Lecture 5

The wave equation

Pre-reading: §15.3–15.4

General Properties of Mechanical Waves

- Need to distinguish **medium** from **particles**
- shape of pattern (pulse, continuous, standing wave)
- speed of wave (or pattern)
- energy transmitted (related to amplitude)
- number of dimensions (rope; pond; speakers)

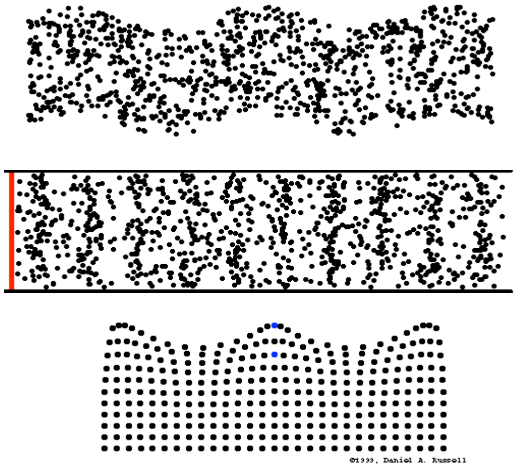
§15.2

Mechanical Waves

Transverse

Longitudinal

Trans. & Long.



Figures courtesy D. Russell

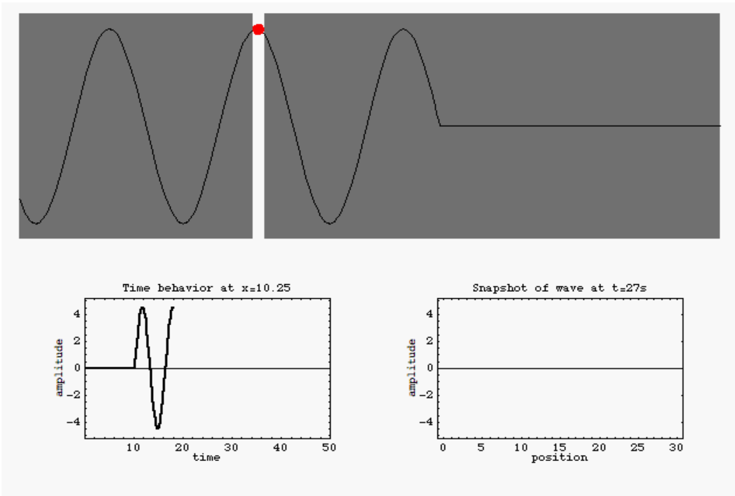
§15.1

Periodic Waves

- Created by continuous, sinusoidal pulses
- restoring force could be tension, pressure, etc.
- Characterised by
  - wavelength (λ) or angular wavenumber (k)
  - period (T) or frequency (f) or ang. freq. (ω)
- Speed of wave pattern is $v = f\lambda = \omega/k$
- We want an expression for how displacement varies in space (x) **and** time (t)

§15.3

Wave Function



Sample Exam Question

A swimmer is 100 meters offshore from Bondi and notices the ocean is well described by sinusoidal periodic waves. The waves are not crashing, and there is no current. The swimmer is vertically displaced a total of 5.0 meters between a wave crest and trough. The swimmer returns to the crest of a wave every 45 seconds and sees that the crests are uniformly separated by 10.0 meters.

a) Write down an expression for the swimmer's vertical displacement as a function of time t and horizontal displacement x.

b) At 12:00pm, the swimmer is at a crest of a wave and starts swimming against the waves. In five minutes, the swimmer has travelled 150 meters. What is the vertical displacement of the swimmer?

Wave Function and Wave Equation

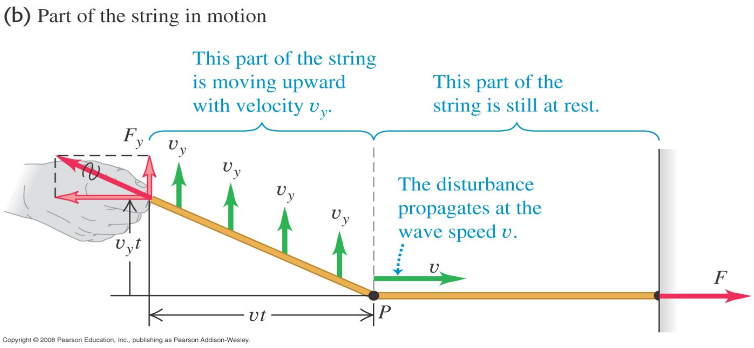
- Wave function gives displacement as function of space and time
- 1-D periodic wave: $y(x,t) = A\cos(\omega t \pm kx)$
- Wave equation relates changes in wave shape to its speed
- Wave equation is true statement for all waves

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2}$$

§15.3

Speed of Mechanical Waves

- To find v, consider the forces, use Newton's 2nd law, calculate derivatives (complicated!)



§15.4

Speed of Mechanical Waves

- To find v, consider the forces, use Newton's 2nd law, calculate derivatives (complicated!)
- From wave eqn:
  $v \approx \sqrt{}$(Acceleration / Curvature)
- Another way:
  $v \approx \sqrt{}$(Restoring force / Inertia)
- 1-D transverse wave on string: $v = \sqrt{(F/\mu)}$
- Longitudinal wave in fluid: $v = \sqrt{(B/\rho)}$
- Sound wave in a gas: $v = \sqrt{(\gamma RT/\mathrm{M})}$

§15.4, 16.2

Next lecture

Interference and superposition

Read §15.6