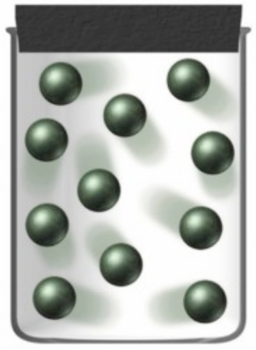
Lecture 5

Ideal gases and the kinetic theory model

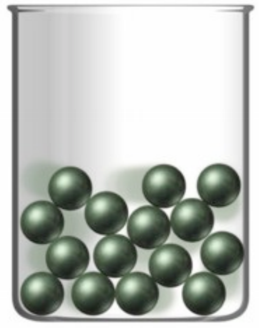
Pre-reading: §18.1

Phases of matter

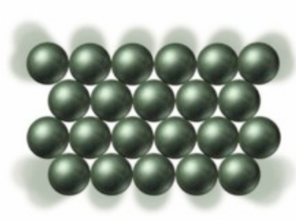
Matter can exist in different *phases*:



– **gas**: very weak intermolecular forces, rapid random motion



– **liquid**: intermolecular forces bind closest neighbors



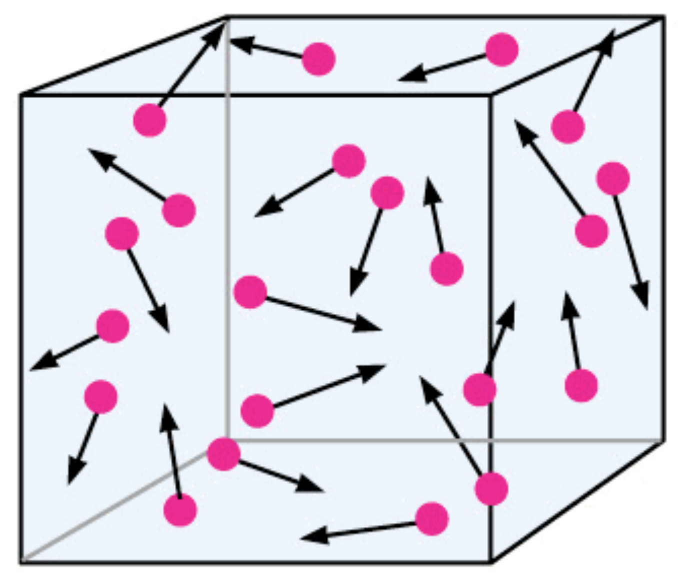
– **solid**: strong intermolecular forces

A transition from one phase to another is called phase change.

Ideal gas

We can understand the properties of a gas by making some simplifying assumptions:

- Volume V contains a large number of identical molecules
- The molecules behave as point particles; their size is very small.
- The molecules are in constant motion; they obey Newton's law of motion; collectively random motion (only kinetic energy for each particle)
- Collisions of molecules with walls of container (elastic collisions).



Ideal gas equation

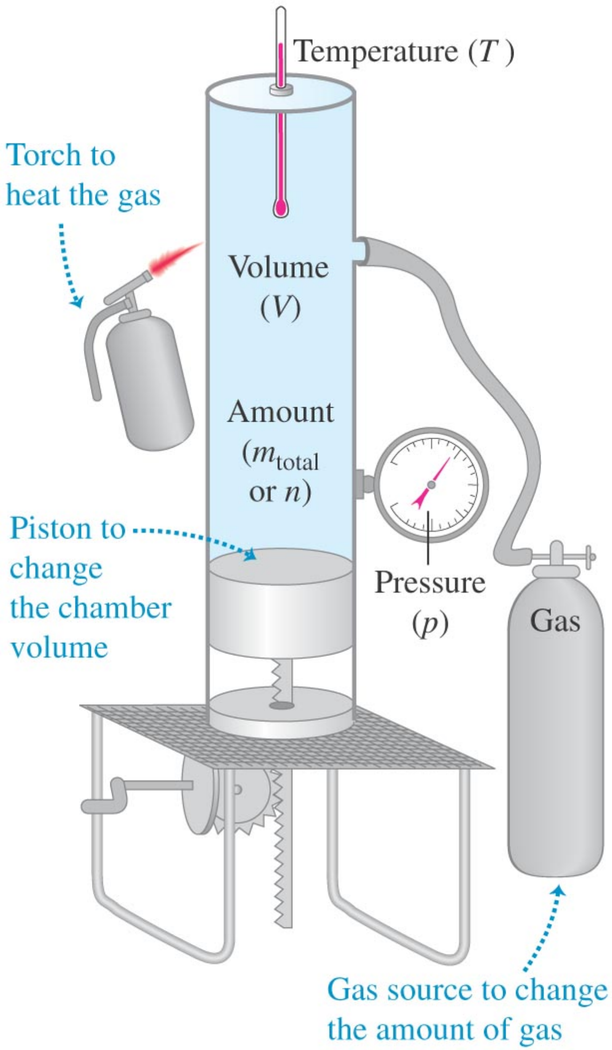
Experiments have found that the pressure, temperature and volume of a gas are related:

$$pV = nRT$$

R is the *gas constant*:

$R = 8.314\ \mathrm{J.mol^{-1}.K^{-1}}$

Note: make sure you express T in K!



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Ideal gas equation: another form

Recall that the number of moles is related to the number of molecules by

$$N = nN_{\mathrm{A}}$$

where N_{A} = Avagadro's number = 6.022×10^{23}

Define the *Boltzmann constant* k

$$k = \frac{R}{N_A} = \frac{8.314 J.mol^{-1}.K^{-1}}{6.022\times10^{23} molecules.mol^{-1}} = 1.381\times10^{-23} J molecule^{-1}.K^{-1}$$

then

$$pV = NkT$$

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Ideal gas equation

$$pV = nRT$$

For a constant mass (= constant n), the product nR is constant, so pV/T is also constant. Hence for any two states of the gas,

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

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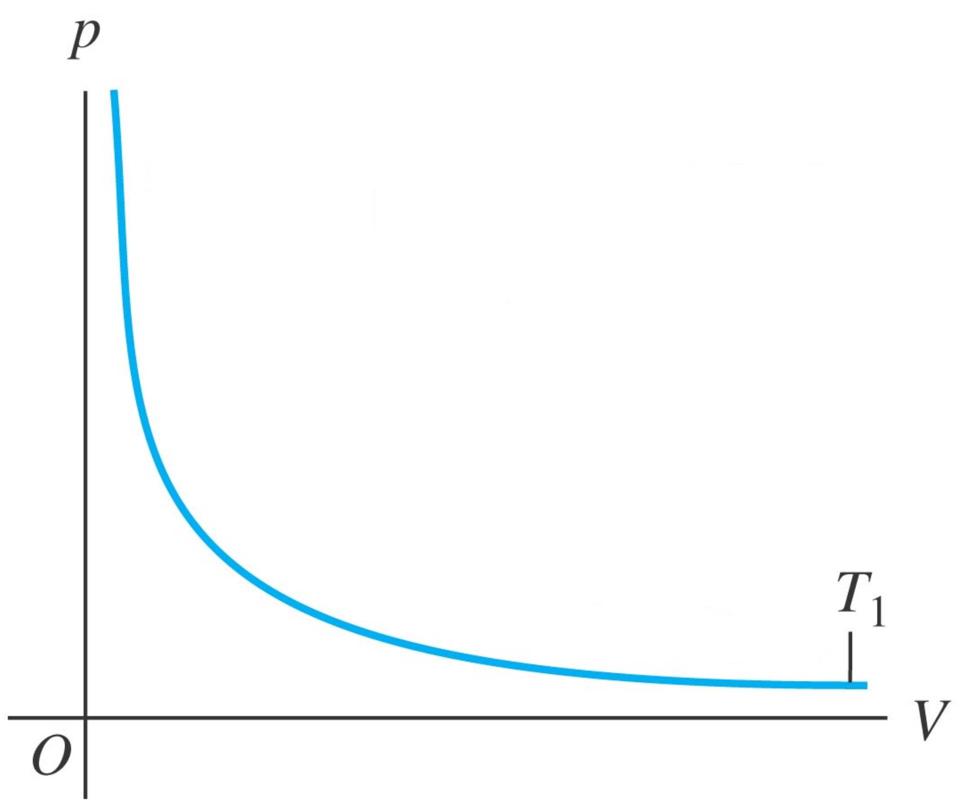
pV diagrams

For fixed temperature, $p \propto \dfrac{1}{V}$ so we can draw

a curve in a *p* vs. *V* graph which represents pressure as a function of volume at a single temperature.

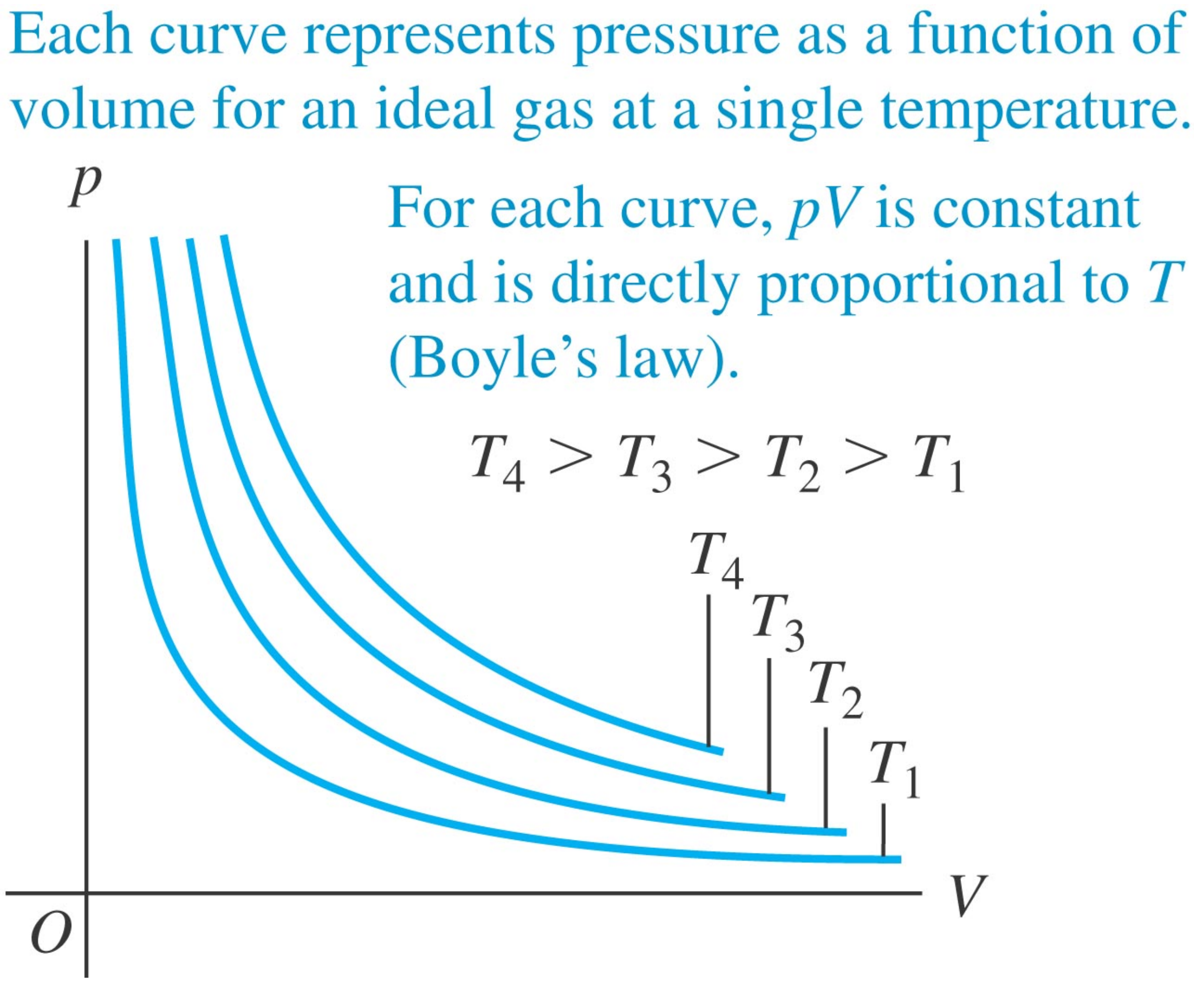
(For a constant amount of ideal gas)



pV diagrams

Different temperatures give us different curves.

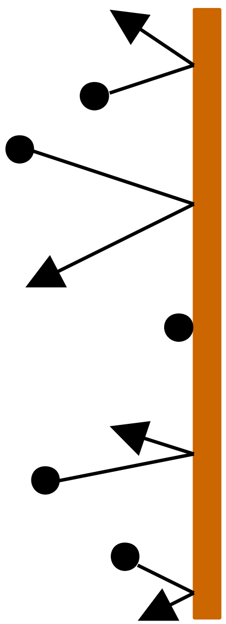
Each curve is an *isotherm*.



Kinetic-molecular model

Pressure *P* (Pa)

Impact of a molecule on the wall of the container exerts a force on the wall and the wall exerts a force on the molecule. Many impacts occur each second and the total average force per unit area is called the **pressure**.



$P = F / A$

force F (N)
area A (m^2)
pressure P (Pa)

$P_{atm} = 1.013 \times 10^5$ Pa

~10^{32} molecules strike our skin every day with v_{avg} ~1700 km/s

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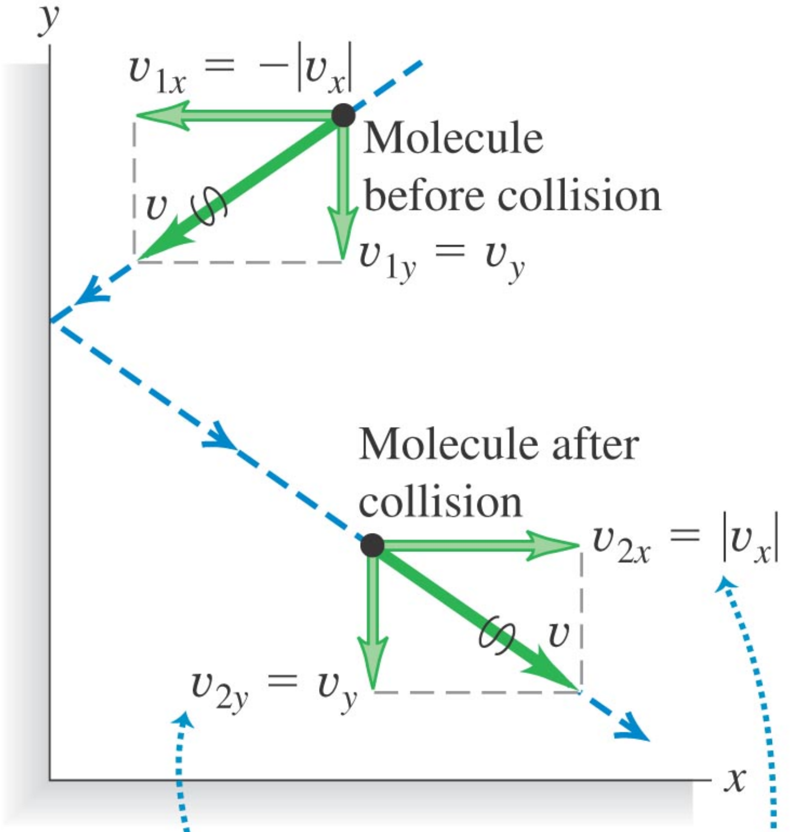
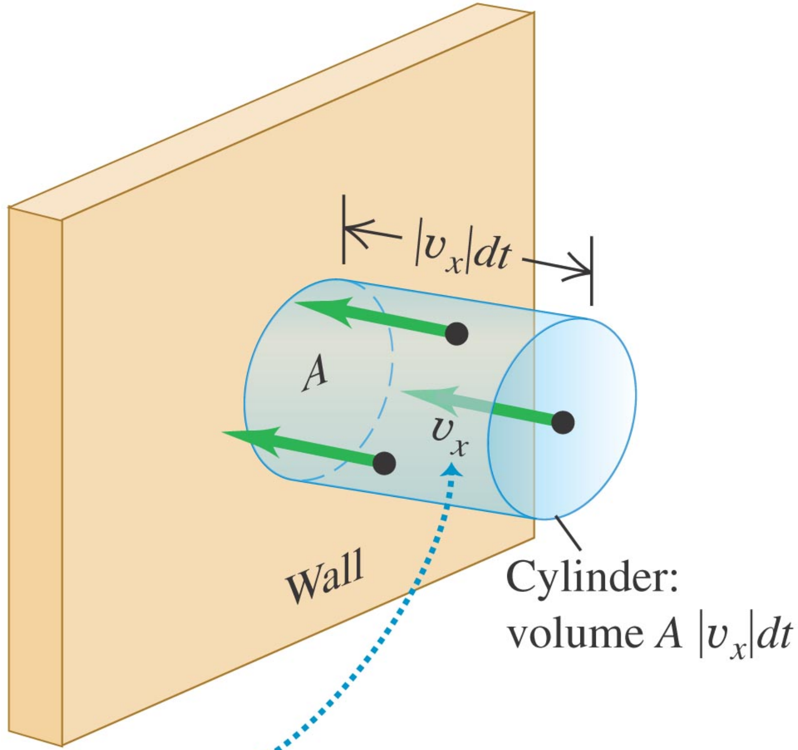
Kinetic-molecular model



- Velocity component parallel to the wall (y-component) does not change.
- Velocity component perpendicular to the wall (x-component) reverses direction.
- Speed v does not change.



All molecules are assumed to have the same magnitude $|v_x|$ of x-velocity.

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Kinetic-molecular model

Find

$$p = \frac{F}{A} = \frac{Nmv_x{}^2}{V}$$

i.e. the pressure depends on the number of molecules per volume, the mass per molecule and the speed of the molecules.

Also:

$$K_{tr} = N\left(\frac{1}{2}m\left(v^2\right)_{av}\right)$$

and so

$$p\,V = \frac{2}{3}K_{tr}$$

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Kinetic-molecular model

- Experimental law: $pV = nRT = NkT$

- Kinetic-Molecular Model (Theory) $pV = \frac{2}{3} KE_{tr}$

For the two equations to agree, we must have:

$$\boxed{KE_{tr} = \frac{3}{2} nRT = \frac{3}{2} NkT}$$

For an ideal gas, temperature is a direct measure of the average kinetic energy of its molecules

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Molecular speeds

We can now write an expression for the average speed of a molecule:

$$v_{\text{rms}} = \sqrt{(v^2)_{\text{av}}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}}$$

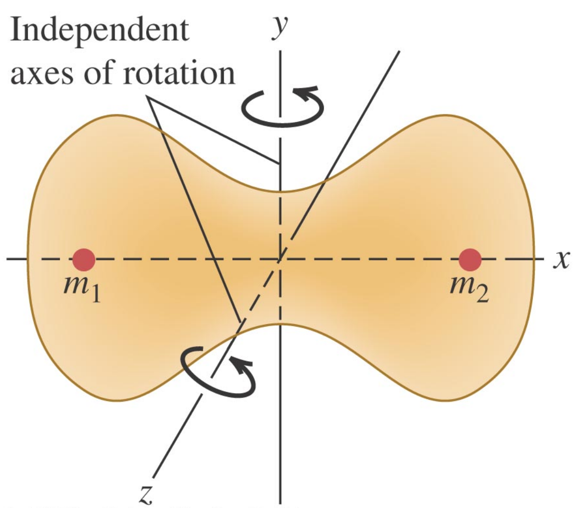
the root-mean-square speed of a gas molecule.

Note that molecules of different mass *m* will have the same average KE but different speeds.

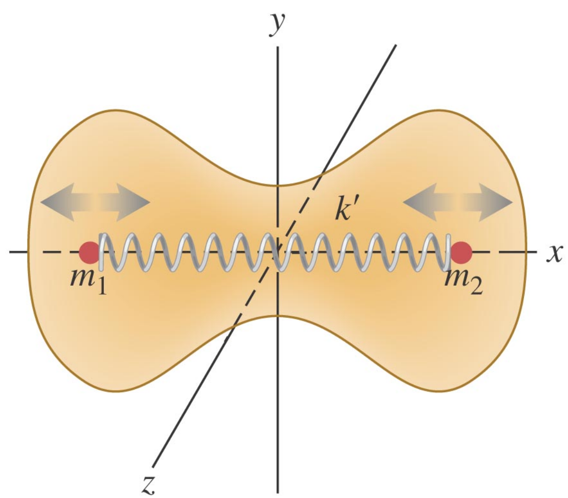
Equipartition of energy

Translation is not the only sort of motion a gas particle can have. In the case of a diatomic molecule, it can also have *rotational* motion and *vibrational* motion.

(b) **Rotational motion.** The molecule rotates about its center of mass. This molecule has two independent axes of rotation.



(c) **Vibrational motion.** The molecule oscillates as though the nuclei were connected by a spring.



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Equipartition of energy

So we can say

– a monatomic gas has 3 degrees of freedom

– a diatomic gas has 5 degrees of freedom

(the number of velocity components needed to describe the motion of molecule completely).

Each degree of freedom has, on average, an associated kinetic energy per molecule of $\frac{1}{2} kT$

- For a monatomic gas, average KE of a molecule is: $3/2kT$
- For a diatomic gas, average KE of a molecule is: $5/2kT$

YF §18.4

Next lecture

The first law of thermodynamics

Read: YF §19.1