Lecture 3

Pendulums

and

Resonance

Pre-reading: §14.5–14.8

SHM: Pendulums

- Simple pendulum: point mass on a massless, unstretched string
- Exhibits SHM with
  $$\omega = \sqrt{(g/L)}$$
  provided the amplitude (angle) is small ($\theta \lesssim 15°$)
- For **real** pendulum, need to know **mass** (m), distance to **center of mass** d, and **moment of inertia** I about rotation axis
  $$\omega = \sqrt{(mgd/I)}$$
  provided the amplitude (angle) is small ($\theta \lesssim 15°$)

§14.5–14.6

Damped Oscillations

- In real world, friction causes oscillations to decrease in amplitude
- If friction force varies linearly with speed, we can solve for motion:

$$x = Ae^{-(b/2m)t}\cos(\omega' t + \phi)$$

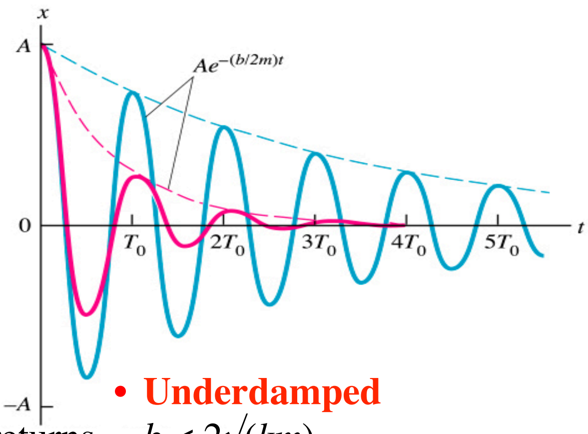
$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

with b describing the amount of damping

§14.7

Effects of damping

- Amplitude decays exponentially.
- Energy decays exponentially.
- Angular frequency decreases.



- **Critical damping**
  if $b=2\sqrt{(km)} \Rightarrow \omega'=0$ returns to equilibrium without oscillating!
- **Underdamped**
  $b < 2\sqrt{(km)}$
- **Overdamped**
  $b > 2\sqrt{(km)}$

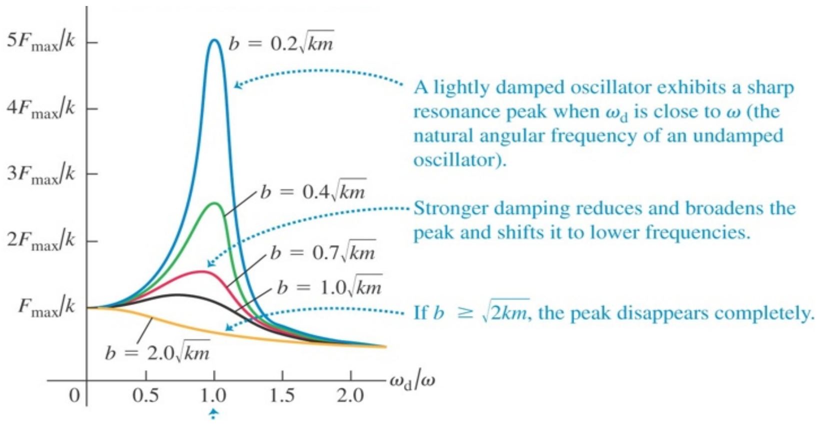
Forced Oscillations

- Now we add a **driving force** to a damped harmonic oscillator
- Suppose driving force is sinusoidal with driving frequency ω_d
- Compare driving frequency with 'natural frequency'

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

- If $\omega_d \simeq \omega'$ then system oscillates with **resonant** behaviour: amplitude gets very large

§14.8



Next lecture

Mechanical waves

Read §15.1–15.2