Steam condensing

What mass of steam at 130 °C must be condensed onto a 0.100 kg glass cup to warm the cup and the 0.200 kg of water it contains from 20.0 °C to 50.0 °C?

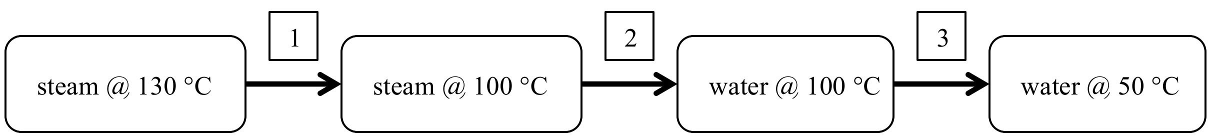
Data:

specific heat capacity of steam $c_s = 2.01 \times 10^3$ J.kg^{-1}.K^{-1}
specific heat capacity of water $c_w = 4.19 \times 10^3$ J.kg^{-1}.K^{-1}
specific heat capacity of glass $c_g = 8.37 \times 10^2$ J.kg^{-1}.K^{-1}
latent heat of vaporisation of water $L = 2.26 \times 10^6$ J.kg^{-1}

Solution

The final state will have all the components in thermal equilibrium, so they will all reach a temperature of 50 °C.
The steam has three separate heat transfers:

- first the steam cools from 130 °C to 100 °C
- then the steam at 100 °C condenses into water at 100 °C
- then the water at 100 °C cools to 50 °C



Taking each step in turn, where the unknown mass of steam is m_s:

1. Steam cools: $Q_1 = m_s c_s \Delta T = m_s(4.19\times10^3)(30) = 6.03\times10^4 m_s$
2. Steam condenses: $Q_2 = m_s L_V = 2.26\times10^6 m_s$
3. Water cools: $Q_1 = m_s c_w \Delta T = m_s(4.19\times10^3)(50) = 2.09\times10^5 m_s$

So the total heat lost by the steam is

$$Q_{\text{lost}} = Q_1 + Q_2 + Q_3 = 6.03\times10^4 m_s + 2.26\times10^6 m_s + 2.09\times10^5 m_s = 2.53\times10^6 m_s$$

The heat gained by the water and glass is

$$\begin{aligned} Q_{\text{gained}} &= (mc\Delta T)_w + (mc\Delta T)_g \\ &= (0.2)(4.19\times10^3)(30) + (0.1)(8.37\times10^2)(30) \\ &= 2.77\times10^4 \text{ J} \end{aligned}$$

From conservation of energy, $Q_{\text{lost}} = Q_{\text{gained}}$, so

$$2.53\times10^6 m_s = 2.77\times10^4 \text{ J}$$

and hence

$$m_s = \frac{2.77\times10^4}{2.53\times10^6} = 1.09\times10^{-2} \text{ kg} = 11 \text{ g}$$