Gas in a piston

A cylinder with a movable piston contains 0.25 mole of monatomic idea gas at pressure 2.40×10^5 Pa and temperature 355 K. The ideal gas first expands isobarically to twice its original volume. It is then compressed adiabatically back to its original volume, and finally it is cooled isochorically to its original pressure.

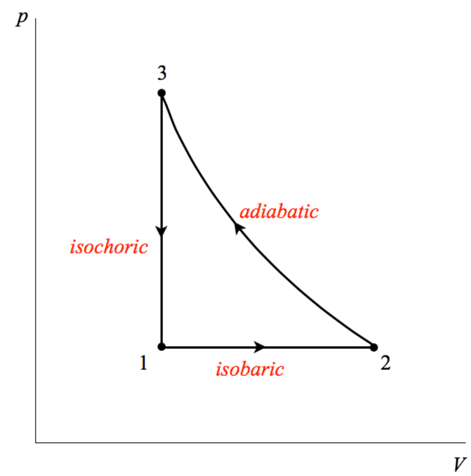
a) Compute the temperature after the adiabatic compression.

b) Compute the total work done by gas on the piston during the whole process.

Take $C_V = 12.47\ \mathrm{J.mol^{-1}.K^{-1}}, \gamma = 1.67$

Solution

Draw the pV diagram for this process:



We will solve this problem by finding T_1 and T_2 in turn, by looking at the individual processes.

Process 1–2: We know $p_2 = p_1$ (isobaric) and $V_2 = 2V_1$, so

$$p_1 = \frac{nRT_1}{V_1} = p_2 = \frac{nRT_2}{V_2}$$

Hence

$$T_2 = \left(\frac{V_2}{V_1}\right)T_1 = 2T_1 = 2\times355 = 710\ \mathrm{K}.$$

The work done is

$$W_{1\to2} = p\Delta V = p(V_2 - V_1) = pV_1$$

and so, since $pV_1 = nRT_1$, we have

$$W_{1\to2} = nRT_1 = (0.25)(8.314)(355) = 738\ \mathrm{J}.$$

Process 2–3: is adiabatic, so $TV^{\gamma-1}$ is constant. So

$$T_2V_2^{\gamma-1} = T_3V_3^{\gamma-1}$$

and $T_2 = 2T_1, V_2 = 2V_1, V_3 = V_1$, so

$$(2T_1)(2V_1)^{\gamma-1} = T_3(V_1)^{\gamma-1}$$

$$T_3 = \frac{(2T_1)(2V_1)^{\gamma-1}}{(V_1)^{\gamma-1}} = 2^{\gamma}T_1 = 2^{1.67}\times355 = 1130\ \mathrm{K}.$$

Since the process is adiabatic, $Q = 0$ and so $W = -\Delta U$, so

$$W_{2\to3} = -nC_V\Delta T = -(0.25)(12.47)(1130 - 710) = -1309\ \mathrm{J}.$$

There is no work done in process 3 → 1 (isochoric), so

$$W_{\mathrm{tot}} = W_{1\to2} + W_{2\to3} = 738 - 1309 = -571\ \mathrm{J}$$

so work is done *on* the gas.