Flask with water

A 1.000 L glass flask is filled to the brim with water at 10 °C. The flask and water are then heated to 30 °C. How much water spills out of the flask?
$\beta_{water} = 0.207 \times 10^{-3}$ K^{-1} and $\alpha_{glass} = 3.25 \times 10^{-6}$ K^{-1}.

Solution

The volume expansion coefficient for the glass is
$$\beta_{glass} = 3\alpha_{glass} = 9.75 \times 10^{-6} \text{ K}^{-1}$$
Both the water and the glass will expand, but the expansion coefficient for water is much larger, so the water will expand more and will spill.
The volume that spills over will be
$$\begin{aligned} V_{spill} &= \Delta V_{water} - \Delta V_{glass} \\ &= \beta_{water} V_0 \Delta T - \beta_{glass} V_0 \Delta T \\ &= (\beta_{water} - \beta_{glass}) V_0 \Delta T \\ &= (0.207 \times 10^{-3} - 3.25 \times 10^{-6})(1.000)(20) \\ &= (1.97 \times 10^{-4})(20) \\ &= 3.95 \times 10^{-3} \text{ L} = 3.0 \text{ mL} \end{aligned}$$
Note that, just like the ring, the flask expands in all directions, even though there's a hole.