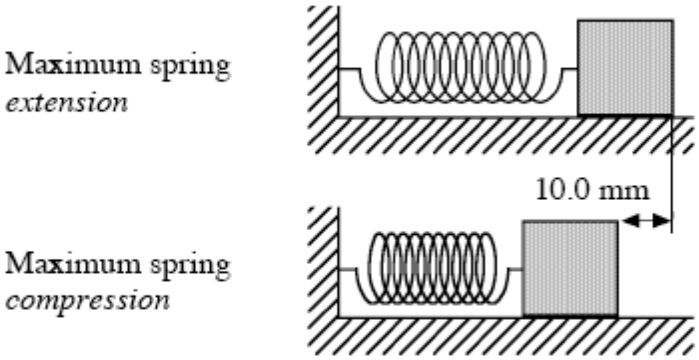
Example: Q11 from 2006 exam



A spring of negligible mass with an unknown force constant k is attached to a block of mass 0.25 kg. The mass is pulled and allowed to oscillate horizontally on a frictionless surface. The frequency of oscillation is 10.0 Hz.

(a) What is the force constant k of the spring?
(b) What is the total mechanical energy of the mass/spring system?
(c) What is the speed of the oscillating mass when it is exactly halfway between the positions of maximum extension and compression?
(d) If the spring is replaced with another with a force constant which is n times larger, what will be the new frequency of oscillation?

Solution:

(a) The frequency of oscillation depends on the spring constant k:

$\omega = 2\pi f = \sqrt{(k/m)}$

so we can use the given frequency of oscillation to determine k:

$k = 4\pi^2 f^2 m$
$= 4\pi^2 \times 10^2 \times 0.250$
$= 987 \text{ N m}^{-1}$

(b) The total energy at maximum extension is all potential:

$E_{tot} = U_{max} = \frac{1}{2} k A^2$
$= \frac{1}{2} \times 987 \times (0.005)^2$
$= 0.0123 \text{ J}$

(c) At the halfway point,

$E_{tot} = K_{max} = \frac{1}{2} m v^2$
so $v = \sqrt{(2\ E_{tot}/m)}$
$= \sqrt{(2 \times 0.0123/0.25)}$
$= 0.314 \text{ ms}^{-1}$

(d) The original spring has frequency

$f_1 = 1/2\pi \sqrt{(k/m)}$

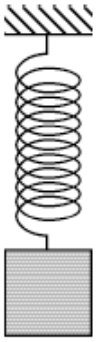
The new spring with spring constant $k_2 = nk$ has frequency

$f_2 = 1/2\pi \sqrt{(nk/m)} = \sqrt{n} \times 1/2\pi \sqrt{(k/m)}$
$= \sqrt{n}\ f_1$

so the new spring oscillates with frequency $\sqrt{n}$ times larger than the old spring.

Question, continued:

Now the mass is allowed to hang vertically (using the *original* spring from part a):



(e) What effect will the force of gravity have on the frequency of oscillation? Explain in 2 or 3 sentences.

Solution:

(e) The frequency of oscillation will not change. Although there are now two forces acting on the mass – the spring force and gravity – the extra force is constant with displacement so the effective spring constant is still the same. The only thing that changes is the position of equilibrium.