture-1.html

Core H exhaustion

- When the hydrogen in the core is exhausted, energy production via the pp chain stops. By now, the temperature has increased enough that H fusion can begin in a shell around the inert He core: H shell burning. With no energy being produced in the core, it is isothermal, so the only way for it to support the material above it in hydrostatic equilibrium is for the density to increase towards the centre. As the H-burning shell continues to fuse H to He, the core continues to grow in mass, while the star moves redward in the HR diagram.

Core H exhaustion

- There is a limit to how much mass can be supported by the isothermal core: we will not derive it here, but quote the result

$$\frac{M_c}{M} \simeq \left(\frac{\mu_{\rm env}}{\mu_c}\right)^2$$

  the **Schönberg-Chandrasekhar limit**

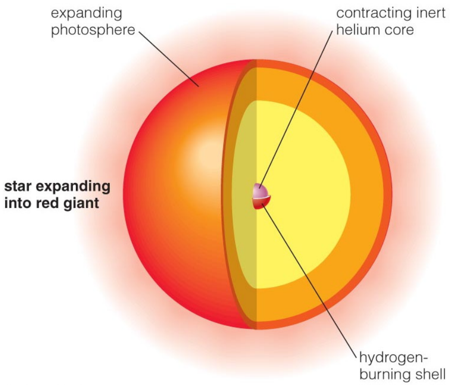
- $\mu_{\rm env}$ and μ_c are the mean molecular weights of the envelope and core. For reasonable values $\mu_{\rm env} = 0.6$, $\mu_c = 1.3$, the limit is $M_c < 0.1M$, i.e. the isothermal core will collapse if it exceeds 10% of the star's total mass.

Core H exhaustion

- Once core starts collapsing, it heats up, which in turn heats up hydrogen surrounding the core, so rate of H-burning in the shell increases.
- Higher T causes higher P outside the core $\rightarrow$ H envelope expands. Luminosity L remains $\sim$ constant so T must decrease $\rightarrow$ star moves to the right along the **subgiant branch**.

Core H exhaustion

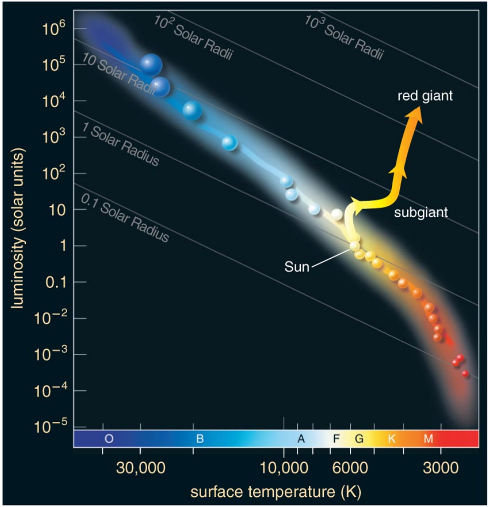
The star continues to expand and cool. But when $T \sim 5000$ K, the opacity of the envelope suddenly increases and convection sets in. This greatly increases the energy transport, and the luminosity increases dramatically.



Red giant branch

- The core continues to collapse, so the temperature in the H-burning shell continues to rise and thus the luminosity in the shell increases as does the pressure. The star is no longer in a hydrostatic equilibrium, and the envelope begins to expand. As it expands, the outer layers cool, so the star becomes redder as its luminosity increases. The star moves up the **red giant branch**, almost vertically in the HR diagram. This imbalance continues until the star finds a new way to generate energy in its core.

Red giant branch



Why do stars become red giants?

- There is no simple explanation for why stars become red giants. We can make a plausibility argument: Suppose the core contraction at the end of hydrogen burning occurs on a timescale shorter than the Kelvin-Helmholtz time of the whole star. Then:

$$\begin{aligned} \textit{Energy conservation}: \quad & \Omega + U = \text{constant} \\ \textit{Virial theorem}: \quad & \Omega + 2U = \text{constant} \end{aligned}$$

... must both hold. Only possible if Ω and U are conserved separately.

Why do stars become red giants?

- Assume star has core ($R = R_c$, $M = M_c$), and envelope ($R = R_*$, $M = M_{\rm env}$). If $M_c \gg M_{\rm env}$,

$$|\Omega| \approx \frac{GM_c^2}{R_c} + \frac{GM_cM_{\rm env}}{R}$$

- Now, assume division between core and envelope is fixed, and differentiate wrt time:

$$0 = -\frac{GM_c^2}{R_c^2}\frac{dR_c}{dt} - \frac{GM_cM_{\rm env}}{R^2}\frac{dR}{dt}$$

  or

$$\frac{dR}{dR_c} = -\left(\frac{M_c}{M_{\rm env}}\right)\left(\frac{R}{R_c}\right)^2$$

  i.e. envelope **expands** as core **contracts**

Helium flash

- When the star reaches the tip of the RGB, T_c becomes high enough to allow the triple-α process to begin. In low-mass stars like the Sun, this does not take place until the core is at such high densities that the core is **degenerate**.
- We will discuss degeneracy pressure in a couple of lectures, in the context of white dwarfs. For now, we just note that under these conditions, the pressure is independent of the temperature.

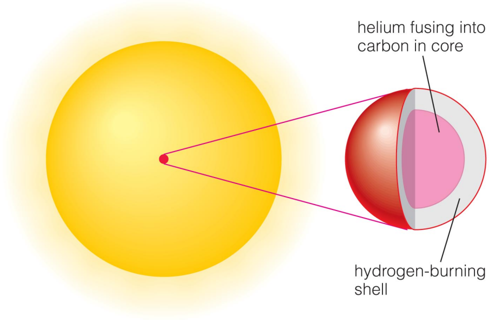
Helium flash

- When He fusion begins, this produces extra energy and T rises, but P does not $\rightarrow$ core does not expand. Higher T increases rate of He fusion even further, resulting in a runaway explosion: the **helium flash**. This energy doesn't escape the star: it all goes into removing the electron degeneracy. Now the core can behave like a perfect gas again, so the star's thermostat has been restored: the core can *expand* and *cool*.
- He flash is *very* difficult to follow computationally, so models of low-mass stars stop when the star reaches the tip of the RGB.

He burning

- Now the star is burning He in the core and H in a shell. The core expands, which pushes the H-shell outwards and cools it $\rightarrow L$ decreases. The envelope then contracts and T_{eff} starts to rise again.

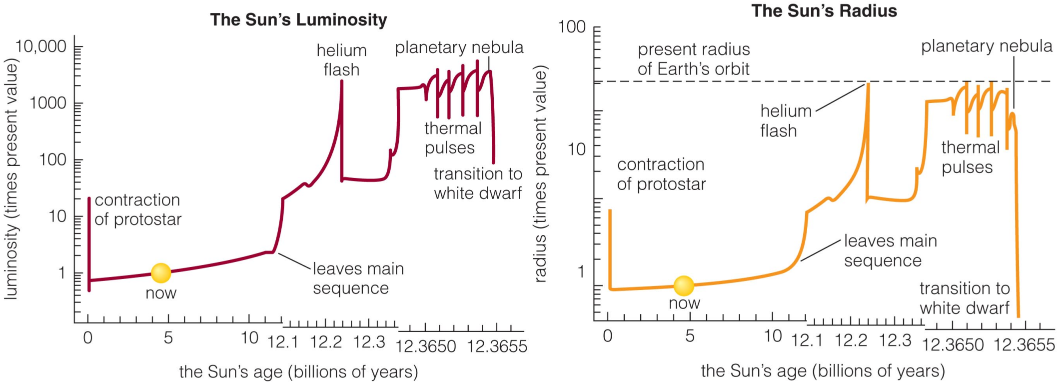
- Star is now fusing He into C steadily in its core, so is once again in quasi-static equilibrium. Lifetime as a red giant is $\sim 10\%$ of its main-sequence lifetime.



Core He exhaustion

- When core runs out of He, it stops producing energy $\rightarrow$ begins to collapse again. Now have inert C core, surrounded by He burning shell and H burning shell. Star moves up the giant branch a second time: the **asymptotic giant branch** (AGB).
- Low/intermediate mass stars ($M < 8M_{\odot}$) do not proceed beyond He burning.
- H burning and He burning is occurring in thin shells surrounding the core. These do not occur simultaneously, but alternate in thermal pulses, which act to eject the outer layers of the star. As the stellar mass diminishes, the mass loss increases, until the entire envelope is ejected. The CO core is exposed and the star's evolution ends.

Core He exhaustion



a Changes in the Sun's luminosity over time.

b Changes in the Sun's radius over time.

Planetary nebulae

- The CO core cools and becomes degenerate: it turns into a **white dwarf**. The ejected envelope expands into the ISM. The new-born WD ionises the gas and we see the expanding shell as a **planetary nebula**.

- The planetary nebula phase only lasts $\sim 10^4$ y. The gas ploughs into the ISM, contributing to its chemical enrichment.

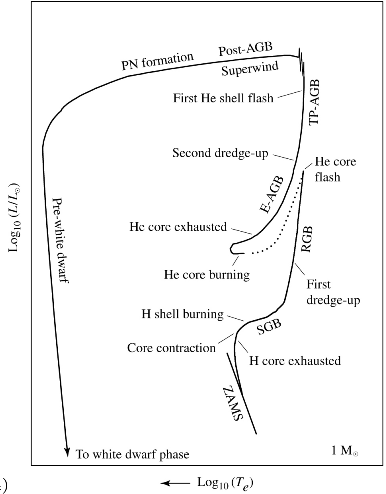
The Helix nebula, NGC 7293

Planetary nebula NGC 2440, with its central white dwarf, one of the hottest WDs known.

Summary of $1M_\odot$ evolution

Approximate typical timescales

Phase	τ (yrs)
Main sequence	9×10^9
Subgiant	3×10^9
Red giant	1×10^9
AGB evolution	$\sim 5 \times 10^6$
PN	$\sim 1 \times 10^5$
WD cooling	$> 8 \times 10^9$



Schematic diagram of the evolution of a $1M_\odot$ star. (CO Fig.13.4)

Next lecture

- Lab 2 on Friday in SNH Learning Studio
  - Evolution of a low mass star
- Next lecture: Evolution of massive stars
  - The evolution of a massive star
  - Convection
  - Mass loss