

Sample exam question

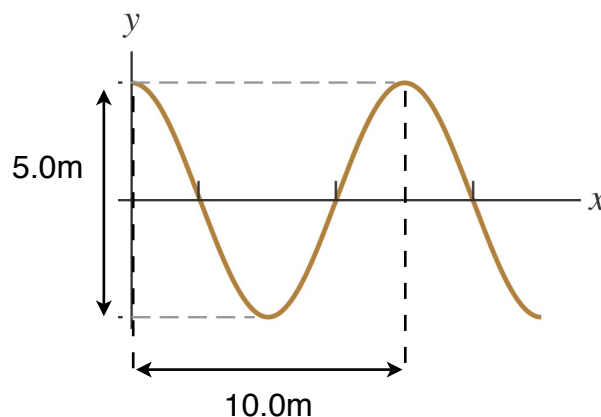
(similar to Q5 from 2009 exam)

A swimmer is 100 metres offshore from Bondi and notices that the ocean is well described by sinusoidal periodic waves. The waves are not crashing and there is no current. The swimmer is vertically displaced a total of 5.0 metres between a wave crest and a trough. The swimmer returns to the crest of the wave every 45 seconds and see that the crests are uniformly separated by 10.0 metres.

- (a) Write down an expression for the swimmer's vertical displacement as a function of time t and horizontal displacement x .
- (b) At 12:00 pm, the swimmer is at the crest of a wave and starts swimming against the waves. In five minutes, the swimmer has travelled 150 metres. What is the vertical displacement of the swimmer?

Solution:

The vertical distance between crest and trough is 5m so the amplitude is half this, $A = 2.5\text{m}$. The wavelength is the horizontal distance between crests, $\lambda = 10\text{m}$. The time between crests is 45 seconds, so this is the period, $T = 45\text{s}$.



Hence

$$\omega = 2\pi/T = 2\pi/45 = 0.140 \text{ rad s}^{-1}$$

$$k = 2\pi/\lambda = 2\pi/10 = 0.628 \text{ rad m}^{-1}$$

- (a) Hence the wave function is

$$y(x,t) = 2.5 \cos(0.63x \pm 0.14t)$$

- (b) Assume the waves are travelling in the $+x$ direction; then the wave function is

$$y(x,t) = 2.5 \cos(0.63x - 0.14t)$$

Set $t=0$ to be 12:00 pm, when the swimmer (at $x=0$) is at a crest, so the vertical displacement is maximum: $y(0,0) = 2.5\text{m}$. The swimmer travels to $x = -150\text{ m}$ at $t = 300\text{s}$. At this time and position, the vertical displacement is

$$\begin{aligned} y(-150,300) &= 2.5 \cos(0.63x - 0.14t) \\ &= 2.5 \cos(-94.5 - 42.0) \\ &= 2.5 \cos(-136.5 \text{ radians}) \\ &= 2.5 \times -0.159 \\ &= -0.40 \text{ m} \end{aligned}$$