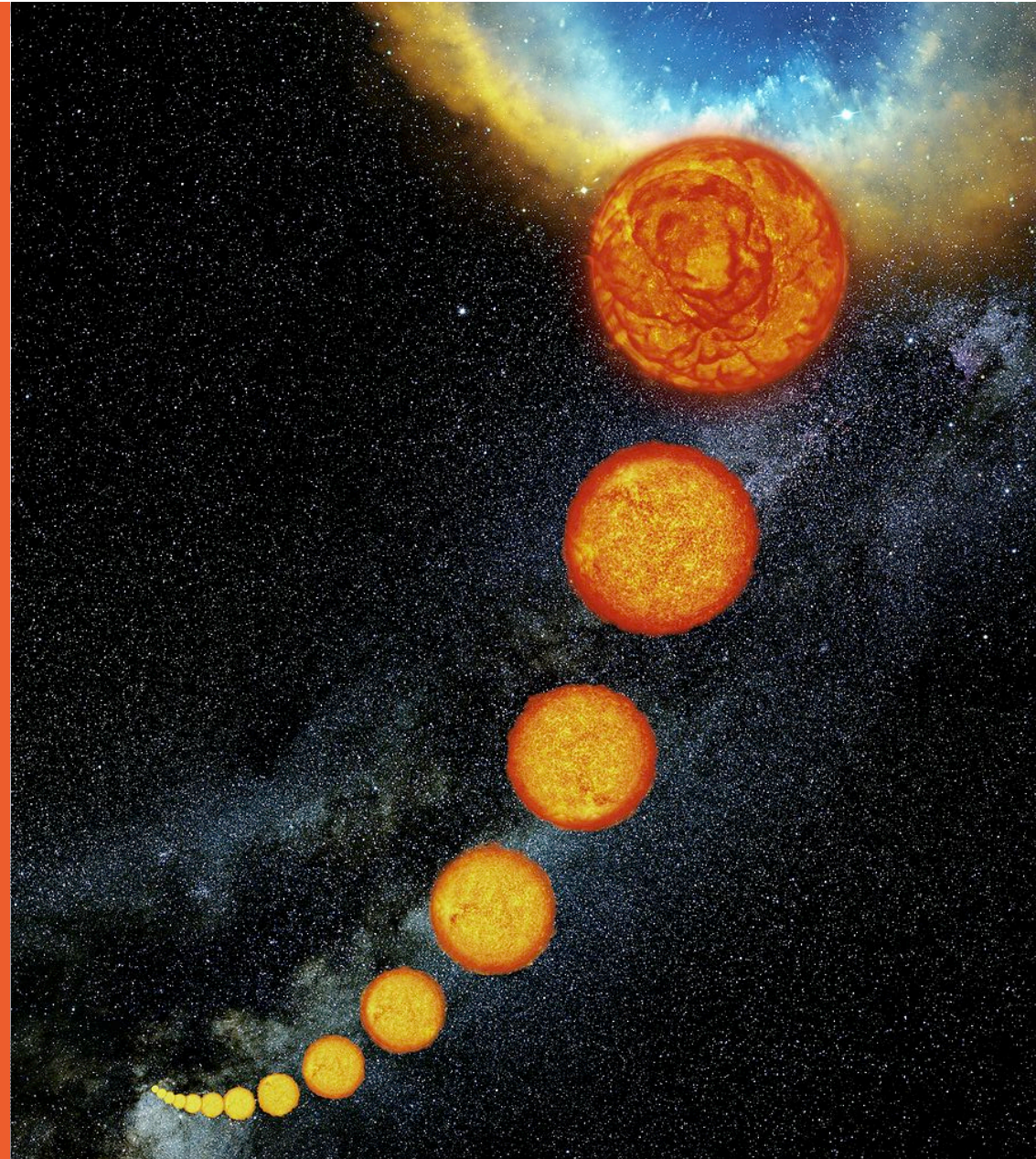


# PHYS3xxx Astrophysics Review

22 June 2018



# Exam

Three sections to the course:

- Radiation
- Stellar evolution
- Binaries

Exam consists of three questions: one question on each topic.

Two common questions, one split Advanced/Normal

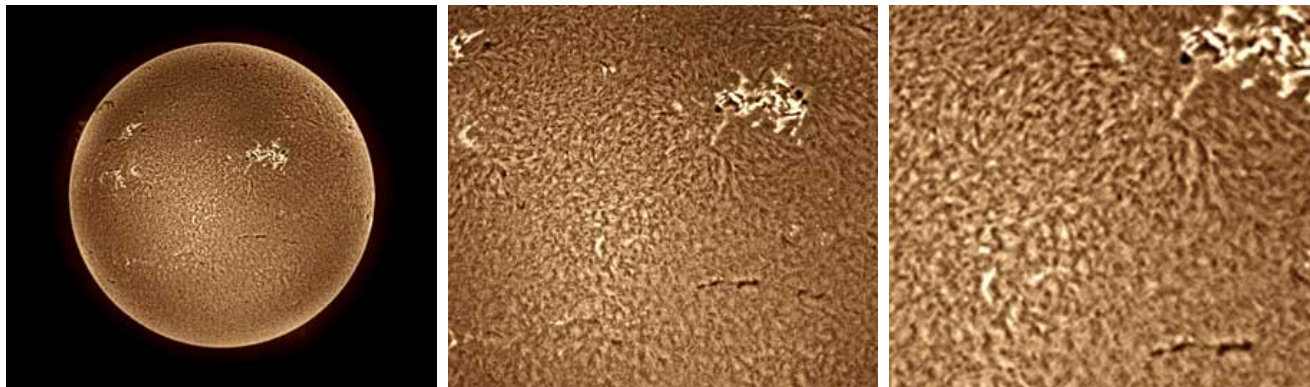
*(1 hr per option = 20 min per question)*

Note **TWO** constants lists

# 1. Radiation

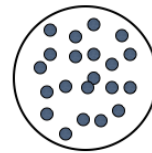
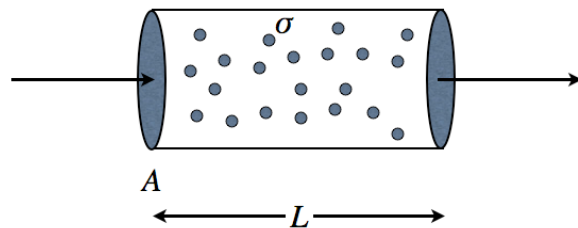
Major topics:

- **Brightness** (specific intensity)
  - amount of light per unit solid angle (per pixel)



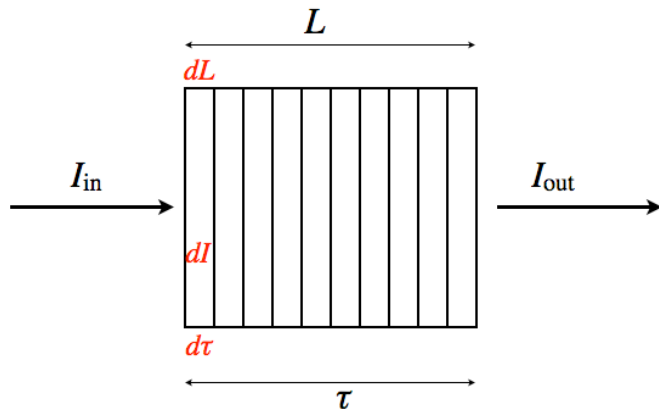
# 1. Radiation

- **Optical depth:** distance over which light decreases by  $1/e$



$$\tau = nL\sigma \quad \text{for small } \tau$$

$$I_{\text{out}} = I_{\text{in}}(1 - \tau)$$



For large  $\tau$  (optically thick)

$$I_{\text{out}} = I_{\text{in}}e^{-\tau}$$

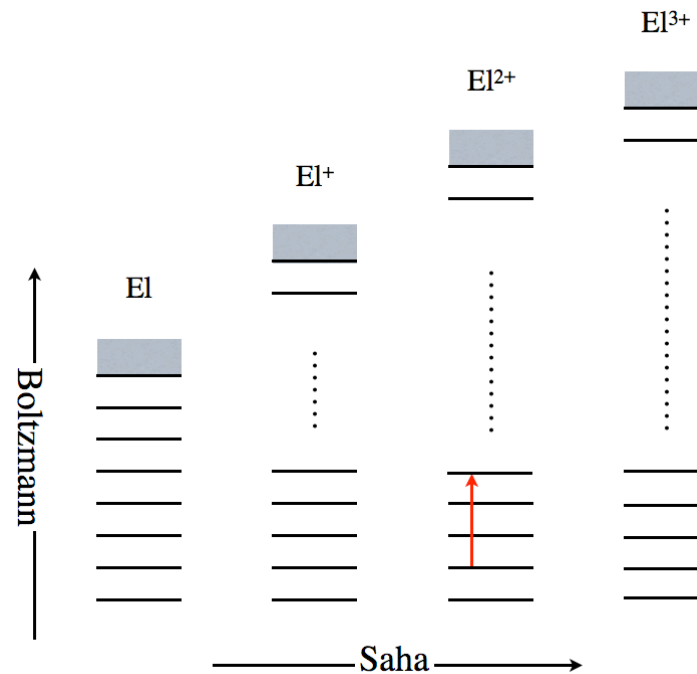
The Sun Typically see to a depth  $\tau \sim 1$  into an atmosphere



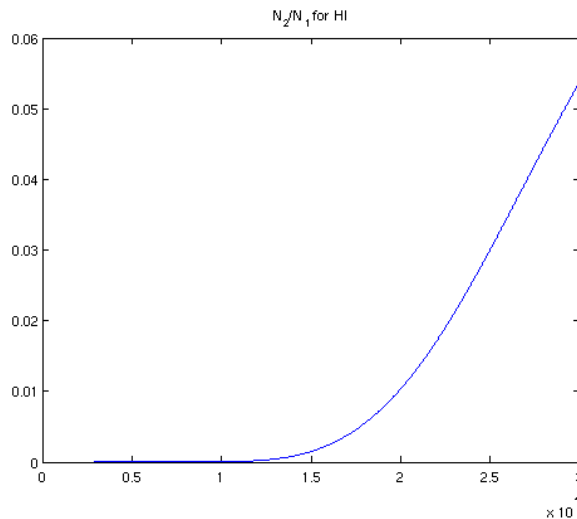
# 1. Radiation

- **Saha-Boltzmann equations**

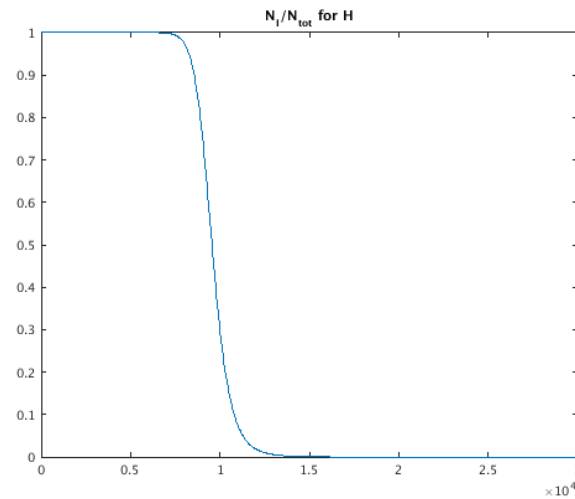
- Boltzmann equation  $\Rightarrow$  distribution of electrons in different energy levels
- Saha equation  $\Rightarrow$  distribution of atoms in different ionisation states



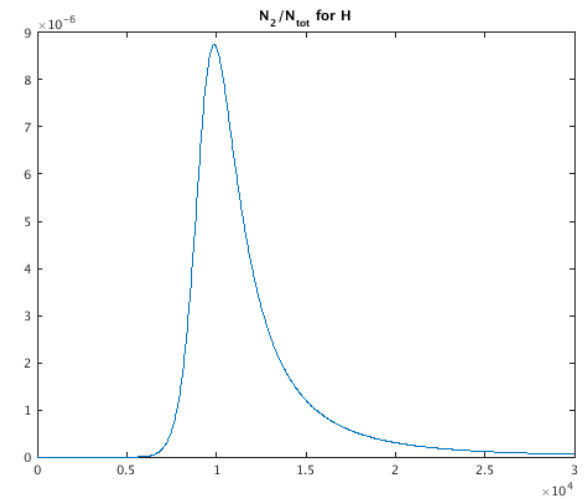
# 1. Radiation



$\times$



$=$



**Boltzmann:**

$N$  in correct energy level  $\uparrow$  with  $T$

**Saha:**

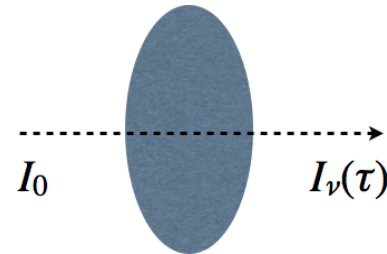
$N$  in correct ionisation state  $\downarrow$  with  $T$

**Result:**

line strength peaks at particular  $T$   
(What determines  $T$ ?)

# 1. Radiation

- Emission/absorption lines**

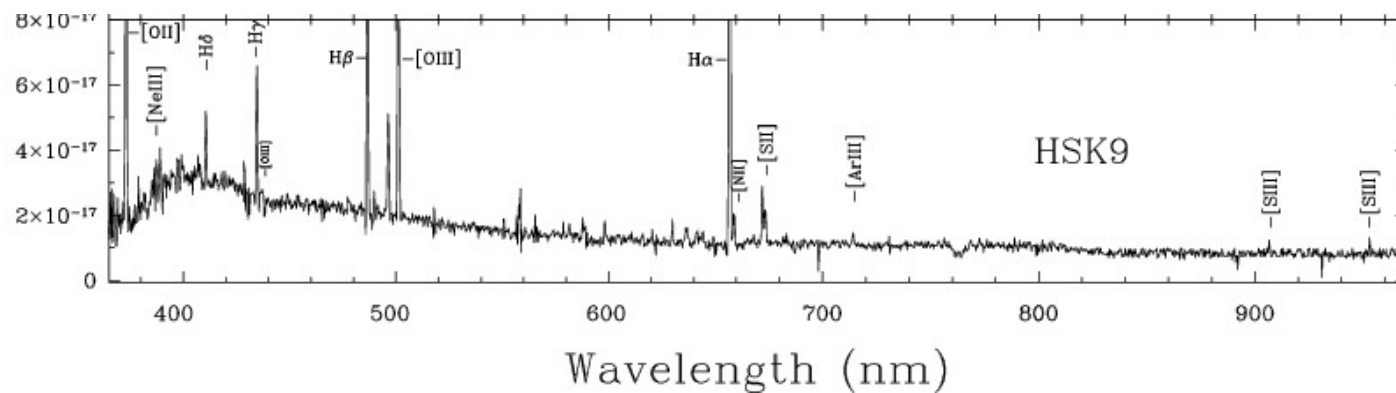


$$I_\nu = I_0 e^{-\tau_\nu} + B_\nu(1 - e^{-\tau_\nu})$$

1. Optically thin,  $\tau$  small:

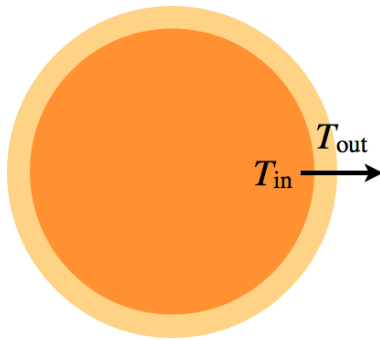
$$I_0 = 0 \quad I_\nu \approx B_\nu(1 - 1 + \tau_\nu) = \tau_\nu B_\nu$$

– large intensity where absorption coefficient is large  
 $\Rightarrow$  **emission** lines



# 1. Radiation

2. Optically thick,  $\tau$  large: spectrum is blackbody in LTE with gas until the outer layer



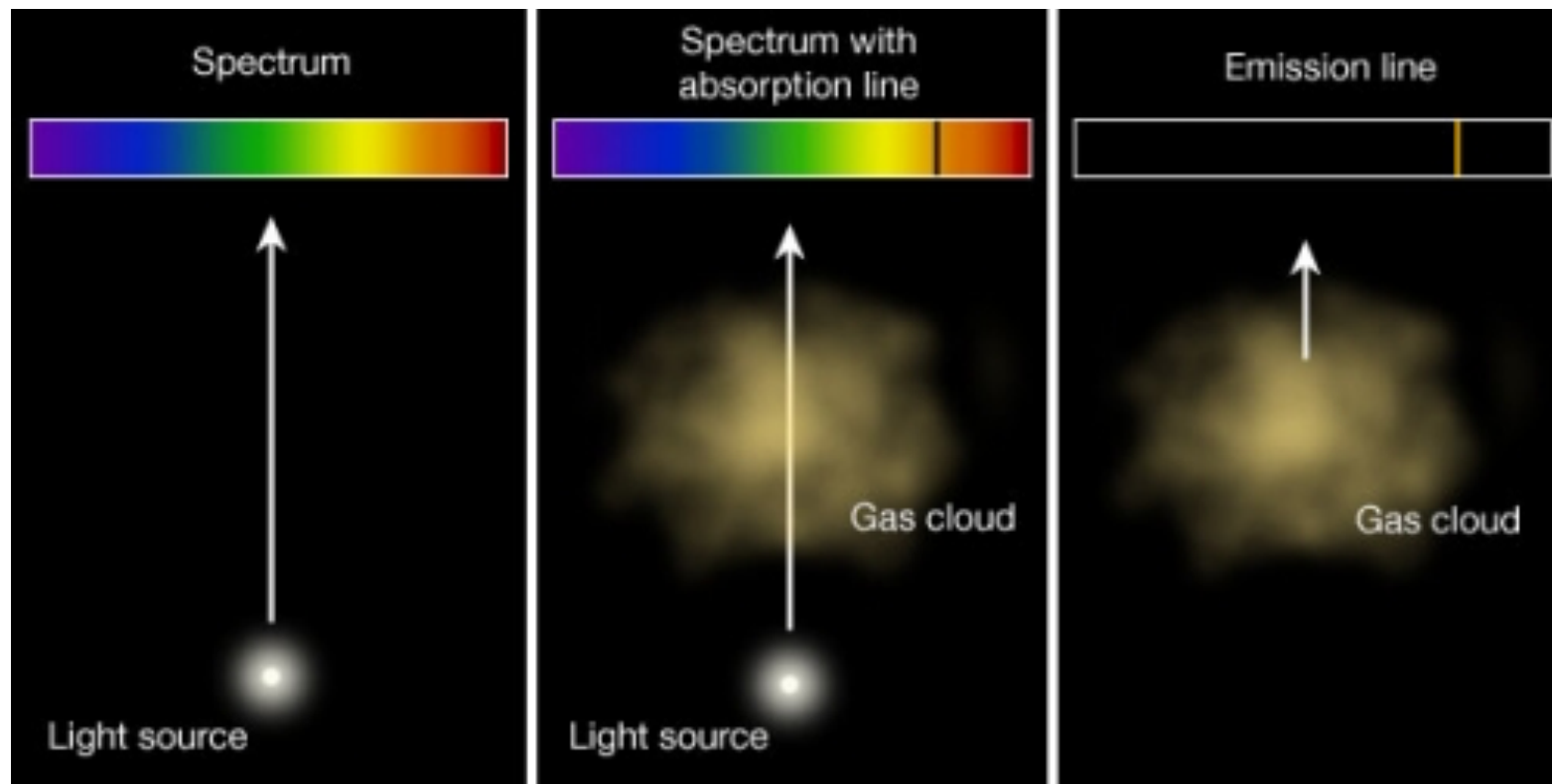
$$I_{\nu} = B_{\nu}(T_{\text{in}}) + \tau_{\nu} [B_{\nu}(T_{\text{out}}) - B_{\nu}(T_{\text{in}})]$$

sign depends on whether  $T_{\text{out}} > T_{\text{in}}$

$\Rightarrow$  continuum + absorption lines if  $T_{\text{out}} < T_{\text{in}}$

continuum + emission lines if  $T_{\text{out}} > T_{\text{in}}$





## 2. Stellar evolution

Major topics:

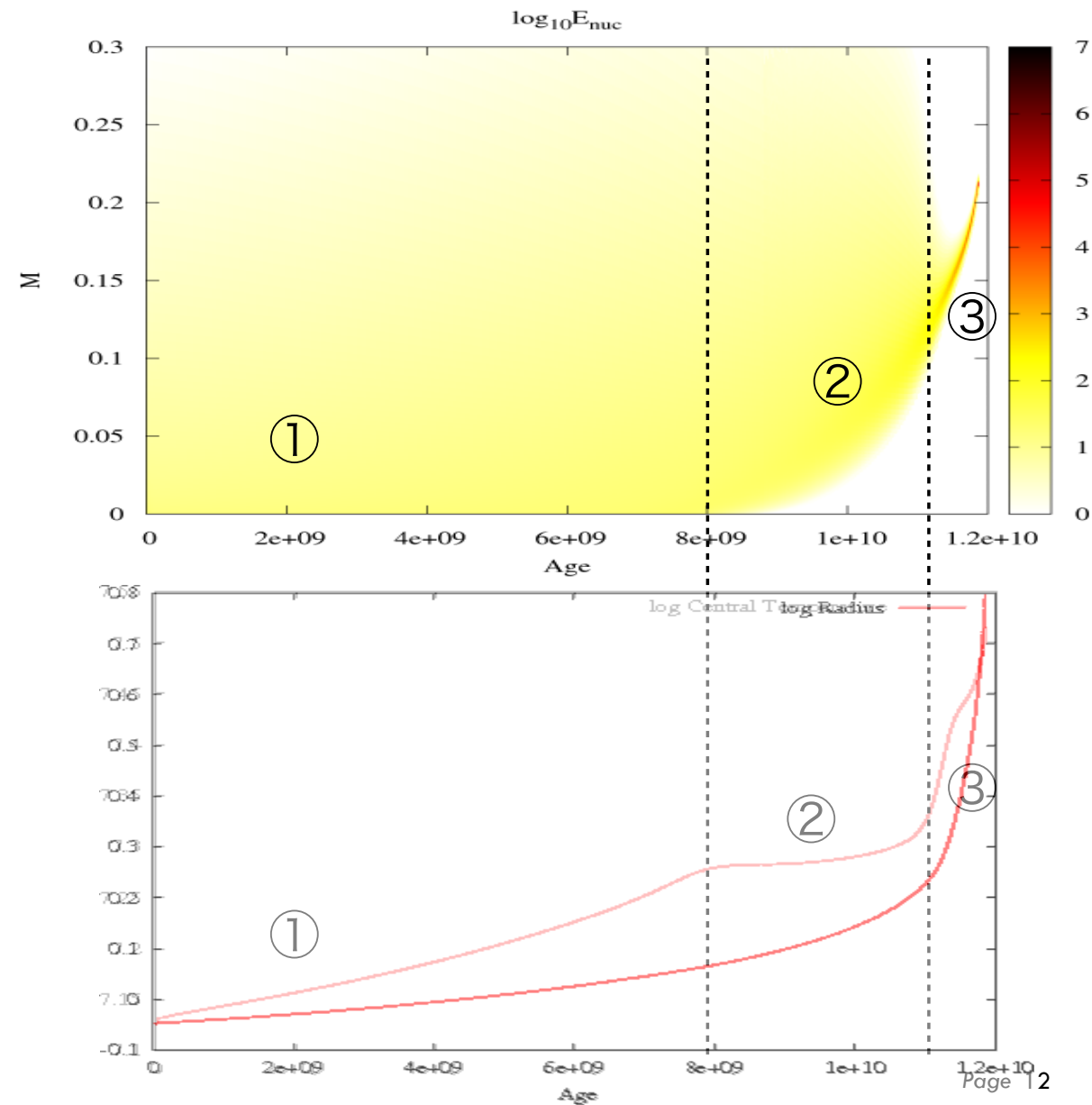
- **Structure** of star: at any time, the structure of a star is the result of balance between inward gravitational pull and outward pressure produced by energy generation in interior
- Stars **evolve** with time as nuclear fuel is consumed
- **Reactions**:  $H \rightarrow He$ ,  $He \rightarrow C$ ,  $C \rightarrow N$  etc. These reactions need higher and higher temperatures  $\Rightarrow$  take place sequentially in cores of stars.
- **Timescales**: estimate 3 timescales:
  - dynamical
  - thermal
  - nuclear

## 2. Stellar evolution

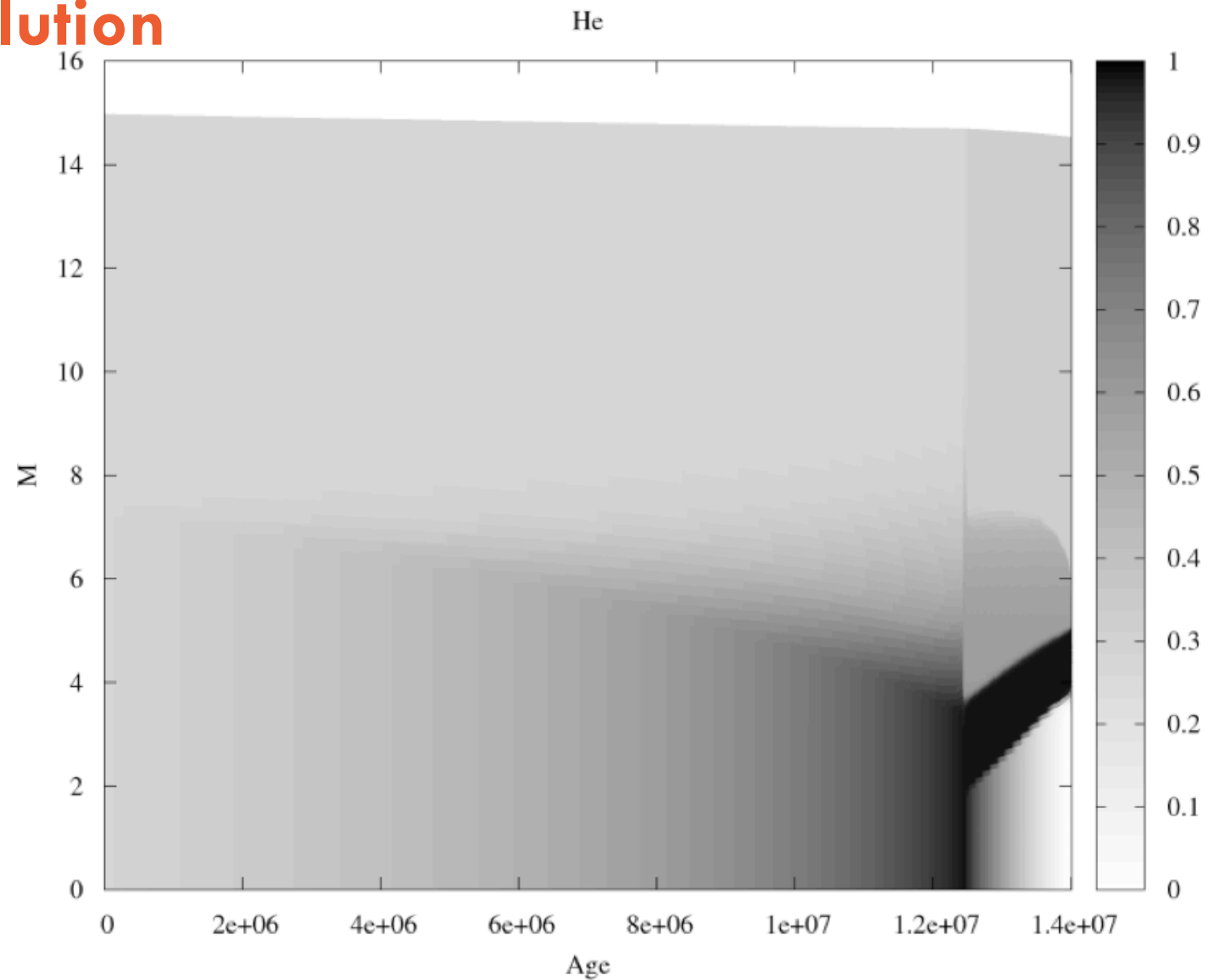
- Four regions of the H-R diagram with different evolution:
  - red dwarfs:  $M_{\odot} < 0.7M_{\odot}$   $\tau_{\text{ms}} > \text{age of Universe}$
  - low-mass:  $0.7M_{\odot} < M < 2M_{\odot}$ . Ends as WD.
  - intermediate-mass:  $2M_{\odot} < M < 8-10M_{\odot}$ ; ends as higher mass WD.
  - massive:  $M > 8-10M_{\odot}$ . Ends as supernova, leaving neutron star or black hole
- Boundaries are uncertain, mass ranges are approximate.
- Be able to discuss the evolutionary tracks of these stars and the internal changes which cause them; remember your WTTS.

## 2. Stellar evolution

1. MS: core H fusion  $\rightarrow$  He
2. Core fusion stops,  $T_c$  plateaus, He core continues to grow
3. Core begins to collapse and heat up. Shell H  $\rightarrow$  He burning starts.
4. Next: threshold  $T$  for He burning is reached (not shown).



## 2. Stellar evolution



## 2. Stellar evolution

- Stellar remnants (white dwarfs, neutron stars, black holes)
- WD and NS supported by *degeneracy pressure* (quantum)
- Maximum possible mass for WD = Chandrasekhar mass,  $\sim 1.4 M_{\odot}$ 
  - *electron degeneracy pressure*
- Maximum neutron star mass not well known,  $\sim 3 M_{\odot}$ 
  - *neutron degeneracy pressure*



### 3. Binaries

- Two stars born together, orbiting according to Kepler's law

$$\left(\frac{P}{2\pi}\right)^2 = \frac{a^3}{G(M_1 + M_2)}$$

- Consider rotating reference frame:

$$\Phi = -G \left( \frac{M_1}{s_1} + \frac{M_2}{s_2} \right) - \frac{1}{2} \omega^2 r^2$$

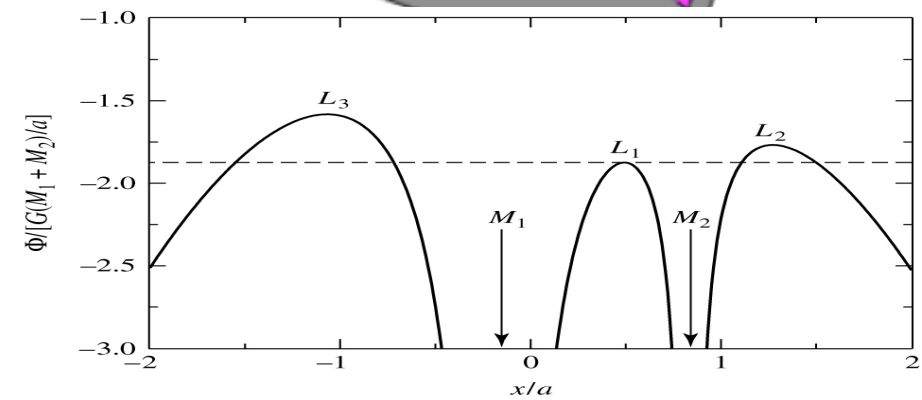
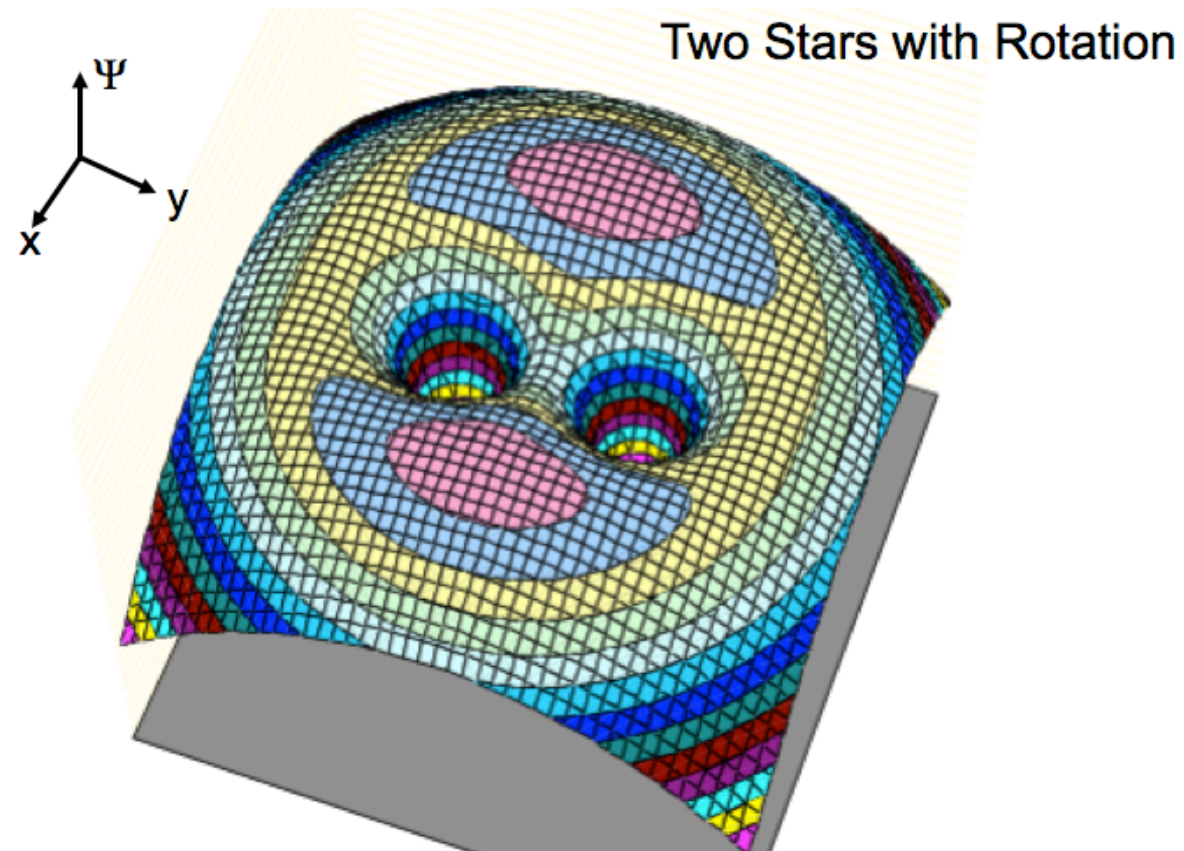
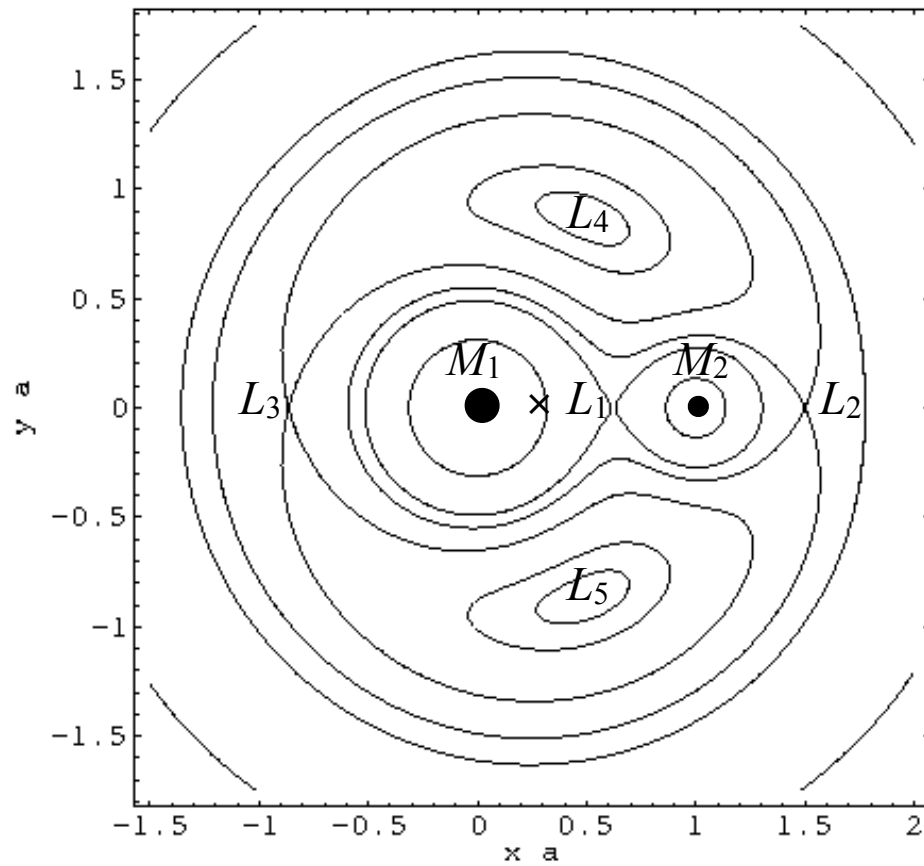
gravity  
star 1



gravity  
star 2

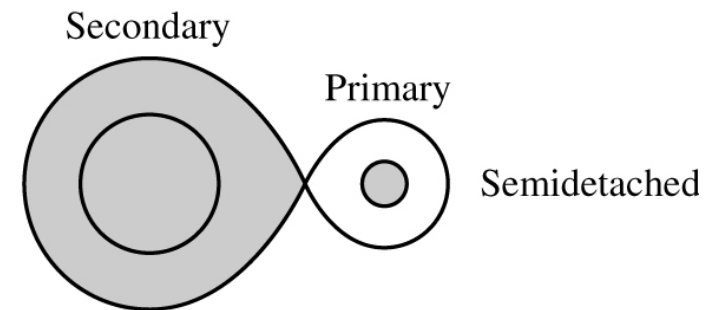
centrifugal  
term

### 3. Binaries



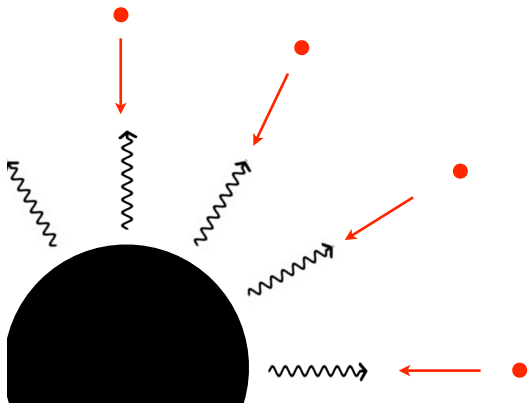
### 3. Binaries

- $L_1$  is the inner Lagrange point: if one star expands to fill its Roche lobe (= region defined by potential through  $L_1$ ), matter will cross this point and transfer to companion  $\Rightarrow$  **accretion**.
- Accretion liberates energy: turns GPE into radiation.
- $E \propto \dot{M}$ : the more mass is accreted, the more energy is produced  $\Rightarrow$  there is a limit to the rate which mass can be accreted



### 3. Binaries

- Eddington limit: balance outward force of radiation with the inward force of gravity.



$$L_{\text{Edd}} = \frac{4\pi c G m_p M}{\sigma_T} = 1.26 \times 10^{31} \left( \frac{M}{M_{\odot}} \right) \text{ W}$$

$L_{\text{Edd}}$  depends **only** on the mass of the accretor

Assumptions?

### 3. Binaries

- Mass transfer can lead to changes in the orbit of the two stars. If star 1 transfers mass to star 2, angular momentum is conserved, and all mass lost by star 1 is gained by star 2, then the change in the orbital separation depends on the mass ratio

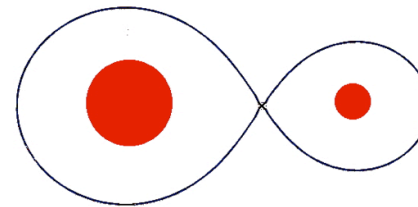
$$\frac{\dot{a}}{a} = -\frac{2\dot{M}_1}{M_1} \left(1 - \frac{M_1}{M_2}\right)$$

- $M_1 < M_2 \quad \Rightarrow \quad \frac{\dot{a}}{a} > 0$  orbit *widens*
- $M_1 > M_2 \quad \Rightarrow \quad \frac{\dot{a}}{a} < 0$  orbit *shrinks*; get *runaway mass transfer*

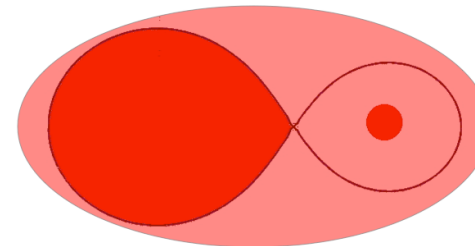
### 3. Binaries

- Types of binaries you should be able to describe/explain:
  - ultra-compact binaries, with orbital separations much less than original star sizes
  - Algol-like systems, where less massive star is more evolved
  - cataclysmic variables
  - (ADV only) Type Ia supernovae and GW binaries

1. main-sequence binary,  $M_1 > M_2 \sim 0.3 M_\odot$



2. primary evolves to RG  $\Rightarrow$  CE



UNSTABLE mass transfer

3. WD + MS star in short  $P$  orbit;  
 $M_{\text{wd}} > M_{\text{ms}}$



4. secondary evolves + transfers mass  $\Rightarrow$  **cataclysmic variable**



STABLE mass transfer