Lecture 10

The Carnot cycle

Pre-reading: §20.6

Review

Engine efficiency e: fraction of the heat input that is converted to work

$$e = \frac{W}{Q_H} = 1 + \frac{Q_C}{Q_H} = 1 - \left|\frac{Q_C}{Q_H}\right|$$

ALL heat engines have $e < 1$

What is the *greatest* efficiency an engine can have?

YF §19.1

The Carnot cycle

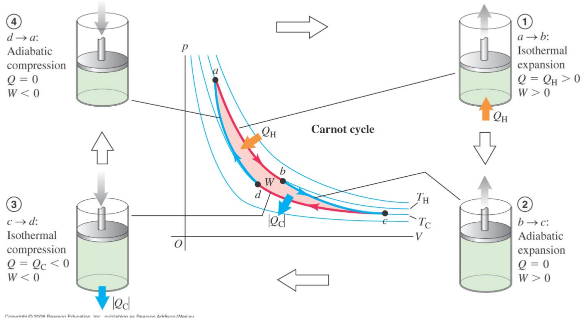
The *Carnot cycle* is a hypothetical, idealised heat engine that has the maximum possible efficiency.

In order to maximise efficiency, we have to avoid *irreversible* processes such as heat transfer with a temperature change.

Thus every heat transfer must be *isothermal* at either T_H or T_C.

The Carnot cycle

The Carnot cycle has two *isothermal* and two *adiabatic* processes, both reversible.



Efficiency of a Carnot engine

In order to calculate the efficiency, we need to find the ratio Q_C/Q_H.

1. Isothermal expansion *ab*:

   heat Q_H supplied from hot reservoir at constant temperature T_H

   $$Q_H = W_{ab} = nRT_H \ln\frac{V_b}{V_a}$$

2. Isothermal compression *cd*:

   heat Q_C rejected to cold reservoir at constant temperature T_C

   $$Q_C = W_{cd} = nRT_C \ln\frac{V_c}{V_d}$$

Efficiency of a Carnot engine

So $$\frac{Q_C}{Q_H} = -\left(\frac{T_C}{T_H}\right)\frac{\ln(V_c/V_d)}{\ln(V_b/V_a)}$$

Now, for the two adiabatic processes $Q = 0$ and

1. Adiabatic expansion *bc*:

   $$T_H V_b^{\gamma-1} = T_C V_c^{\gamma-1}$$

2. Adiabatic compression *da*:

   $$T_H V_a^{\gamma-1} = T_C V_d^{\gamma-1}$$

So $$\left(\frac{V_b}{V_a}\right)^{\gamma-1} = \left(\frac{V_c}{V_d}\right)^{\gamma-1} \quad \text{or} \quad \frac{V_b}{V_a} = \frac{V_c}{V_d}$$

Efficiency of a Carnot engine

Hence the two logarithms cancel out, and we get

$$\frac{Q_C}{Q_H} = -\frac{T_C}{T_H} \quad \text{or} \quad \frac{|Q_C|}{|Q_H|} = \frac{T_C}{T_H}$$

So the efficiency is

$$e = 1 + \frac{Q_C}{Q_H} = 1 - \frac{T_C}{T_H} = \frac{T_H - T_C}{T_H}$$

The efficiency of a Carnot engine depends only on the *temperature difference* of the two heat reservoirs.

Efficiency of a Carnot engine

$$e = 1 + \frac{Q_C}{Q_H} = 1 - \frac{T_C}{T_H} = \frac{T_H - T_C}{T_H}$$

The *bigger* the temperature difference, the greater the efficiency.

e.g. jet engines are made of ceramic, which can withstand temperatures in excess of 1000 °C.

Example

Water at the surface of the ocean near the equator has a temperature of 298 K, whereas 700 m below the surface the temperature is 280 K.

If you build a heat engine using these two layers of water as the heat reservoirs, what is the maximum possible efficiency?

Entropy in a Carnot engine

For a Carnot cycle,

$$\frac{Q_C}{Q_H} = -\frac{T_C}{T_H} \quad \text{so} \quad \frac{Q_H}{T_H} + \frac{Q_C}{T_C} = 0$$

Two adiabatic processes: $\Delta S = 0$

Two isothermal processes: $\Delta S = Q/T$

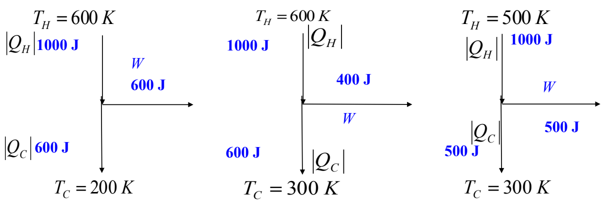
So total entropy change is

$$\Delta S_{\text{total}} = \Delta S_H + \Delta S_C = \frac{Q_H}{T_H} + \frac{Q_C}{T_C} = 0$$

so e_{Carnot} is *maximum* possible efficiency for a heat engine.

Example

Which of the following designs are feasible?



REVIEW