PH625

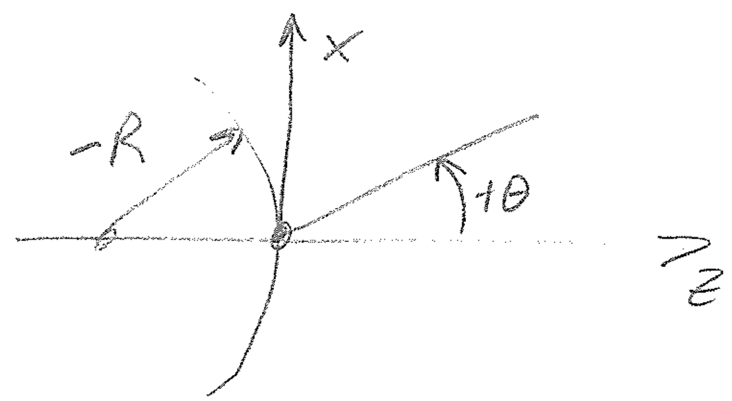
1/27/10 ①

MATRIX OPTICS

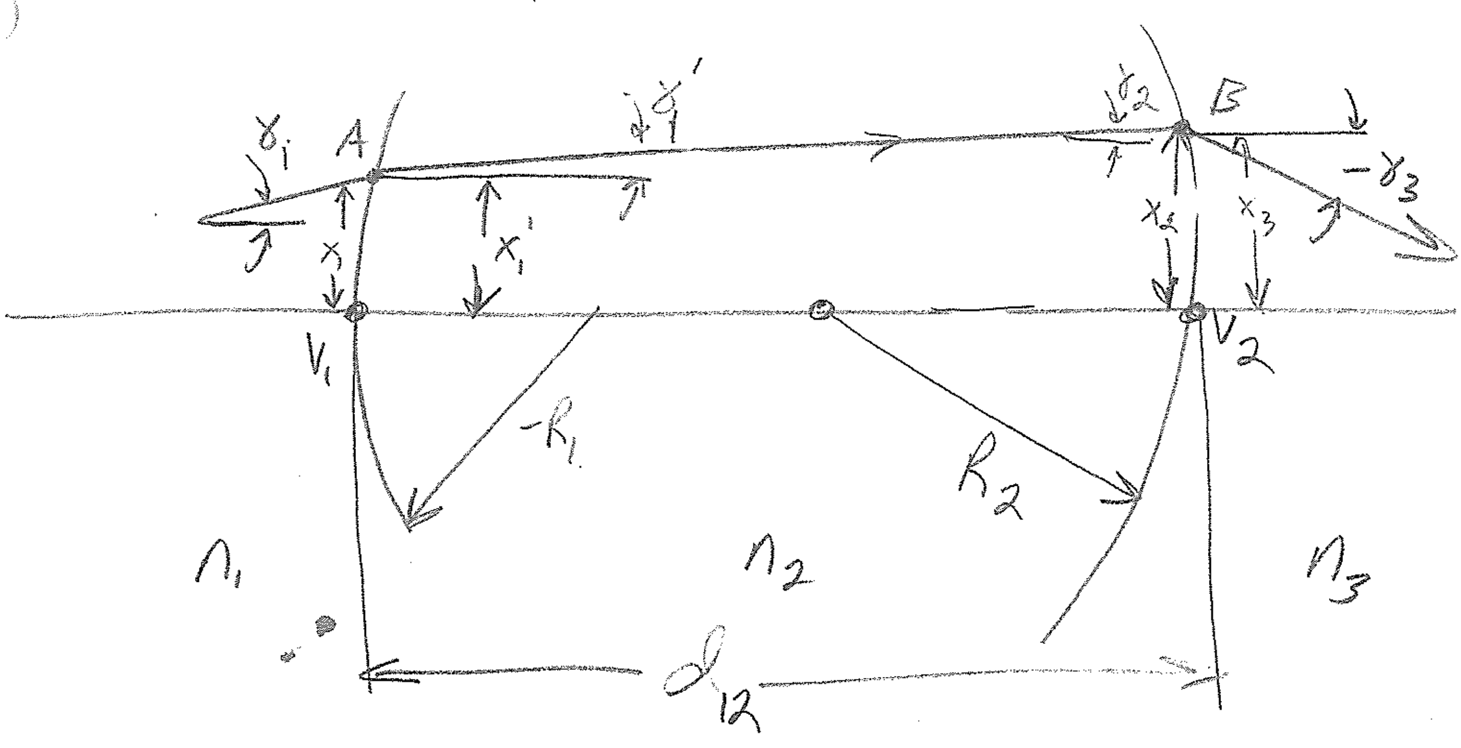
→ TODAY "ABCD" MATRIX

→ FRIDAY "EFGH" MATRIX

Sign CONVENTION:



Consider the case: RAY propagation FROM A TO B, through an optical system



NO Need TO DRAW This!

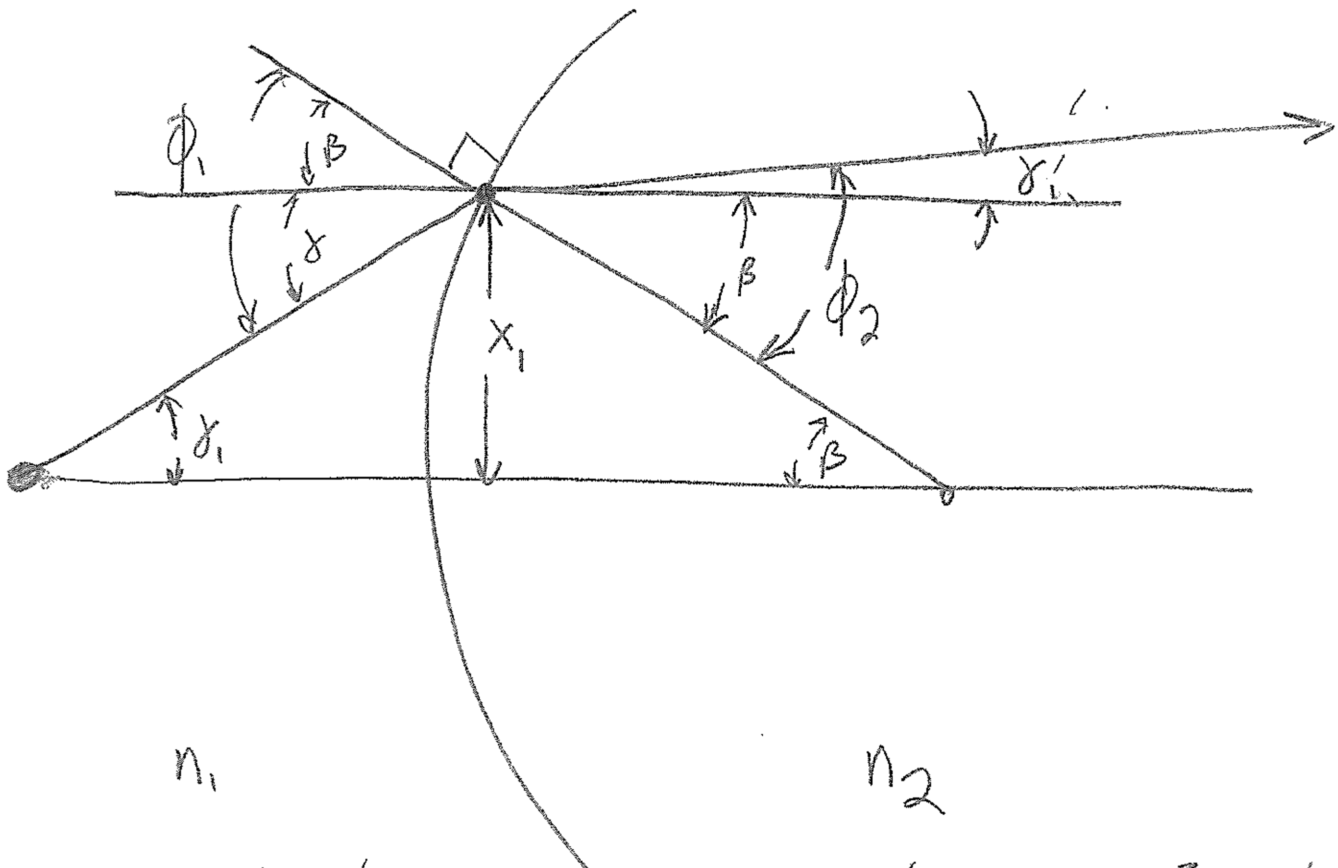
would like to determine:

$$\begin{pmatrix} x_3 \\ \gamma_3 \end{pmatrix} = \begin{pmatrix} \text{given} \end{pmatrix} \begin{pmatrix} x_1 \\ \gamma_1 \end{pmatrix}$$

WHERE x_1 = Ray Height at SURFACE # 1

γ_1 = Ray Angle in medium n_1

START WITH REFRACTION:



SNELL: $n_1 \sin\phi_1 = n_2 \sin\phi_2$

$n_1 \phi_1 = n_2 \phi_2$

$$\sin\phi_1 = \phi_1 - \frac{\phi_1^3}{3!} + \frac{\phi_1^5}{5!}$$

PARAXIAL APPROX = 1ST ORDER

$\sin\phi_1 = \phi_1$

3RD ORDER & 5TH ORDER = ABBERATION THEORY

$$n_1\phi_1 = n_2\phi_2 \Rightarrow n_1(\gamma+\beta) = n_2(\gamma_1'+\beta)$$

$$n_2\gamma_1' = n_1\gamma - (n_2-n_1)\beta$$

But $\beta = \frac{x_1}{R_1}$

$$n_2\gamma_1' = n_1\gamma - \frac{(n_2-n_1)x_1}{R_1}$$

$$\gamma_1' = \frac{n_1}{n_2}\gamma - \frac{(n_2-n_1)}{n_2}\frac{x_1}{R_1}$$

DEFINE $\phi_1 \equiv \frac{n_2-n_1}{R_1}$

= POWER

$= \frac{1}{f}$ FOR THIS LENS

$$\boxed{x_1' = x_1 + 0\gamma}$$

$$\boxed{\gamma_1' = -\frac{\phi_1}{n_2}x_1 + \frac{n_1}{n_2}\gamma}$$

$$\boxed{\begin{pmatrix} x_1' \\ \gamma_1' \end{pmatrix} = R_1 \begin{pmatrix} x_1 \\ \gamma_1 \end{pmatrix} \quad \text{WHERE} \quad R_1 = \begin{pmatrix} 1 & 0 \\ -\frac{\phi_1}{n_2} & \frac{n_1}{n_2} \end{pmatrix}}$$

$$\|R\| = \frac{n_1}{n_2}$$

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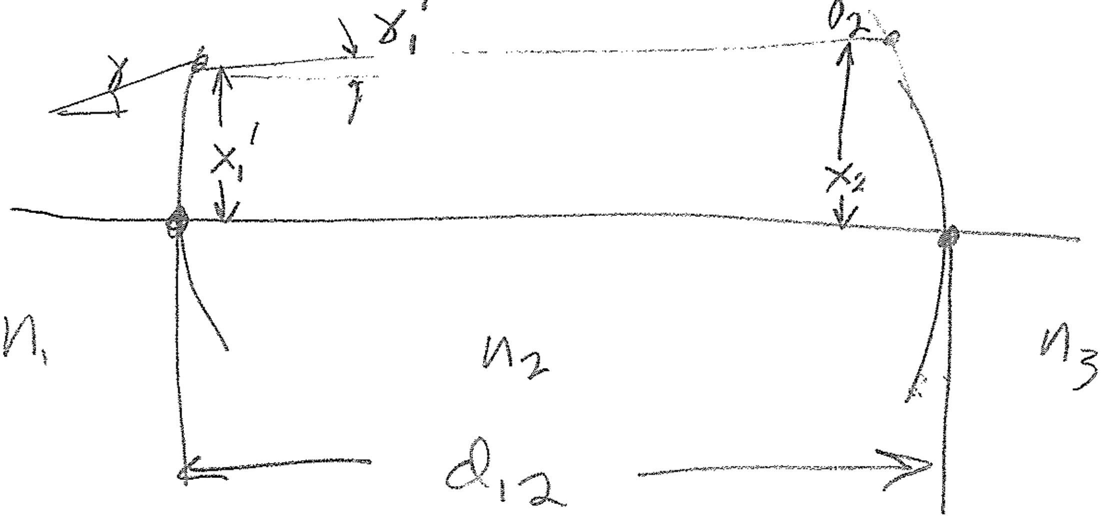

$$x_2 = x_1' + \tan\gamma_2 \cdot d_{12}$$

$$\boxed{x_2 = x_1' + d_{12}\,\gamma_1'}$$

$$\boxed{\gamma_2 = 0\,x_1 + \gamma_1'}$$

$$\begin{pmatrix} x_2 \\ \gamma_2 \end{pmatrix} = T_{12} \begin{pmatrix} x_1' \\ \gamma_1' \end{pmatrix} \quad \text{WHERE} \quad T_{12} = \begin{pmatrix} 1 & d_{12} \\ 0 & 1 \end{pmatrix}$$

$$\| T_{12} \| = 1$$

$$S = R_2 \, T_{12} \, R_1$$

$$= \begin{pmatrix} 1 & 0 \\ -\frac{\phi_2}{n_3} & \frac{n_2}{n_3} \end{pmatrix} \begin{pmatrix} 1 & d_{12} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{\phi_1}{n_2} & \frac{n_1}{n_2} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$A = 1 - \frac{d}{n_2}\phi_1 \qquad B = \frac{d\, n_1}{n_2}$$

$$C = \frac{-1}{n_3}\left(\phi_2 + \phi_1 - \frac{d}{n_2}\phi_1 \phi_2 \right)$$ = SYSTEM POWER

$= \frac{1}{f_{TOTAL}}$

$$D = \frac{n_1}{n_3}\left(1 - \frac{d}{n_2}\phi_2 \right)$$

$$\|S\| = \|R_2\| \cdot \|T_{12}\| \cdot \|R_1\| = AD - BC = \frac{n_1}{n_3}$$

Thick lens in AIR

$n_1 = n_3 = 1$

$n_2 = n$

$$S = \begin{pmatrix} \left(1 - \frac{\phi_1}{n} d\right) & d \\ -\left(\phi_1 + \phi_2 + \frac{d\,\phi_1 \phi_2}{n}\right) & \left(1 - \frac{\phi_2}{n} d\right) \end{pmatrix}$$

$\phi = \frac{(n_2 - n_1)}{R_1}$

Thin lens: special case of thick lens

$d \ll R_1, R_2 \qquad \phi d \ll 1$

$$S = \begin{pmatrix} 1 & 0 \\ -\phi_{TOTAL} & 1 \end{pmatrix}$$

$$\phi_{TOTAL} = \phi_1 + \phi_2 - \frac{\phi_1 \phi_2 d}{n} \simeq \phi_1 + \phi_2$$

$$= \frac{n-1}{R_1} + \frac{1-n}{R_2} = n-1\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\boxed{\phi_{TOT} \simeq \frac{1}{f}}$$

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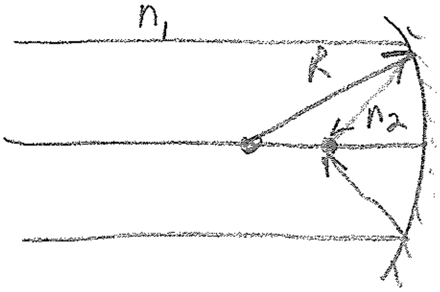
SPHERICAL MIRROR (in AIR)

- TREATED as a Thin lens (mostly)
- AFTER REFLECTION $n \Longrightarrow -n$

$n_2 = -n_1$



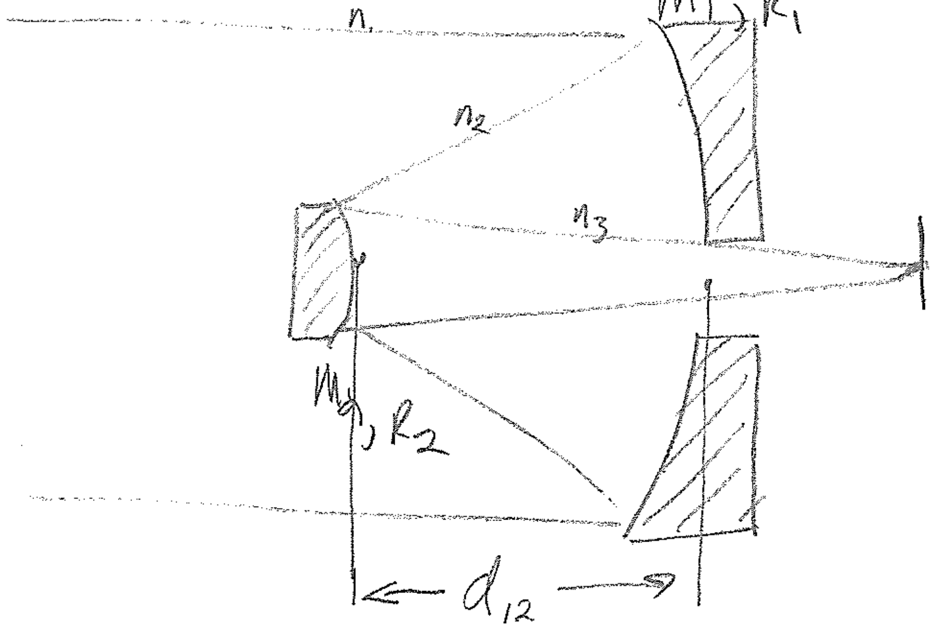
$$\phi \equiv \frac{n_2 - n_1}{R_1}$$

$$\phi_m = \frac{(-1) - 1}{R} = -\frac{2}{R}$$

$$R = \begin{pmatrix} 1 & 0 \\ -\phi & \frac{n_1}{n_2} \end{pmatrix}$$

$$\mathcal{R}_m = M = \begin{pmatrix} 1 & 0 \\ -\frac{2}{R} & -1 \end{pmatrix}$$

$$f = \frac{1}{\phi} = \frac{R}{2}$$



KECK

$R_1 = -34.972$ m

$R_2 = -4.737$ m

$d_{12} = 15.4$ m

Example#1 Sign convention 11/27/10 7

Calculate the effective focal length of the Keck Telescope

$$S_{Keck} = R_2 T_{12} R_1$$

$$= \begin{pmatrix} 1 & 0 \\ \frac{-2}{-4.737} & -1 \end{pmatrix} \begin{pmatrix} 1 & +15.4 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{-2}{-34.972} & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & +15.4 \\ +0.4222 & -7.502 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{-2}{34.972} & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 0.119 & -15.4 \\ 6.821E{-3} & 7.502 \end{pmatrix}$$

$efl = -146.604$ m

$D = 10$-meters

$$F/\# = \frac{efl}{D}$$

$F/\# = 14.6604$

1/27/10 B

Example #1, the Keck Telescope

$$R2 := \begin{pmatrix} 1 & 0 \\ \dfrac{-2}{-4.737} & -1 \end{pmatrix} \qquad R1 := \begin{pmatrix} 1 & 0 \\ \dfrac{-2}{-34.972} & -1 \end{pmatrix} \qquad T12 := \begin{pmatrix} 1 & -15.395 \\ 0 & 1 \end{pmatrix}$$

$$SYS := R2 \cdot T12 \cdot R1$$

$$SYS = \begin{pmatrix} 0.12 & 15.395 \\ -6.7 \times 10^{-3} & 7.5 \end{pmatrix}$$

$$f := \frac{-1}{SYS_{1,0}}$$

$$f = 149.245$$

$$R3 := \begin{pmatrix} 1 & 0 \\ \dfrac{-1}{2.286} & 1 \end{pmatrix} \qquad R4 := \begin{pmatrix} 1 & 0 \\ \dfrac{-1}{0.307} & 1 \end{pmatrix}$$

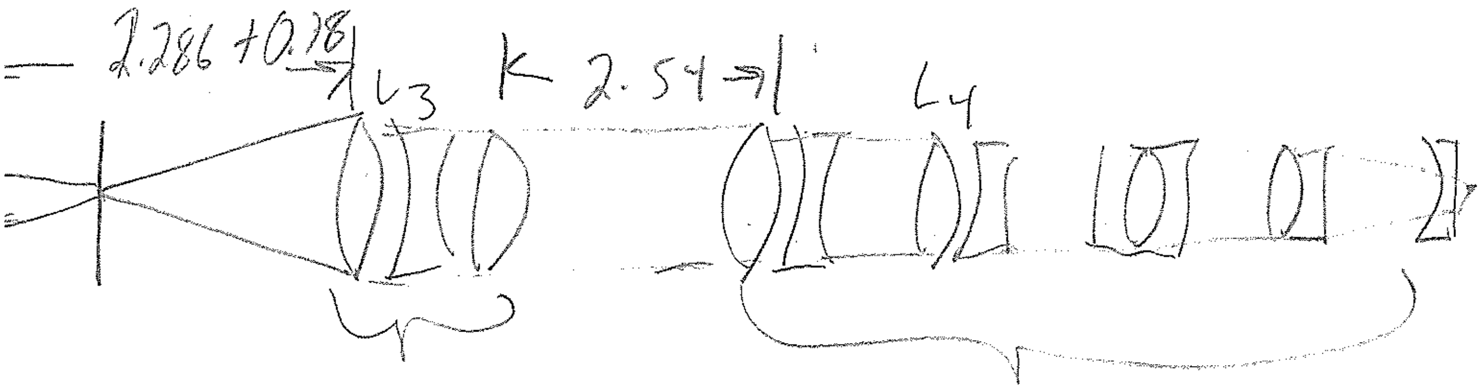
$$T23 := \begin{bmatrix} 1 & (17.8469 + 2.286) \\ 0 & 1 \end{bmatrix} \qquad T34 := \begin{pmatrix} 1 & 2.54 \\ 0 & 1 \end{pmatrix}$$

$$SYS2 := R4 \cdot T34 \cdot R3 \cdot T23 \cdot R2 \cdot T12 \cdot R1$$

$$SYS2 = \begin{pmatrix} -0.015 & 0.562 \\ 0.05 & -67.117 \end{pmatrix} \qquad efl := \frac{-1}{SYS2_{1,0}}$$

$$efl = -20.043$$

Ex #1 B   Keck with Echellette   11/27/10 (8)
Spectrograph; Imager



$efl_{coll} = 2.286\ m \quad efl_{cam} = 0.307\ m$

$$\$ys = \begin{pmatrix} -0.015 & 0.362 \\ -0.049 & -67.117 \end{pmatrix}$$

$efl = 20.043\ m$

$$\boxed{F/\# = 2.0043}$$

Now consider object/image situation 1/29/10

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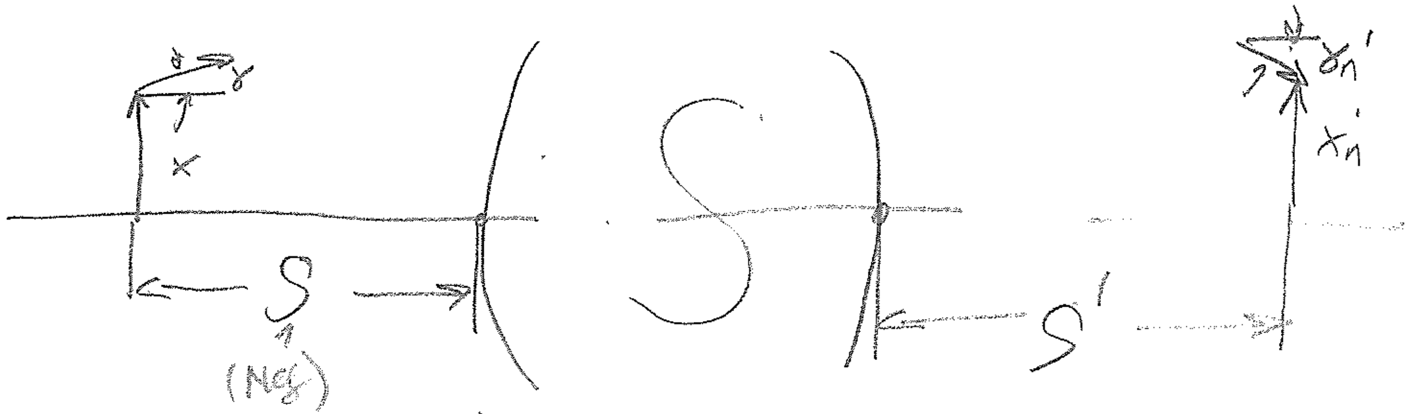


$$\bar{\bar{S}} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$\begin{pmatrix} x_n' \\ \gamma_n' \end{pmatrix} = \begin{pmatrix} 1 & S' \\ 0 & 1 \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} 1 & -S \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ \gamma \end{pmatrix}$$

$$= \begin{pmatrix} A+S'C & B+S'D \\ C & D \end{pmatrix} \begin{pmatrix} 1 & -S \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} A+S'C & -AS-SS'C+(B+S'D) \\ C & D-CS \end{pmatrix}$$

$$\begin{pmatrix} x_n' \\ \gamma_n' \end{pmatrix} = \begin{pmatrix} H & G \\ F & E \end{pmatrix} \begin{pmatrix} x \\ \gamma \end{pmatrix} = \bar{\bar{O}} \begin{pmatrix} x \\ \gamma \end{pmatrix}$$

SPECIAL CASES:

$$x'_n = Hx + G\gamma$$

$$\gamma'_n = Fx + E\gamma$$

1) $\boxed{E = 0} \Rightarrow \boxed{\gamma' \text{ independent of } \gamma}$ $\boxed{\gamma' \sim x}$

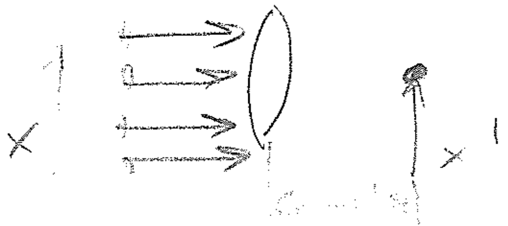
FRONT FOCAL POINT

$$\boxed{s = \frac{+D}{C}}$$

Front focal point

Rays of any angle and fixed height will come out at constant angle and any height

2) $\boxed{H = 0}$ $\boxed{x' \text{ independent of } x}$ $\boxed{x' \sim \gamma}$



Back Focal Point

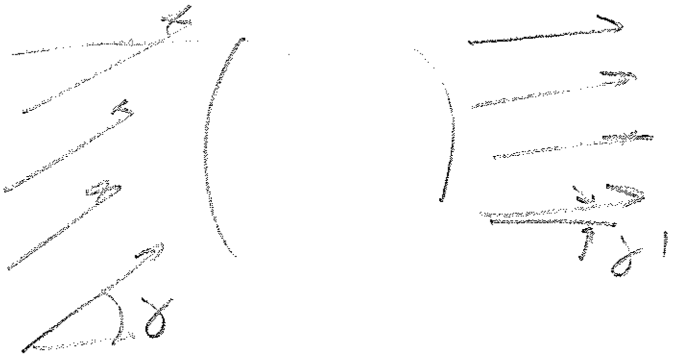
$$\boxed{s' = -\frac{A}{C}}$$

rear focal point

Rays of constant angle and any height will arrive at a fixed height (and any angle)

3) $\boxed{E=0}$ Output angle indep. of Height

DEF OF TELESCOPE



$\frac{\gamma'}{\gamma}$ = Angular Mag.

$$\boxed{\frac{\gamma'}{\gamma} = E}$$

Rays of constant angle and any height will come out at diff constant angle

4) $\boxed{G=0}$

Image Formation

$x'_n = Ex$ Any height

Mag. $= \frac{x_n'}{x} = H$

$$\boxed{s' = \frac{As - B}{D - Cs}}$$

$$\boxed{s = \frac{B + Ds'}{A + Cs'}}$$

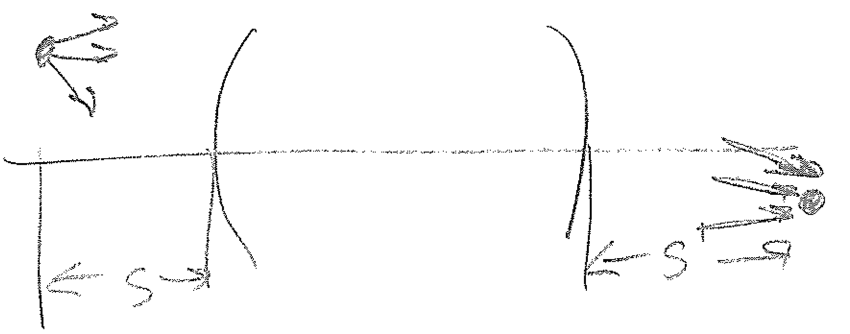
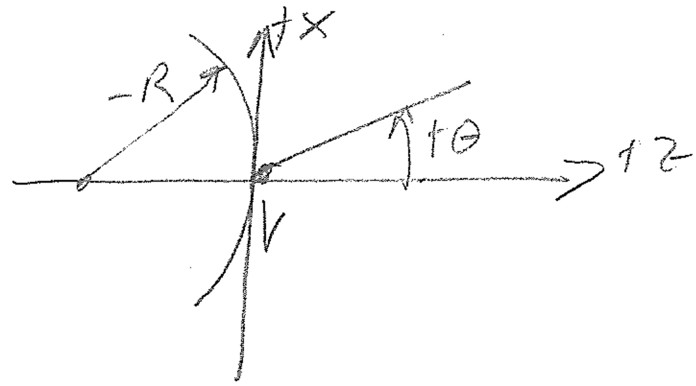


Image condition: Rays emitted from fixed height arrive at fixed height independent of angle.

Sign Convention



1/29/10 (4)

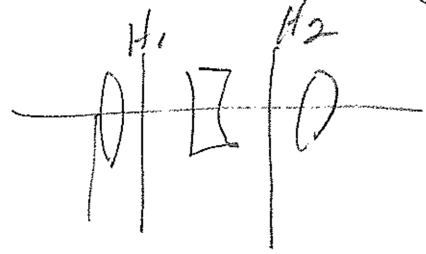


Image Formation $\boxed{G=0}$ $EH=1$

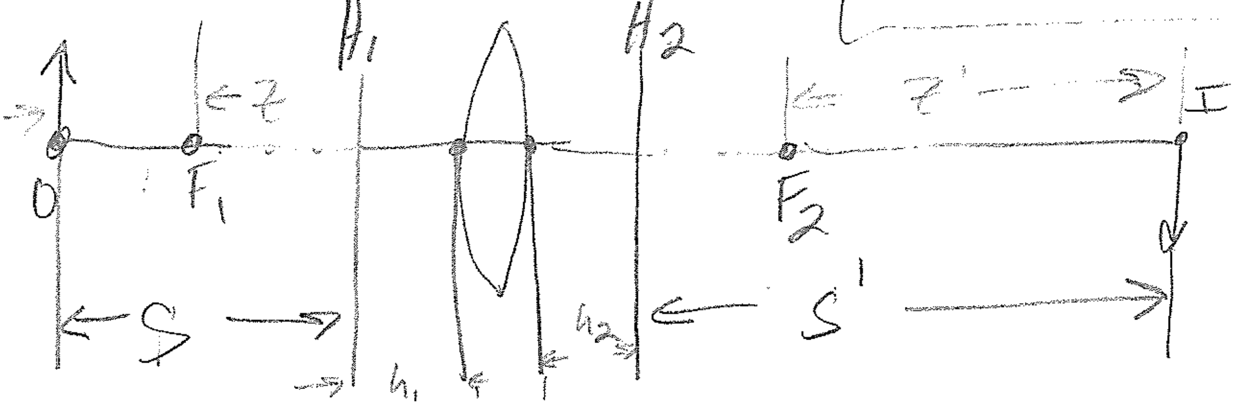
$$M = \begin{pmatrix} H & G \\ F & E \end{pmatrix} \qquad EH - FG = 1$$

$$= \begin{pmatrix} m & 0 \\ -\phi & \frac{1}{m} \end{pmatrix} \qquad E = \frac{1}{H} = \frac{1}{Mag}$$

$$m = \frac{x_1'}{x} = A + Cs' = \frac{1}{D - Cs}$$

UNIT Planes := Principle Planes

DEFINED s.t. $m = +1$



These are the planes that ARE EQUIV. TO VERTICES in THIN lens. i.e. PP ⇒ V EQUIV. lens ⇒ Thin

SET $m=1$ & SOLVE Principle Planes

$$\boxed{h_1 = \frac{D-1}{C}} \qquad \boxed{h_2 = \frac{1-A}{C}}$$

Thin lens:

$$\mathcal{O} = \begin{pmatrix} 1 & s' \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -\phi & 1 \end{pmatrix}\begin{pmatrix} 1 & -s \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1-\phi s' & s' \\ -\phi & 1 \end{pmatrix}\begin{pmatrix} 1 & -s \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1-\phi s' & ss'\phi - s + s' \\ -\phi & 1+\phi s \end{pmatrix}$$

$$\boxed{G=0} \qquad \frac{1}{ss'} \Big\| \; s - s' + ss\phi = 0$$

$$\frac{1}{s'} - \frac{1}{s} = \phi \quad \Rightarrow \boxed{\frac{1}{s'} - \frac{1}{s} = \frac{1}{f}} \checkmark$$

Plug in the power for thick lens

∴ we get lensmakers:

$$\boxed{\frac{1}{s'} - \frac{1}{s} = (n_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

let $z = s - f_1$ $\quad z' = s' - f_2$

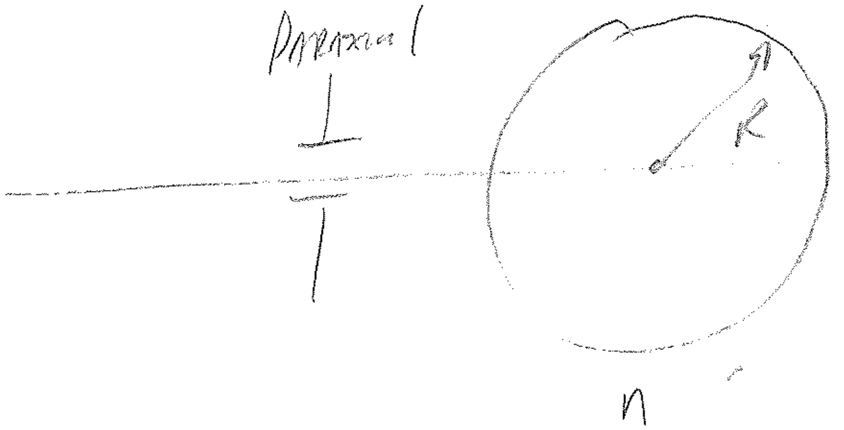
$$\Rightarrow \boxed{zz' = -f_1 f_2}$$

Newton's Form

$\boxed{\text{Ex 2}}$ Glass Ball as thick lens

$n = 1.5 \quad R = 3\,\text{cm}$

$d = 2R$



$$\phi_1 = \frac{n-1}{R} = \frac{1}{6}$$

$(3\,\text{cm})$

$$\phi_2 = \frac{1-n}{R} = \frac{1}{6}$$

$(-3\,\text{cm})$

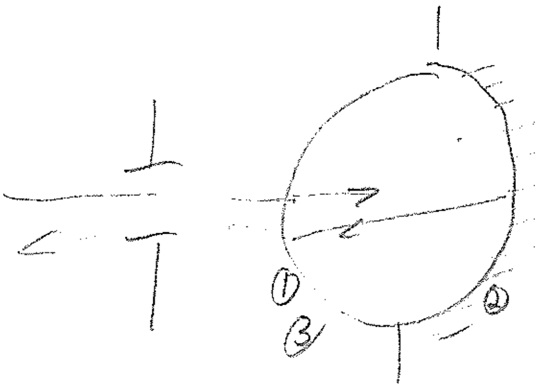
$$S = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$= \begin{pmatrix} 1 - \frac{\phi_1}{n} d & d \\ -\left(\phi_1 + \phi_2 - \frac{D\phi_1\phi_2}{n}\right) & \left(1 - \frac{\phi_2}{n} d\right) \end{pmatrix}$$

Where is Back focal point?

$$S'_{BFL} = \frac{-A}{C} = \frac{1 - \frac{\phi d}{n}}{\phi_1 + \phi_2 - \frac{D\,\phi_1\phi_2}{n}} = 1.5\,\text{cm}$$

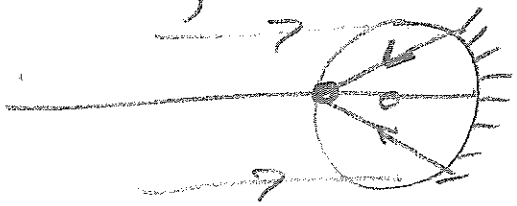
Ex 3 Reflecting glass ball.



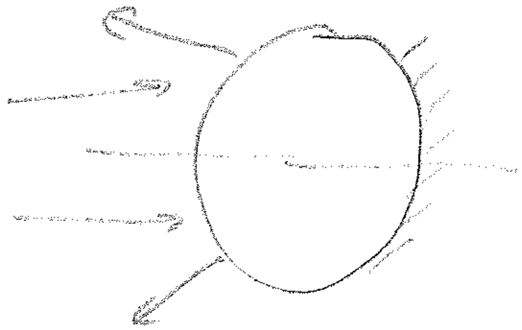
$$S = R_3 T_{23} R_2 T_{12} R_1$$

7/1/10
(3)

2) $n=4$; $A=D=0$, $s=s'=0$

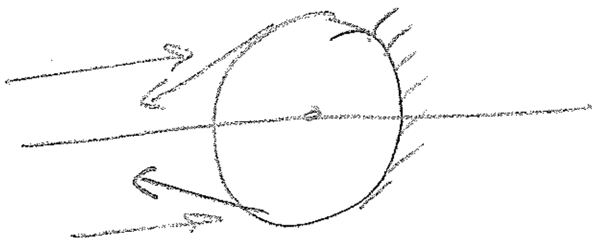


3) $1 < n < 2$ Diverging out



4) $2 < n < 4$

Converging beam



$l = -a$ to 0