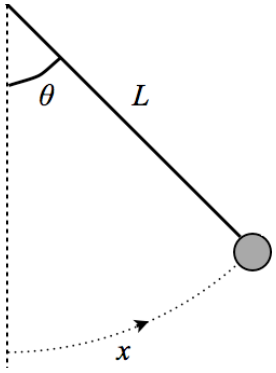
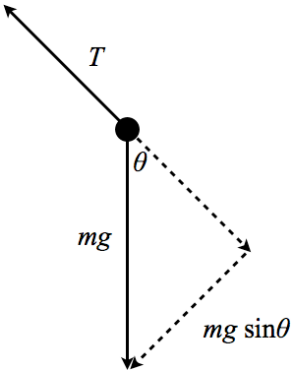
Lecture 3: Simple pendulum

Consider a simple pendulum: a point mass suspended by a massless, unstretched string.



FBD:



Take the length of the arc as our distance x, where $\theta = {}^{x}/_{L}$.
From the FBD, the restoring force is the component of the weight force perpendicular to the tension:

$$F = -mg \sin\theta$$

where the sign is negative because it points opposite to x. This is *not* SHM, since $F \propto \sin x$ instead of $F \propto x$.
However, if θ is small, then $\sin\theta \sim \theta$ (in radians); so then

$$F = -mg\theta = -mg\frac{x}{L}$$

so

$$F = -\frac{mg}{L}x$$

i.e. SHM with $k = {}^{mg}/_{L}$. This will have oscillation frequency

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{mg}{L}.\frac{1}{m}} = \sqrt{\frac{g}{L}}$$

(simple pendulum, small amplitude). The frequency does not depend on m.