Lecture 6

Interference
and
Superposition

Pre-reading: §15.6

Please take a clicker

Wave Function and Wave Equation

- Wave function gives displacement as function of space and time
- 1-D periodic wave: $y(x,t) = A\cos(\omega t \pm kx)$
- Wave equation relates changes in wave shape to its speed
- Wave equation is true statement for all waves

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2}$$

§15.3

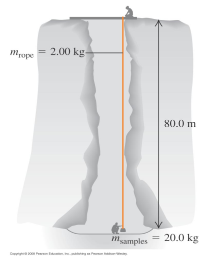
Speed of Mechanical Waves

- To find v, consider the forces, use Newton's 2nd law, calculate derivatives (complicated!)
- From wave eqn:
  $v \approx \sqrt{(\text{Acceleration / Curvature})}$
- Another way:
  $v \approx \sqrt{(\text{Restoring force / Inertia})}$
- 1-D transverse wave on string: $v = \sqrt{(F/\mu)}$
- Longitudinal wave in fluid: $v = \sqrt{(B/\rho)}$
- Sound wave in a gas: $v = \sqrt{(\gamma RT/M)}$

§15.4, 16.2

Example 15.3

- A box with mass 20.0 kg hangs at end of 80.0 m rope. Rope has mass 2.00 kg. Person at bottom sends a transverse wave to top.
- What is speed of wave?
- If wave is periodic with frequency 2.00 Hz, how many cycles are there in the rope length?



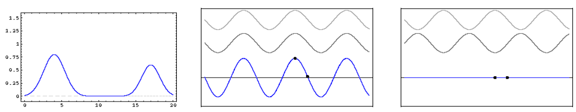
Reflections

- Waves reflect at a boundary
- Hard boundary: wave is inverted
- Soft boundary: wave is NOT inverted

None Hard Soft

Superposition

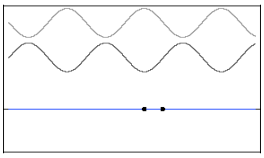
- Waves functions add together linearly, e.g.
  $y(x,t) = y_1(x,t) + y_2(x,t)$
- Waves can go in same or opposite directions
- Interference: Two or more waves pass through same region at same time



6

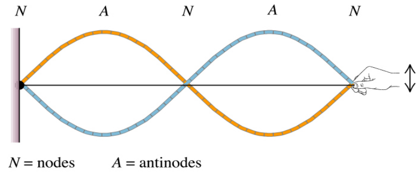
Standing waves

- What happens when two identical waves travelling in opposite directions are superimposed?



Standing Waves

- Formed through reflection + superposition of waves moving in opposite directions
- Contains 'nodes' (no displacement) and 'anti-nodes' (maximum displacement)

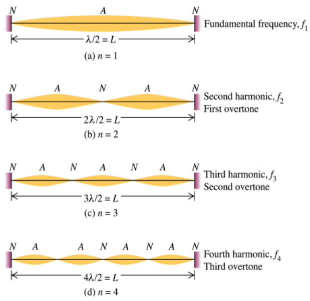


§15.7

Normal modes

- When the string is fixed at *both* ends, only certain standing waves are allowed: **normal modes**
- A pattern with particular λ (or f) is a 'mode'
- Mode with lowest frequency is 'fundamental'; higher frequency modes are 'harmonics' or 'overtones'

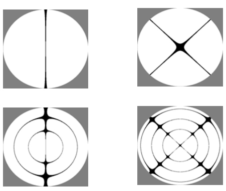
Stretched string



$f_n = n\upsilon/(2L)$, n = 1, 2, 3, …

Chladni plates

- Two dimensional surfaces can vibrate in normal modes



Next lecture

Standing waves
and
Normal modes

Read §15.7–15.8, 16.1