



**Physics 1901**

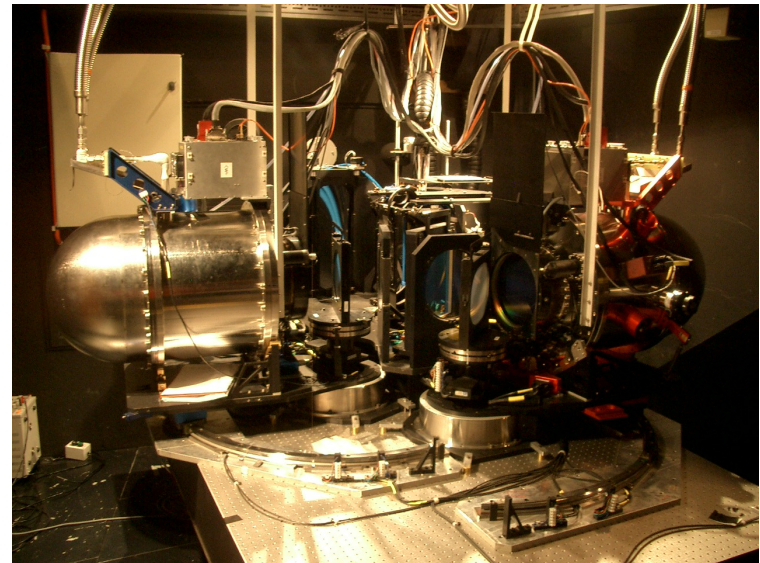
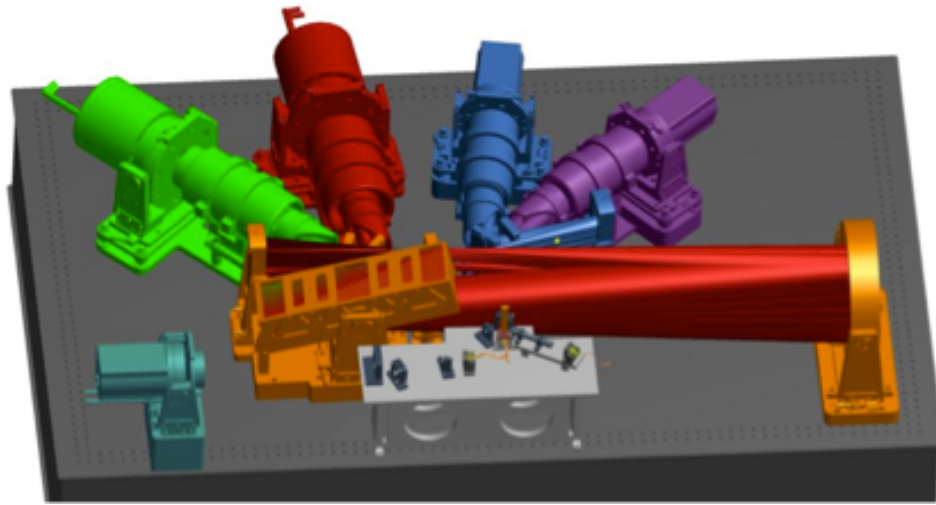
**Experimental Astronomy –  
Graduate Course  
Autumn (Apr-May 2014)**

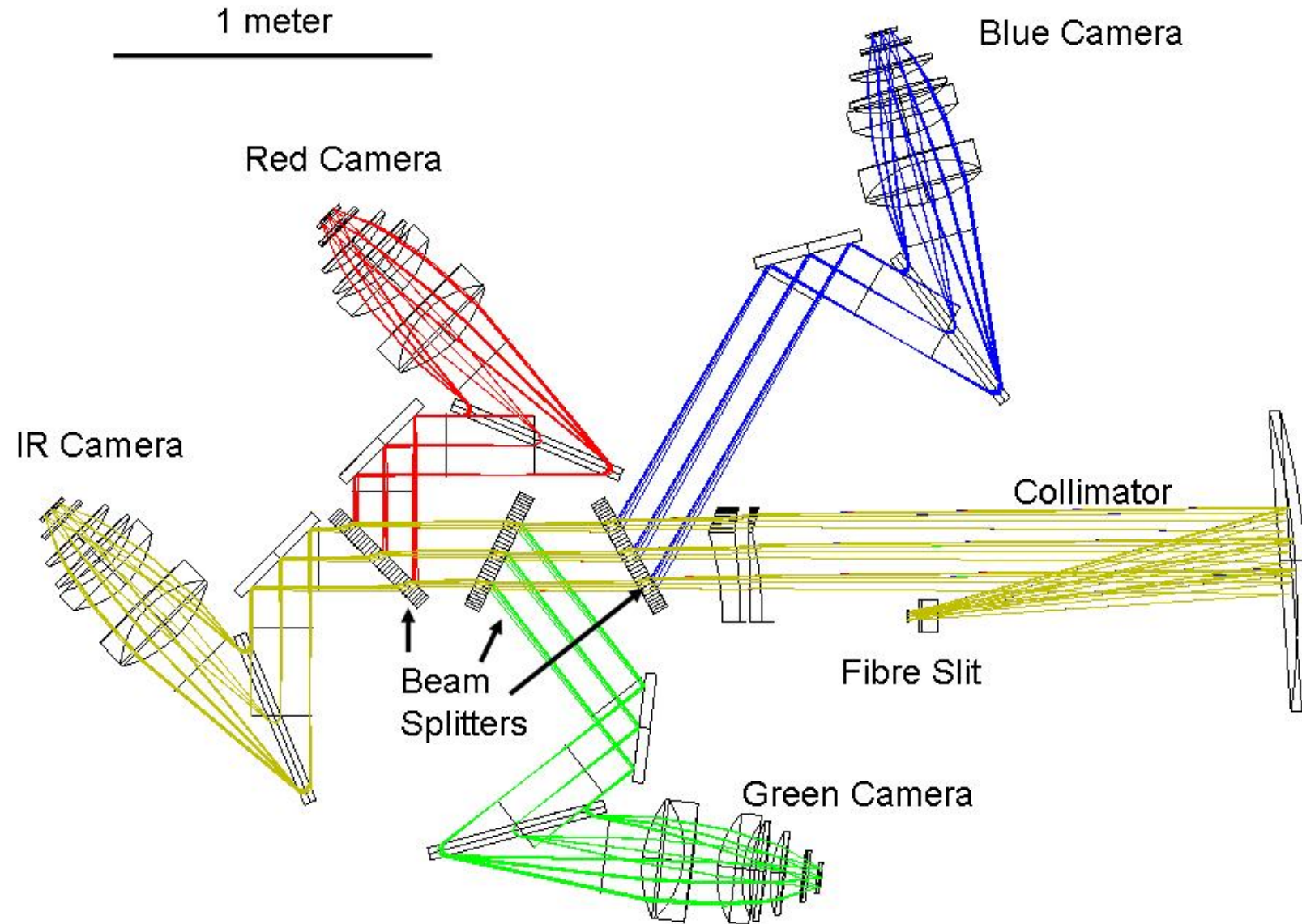
Assoc. Prof. Andrew I. Sheinis ,  
Australian Astronomical Observatory

Prof. Joss Bland-Hawthorn  
Sydney Institute for Astronomy

# Some questions: (which you should be able to answer at the end of the course)

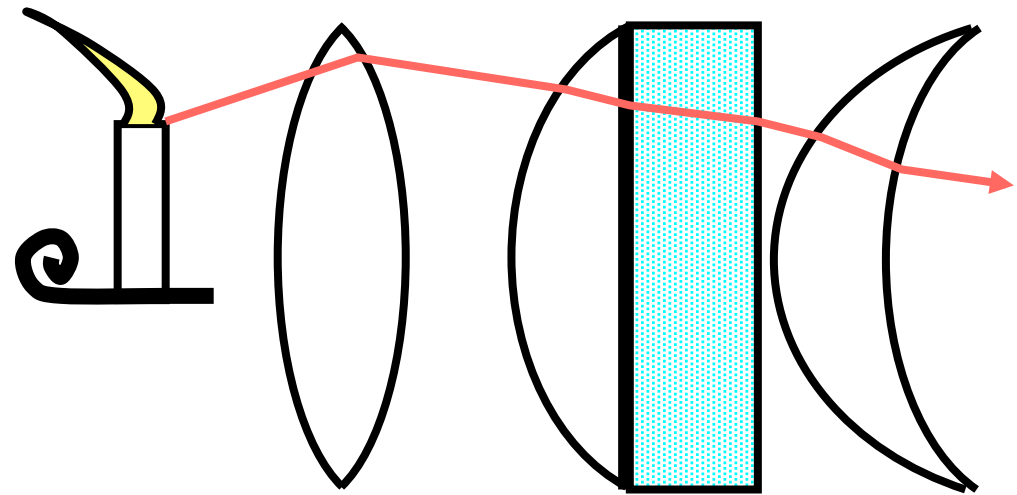
- What are the parts of a spectrograph
- Why are spectrographs so big?
- What sets the sensitivity?
- How do I estimate the exposure time?





# ABCD Matrix Concepts

- Ray Description
  - Position
  - Angle
- Basic Operations
  - Translation
  - Refraction
- Two-Dimensions
  - Extensible to Three



Ray Vector

$$\mathcal{V}_1 = \begin{pmatrix} x_1 \\ \alpha_1 \end{pmatrix}$$

Matrix Operation

$$\mathcal{V}_{out} = \mathcal{M}\mathcal{V}_{in}$$

System Matrix

$$\mathcal{M}_{25} = \mathcal{R}_5 \mathcal{T}_{45} \mathcal{R}_4 \mathcal{T}_{34} \mathcal{R}_3 \mathcal{T}_{23}$$

# Cascading Matrices (1)

Generic Matrix:  $\mathcal{M} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$

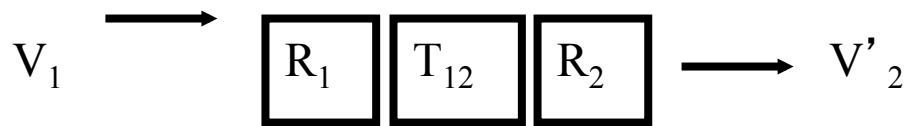
Determinant (You can show that this is true for cascaded matrices)

$$\text{Det } \mathcal{M} = \frac{n}{n'}$$

$$\begin{pmatrix} x'_2 \\ \alpha'_2 \end{pmatrix} = \mathcal{R}_2 \begin{pmatrix} x_2 \\ \alpha_2 \end{pmatrix}$$

$$\begin{pmatrix} x_2 \\ \alpha_2 \end{pmatrix} = \mathcal{T}_{12} \begin{pmatrix} x'_1 \\ \alpha'_1 \end{pmatrix}$$

$$\begin{pmatrix} x'_1 \\ \alpha'_1 \end{pmatrix} = \mathcal{R}_1 \begin{pmatrix} x_1 \\ \alpha_1 \end{pmatrix}$$

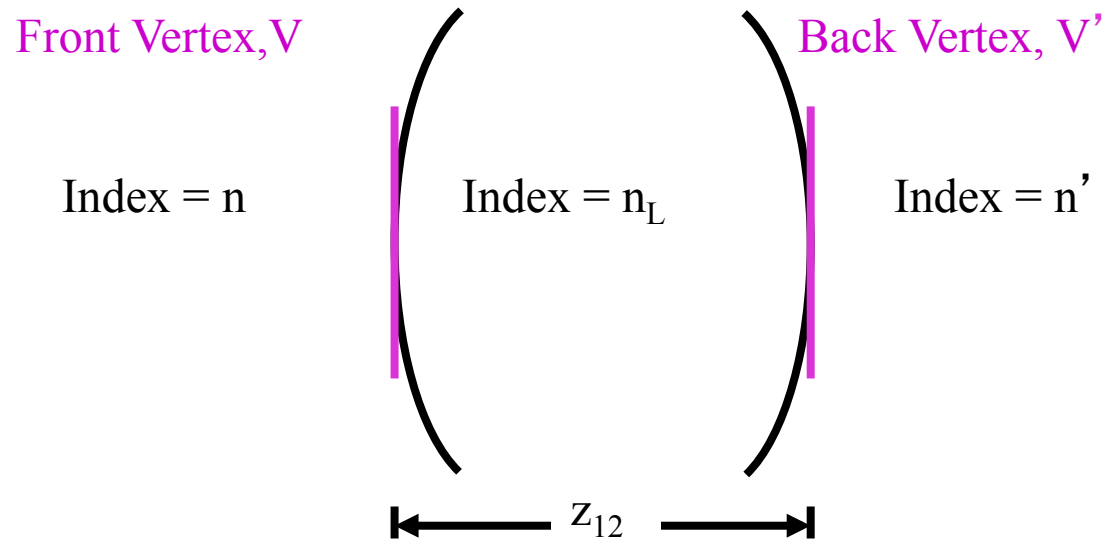


Light Travels Left to Right, but  
Build Matrix from Right to Left

$$\begin{pmatrix} x'_2 \\ \alpha'_2 \end{pmatrix} = \mathcal{R}_2 \mathcal{T}_{12} \mathcal{R}_1 \begin{pmatrix} x_1 \\ \alpha_1 \end{pmatrix}$$

$$\mathcal{M}_{12} = \mathcal{R}_2 \mathcal{T}_{12} \mathcal{R}_1$$

# The Simple Lens (Matrix Way)



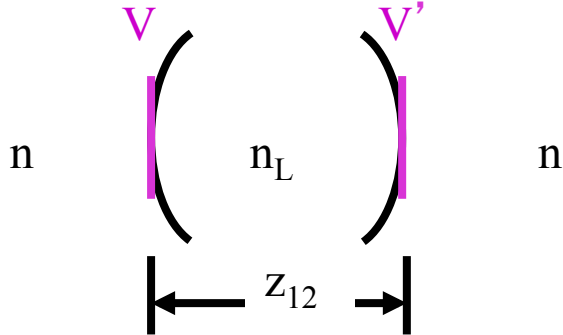
$$\mathcal{M}_{VV'} = \mathcal{R}_2 \mathcal{T}_{12} \mathcal{R}_1$$

# Building The Simple Lens Matrix

$$\mathcal{M}_{VV'} = \mathcal{R}_1 \mathcal{T}_{12} \mathcal{R}_1$$

$$= \begin{pmatrix} 1 & 0 \\ \frac{-P_2}{n'_2} & \frac{n_2}{n'_2} \end{pmatrix} \begin{pmatrix} 1 & z_{12} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{-P_1}{n'_1} & \frac{n_1}{n'_1} \end{pmatrix}$$

$n_1 = n \quad n'_2 = n' \quad n'_1 = n_2 = n_\ell$

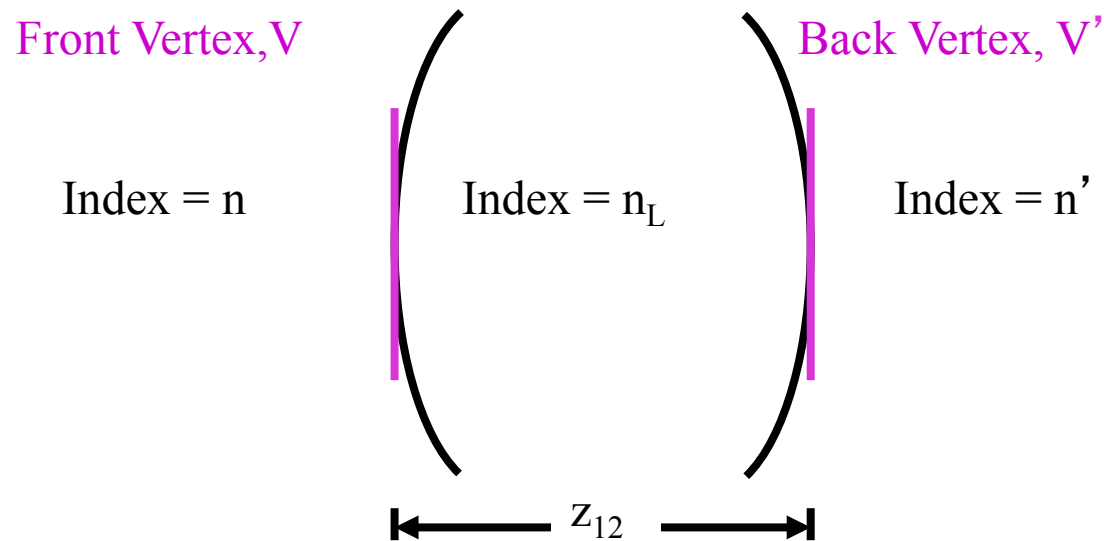


Simple Lens Matrix

$$\begin{pmatrix} 1 & 0 \\ \frac{-P_2}{n'} & \frac{n_\ell}{n'} \end{pmatrix} \begin{pmatrix} 1 & z_{12} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{-P_1}{n_\ell} & \frac{n}{n_\ell} \end{pmatrix}$$

# The Equivalent Lens, Using the System Matrix to get everything you need!

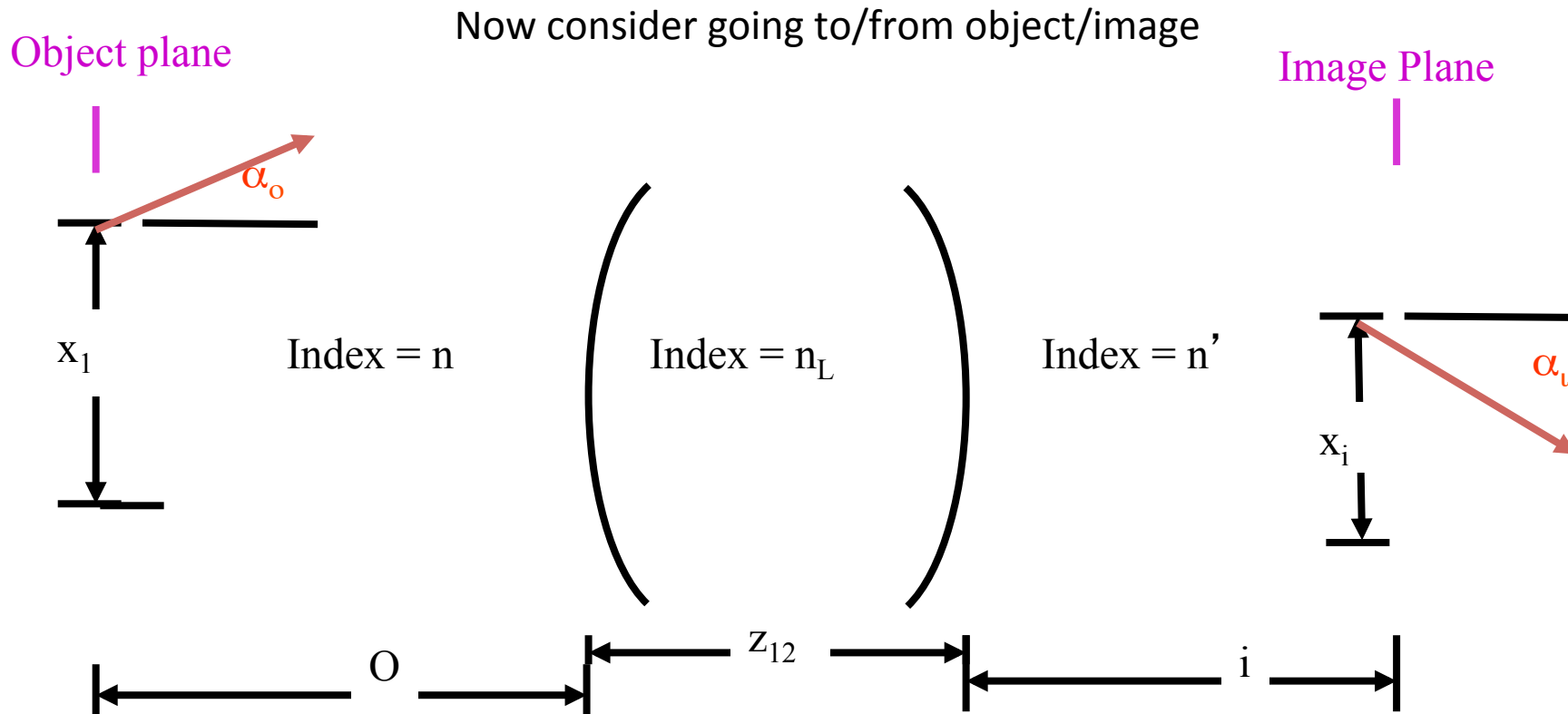
Here is what we did last time for a lens system



$$\mathcal{M}_{VV'} = \mathcal{R}_2 \mathcal{T}_{12} \mathcal{R}_1$$

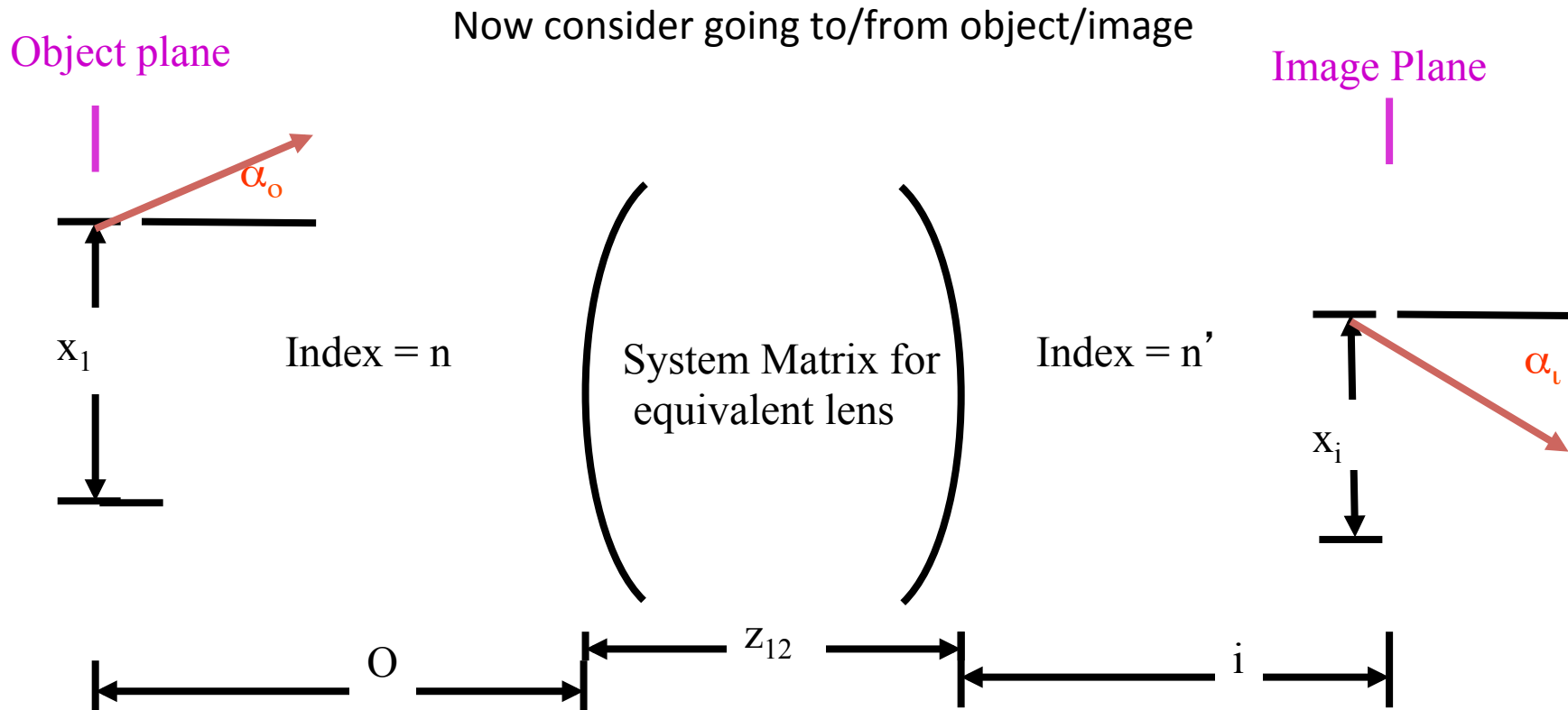


# The Equivalent Lens, Using the System Matrix to get everything you need!



$$M_{oi} = \tau_{32} R_2 \tau_{21} R_1 \tau_1$$

# The Equivalent Lens, Using the System Matrix to get everything you need!



$$M_{oi} = \tau_{32} R_2 \tau_{21} R_1 \tau_1$$

The Equivalent Lens, Using the System Matrix to get everything you need!

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$M_{oi} = \tau_{32} R_2 \tau_{21} R_1 \tau_1$$

The Equivalent Lens, Using the System Matrix to get everything you need!

$$M = \begin{pmatrix} H & G \\ F & E \end{pmatrix} \quad \begin{aligned} x'_n &= Hx + G\gamma \\ \gamma'_n &= Fx + E\gamma \end{aligned}$$

$$M_{oi} = \tau_{32} R_2 \tau_{21} R_1 \tau_1$$

Where

$$H = A + iC$$

$$G = -o - ioC + B + oD$$

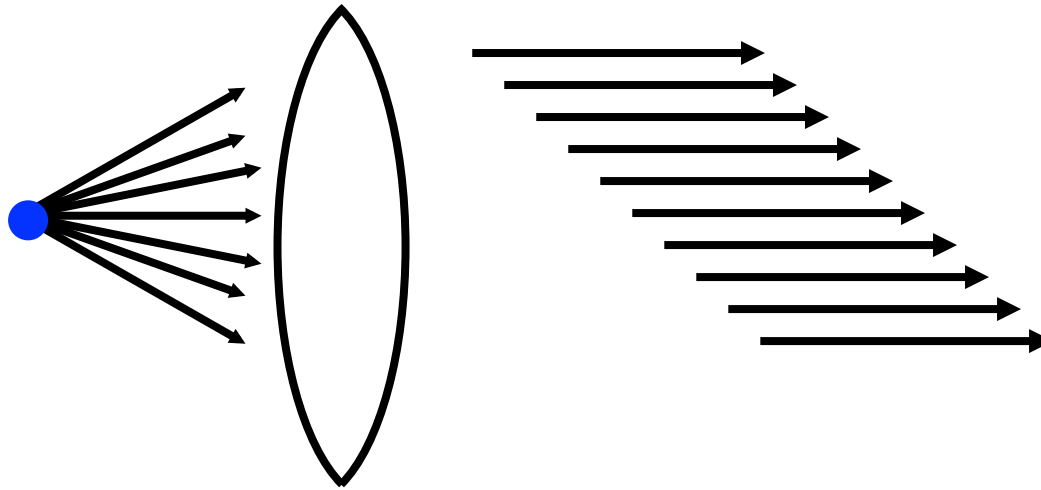
$$F = C$$

$$E = D - Co$$

# Using the System Matrix

## Case 1: $E=0$

Output angle independent of input angle



$$x'_n = Hx + G\gamma$$

$$\gamma'_n = Fx + E\gamma$$

$$o = \frac{D}{C}$$

Front focal distance!

# Using the System Matrix

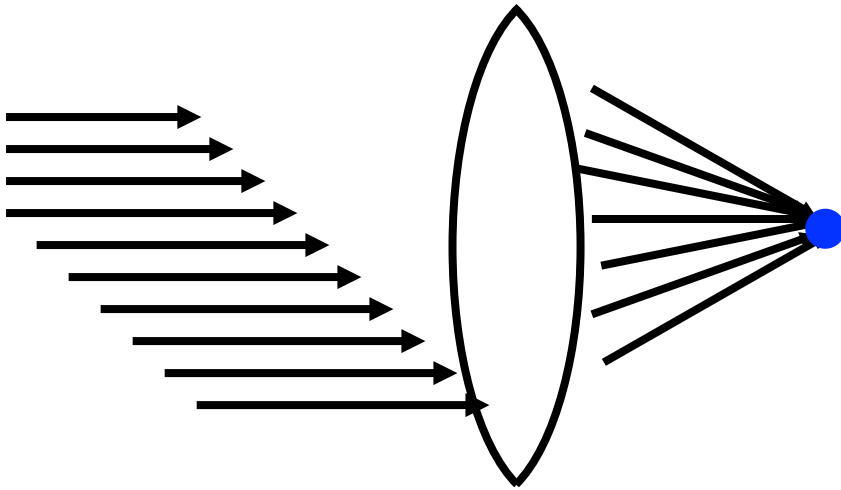
## Case 2: $H=0$

Output height independent of input height

Output height proportional to input angle

$$x'_n = Hx + G\gamma$$

$$\gamma'_n = Fx + E\gamma$$



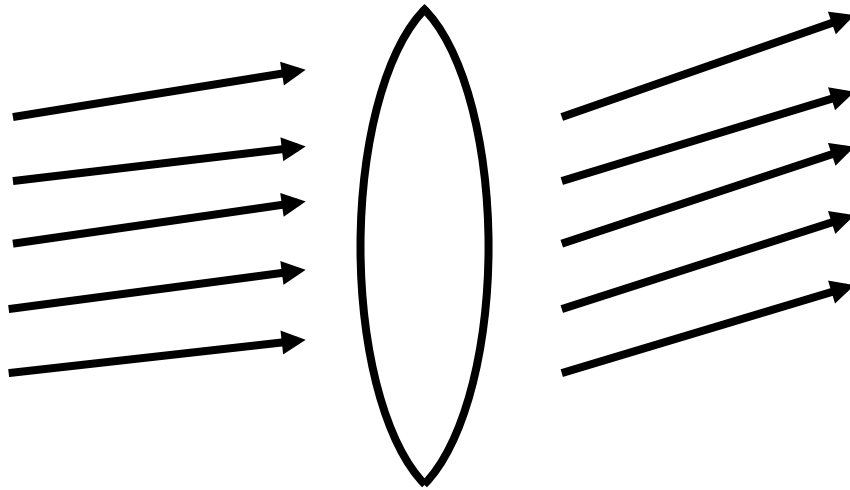
$$i = \frac{-A}{C}$$

Back focal distance!

# Using the System Matrix

## Case 3: $F=0$

Output angle independent of input height  
Telescope condition!



$$x'_n = Hx + G\gamma$$

$$\gamma'_n = Fx + E\gamma$$

$$E = \frac{\gamma'}{\gamma}$$

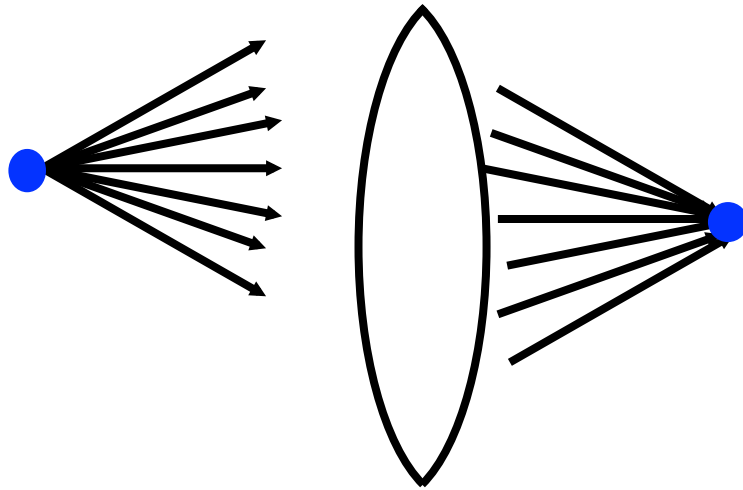
Angular magnification



# Using the System Matrix

## Case 3: $G=0$

Rays emitted at a fixed height  
arrive at a fixed height independent of angle



$$x'_n = Hx + G\gamma$$

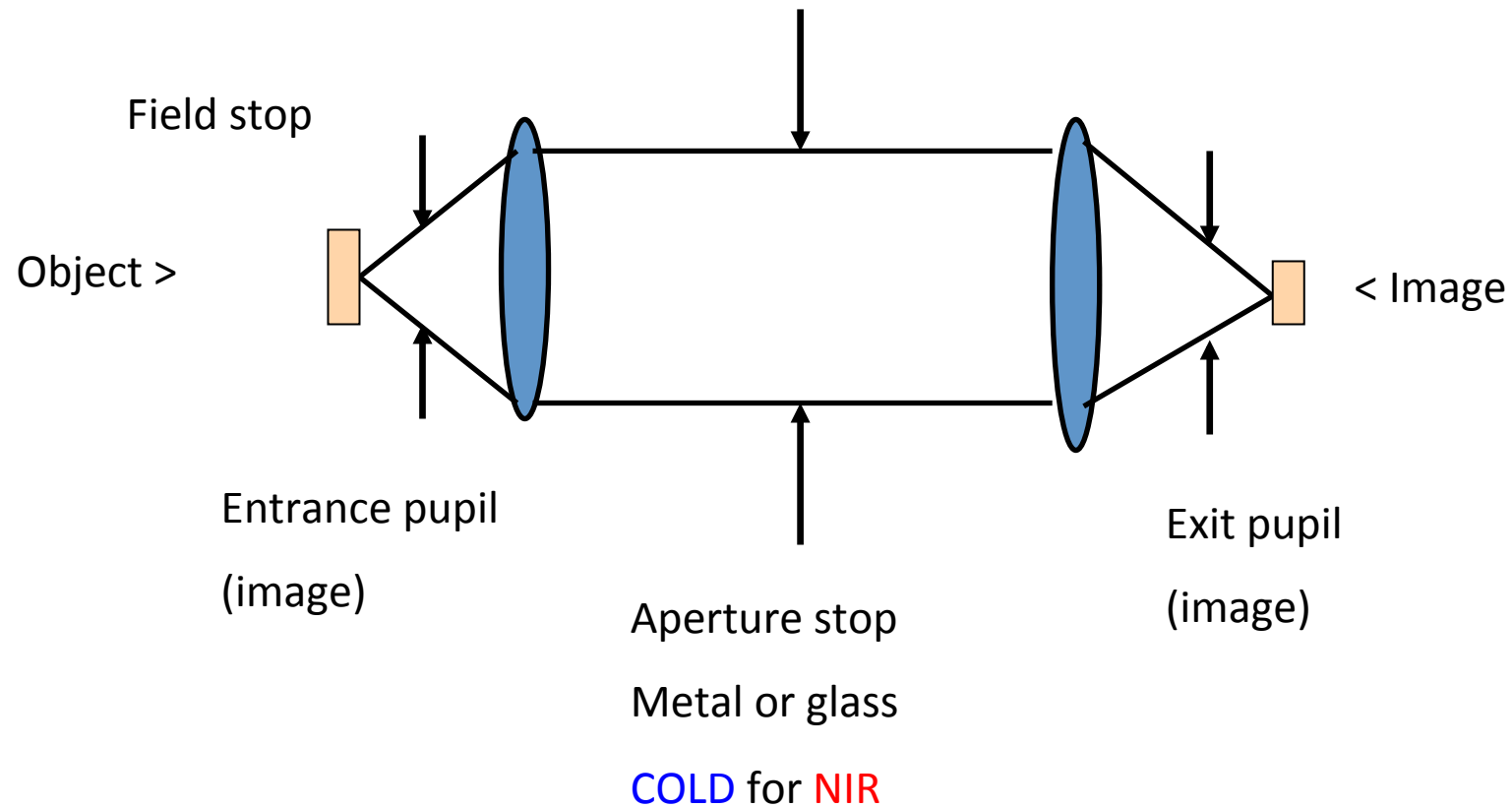
$$\gamma'_n = Fx + E\gamma$$

Object and image distances!

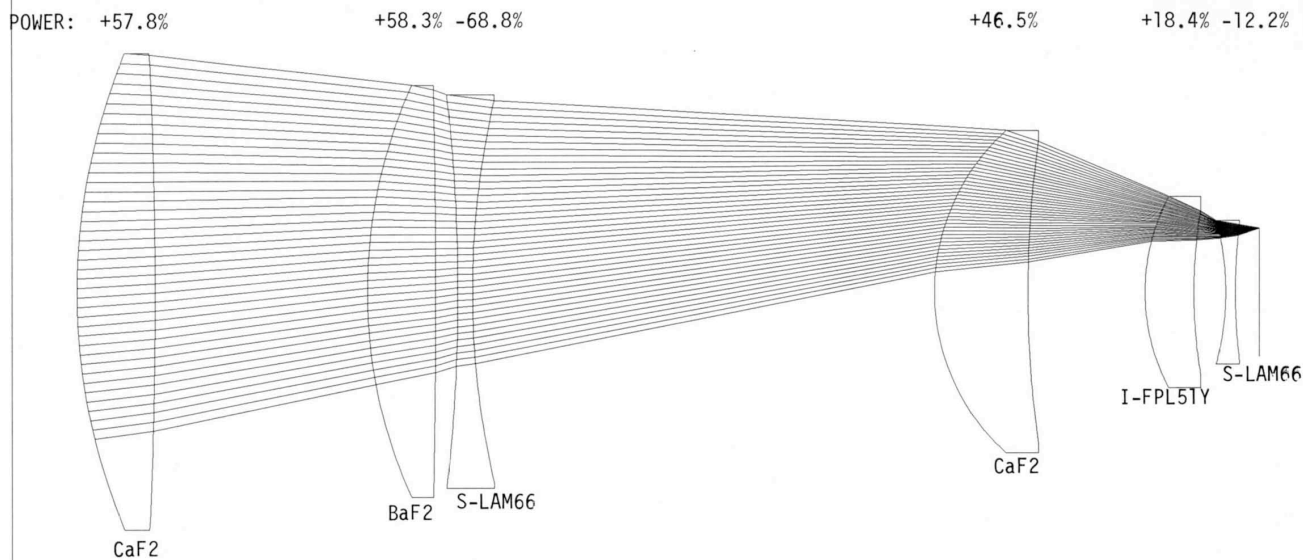
$$i = \frac{Ao - B}{D - Co}$$

$$o = \frac{B + Di}{A + Ci}$$

Now you can use the matrix representation to solve for Entrance Pupil and Exit Pupil for any system!



# Now can calculate the exit pupil.



## LAYOUT

PFIS PRELIMINARY CAMERA X-02 RUN 040705AD  
THU APR 14 2005  
TOTAL TRACK: 484.84793 MM

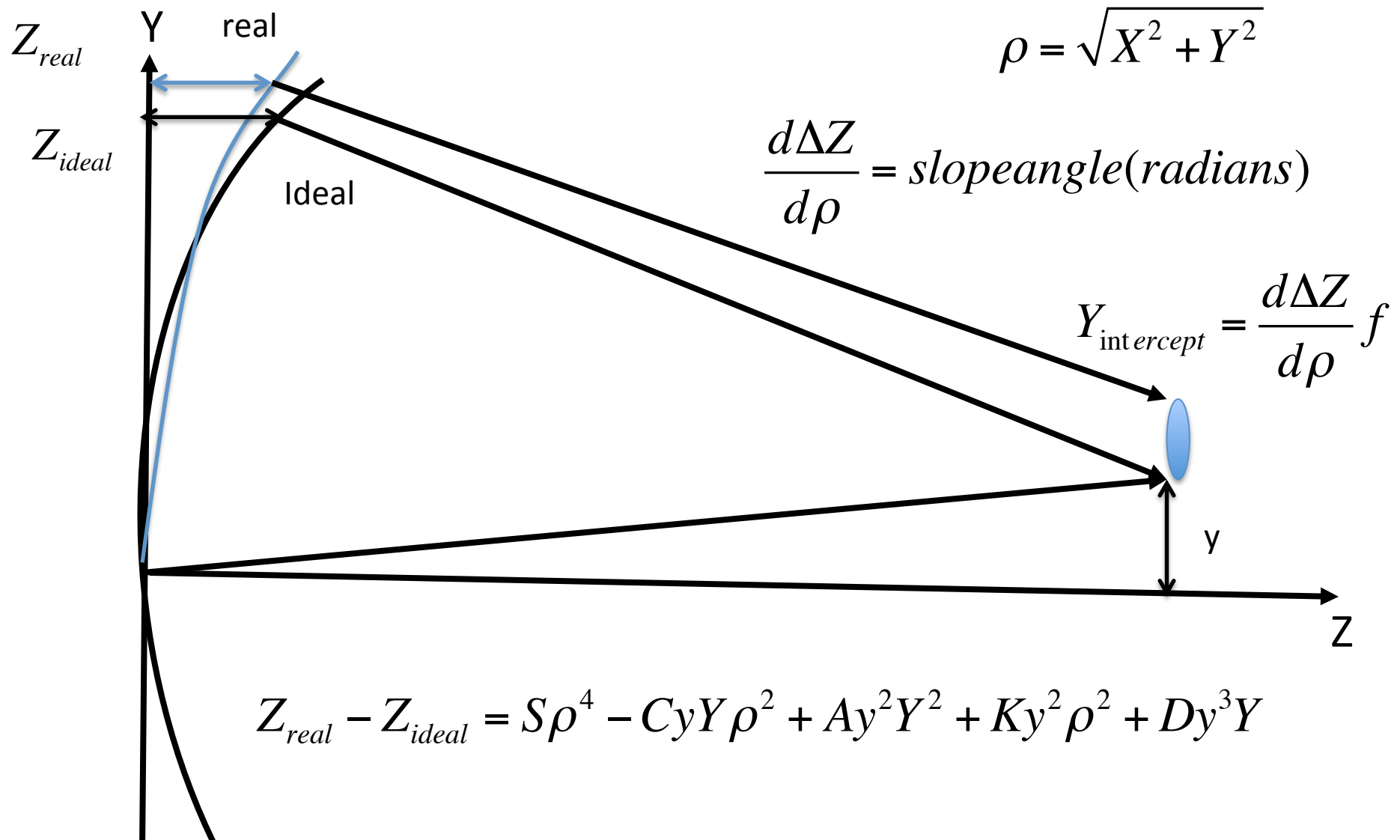
H.W.EPPS-UCO/LICK-UC SANTA CRUZ

\*\*\* DO NOT BUILD \*\*\*

FIGURE 1

# Geometric Theory of Aberrations

## (Eikonal analysis, what's an eikonal?)



# Geometric Theory of Aberrations (Eikonal analysis)

$$Z_{real} - Z_{ideal} = S\rho^4 - CyY\rho^2 + Ay^2Y^2 + Ky^2\rho^2 + Dy^3Y$$

Where

$$Z = OPD$$

$$\frac{d\Delta Z}{d\rho} = slopeangle(radians)$$

S= Spherical

C=Coma

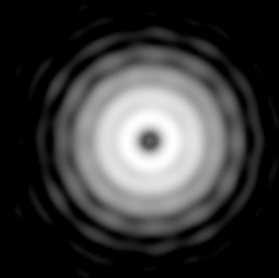
A=Astigmatism

K=Field Curvature

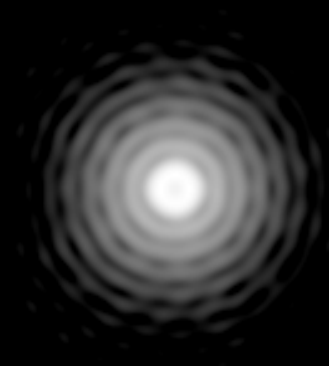
D=Distortion



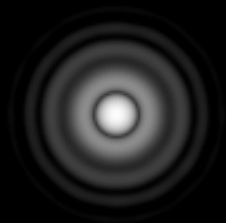
$$\Delta W = 0$$



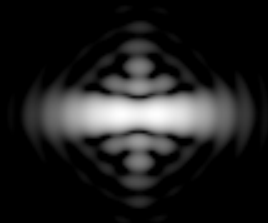
$$\Delta W = \rho^2$$



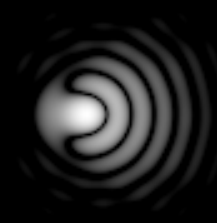
$$\Delta W = \rho^4$$



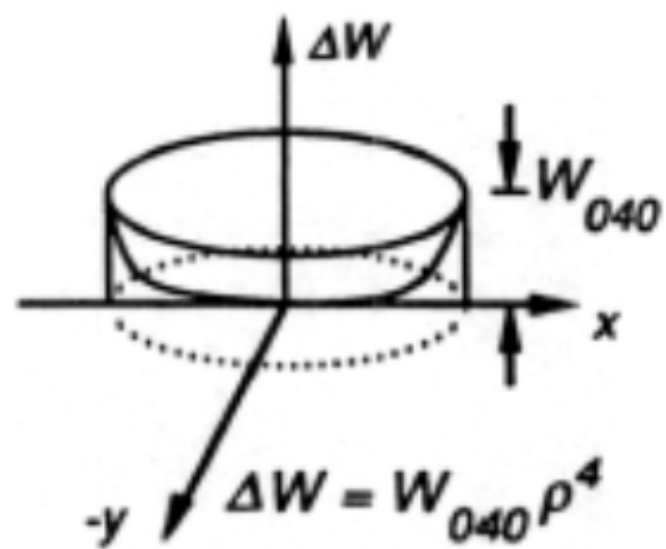
$$\Delta W = \rho^4 - \rho^2$$



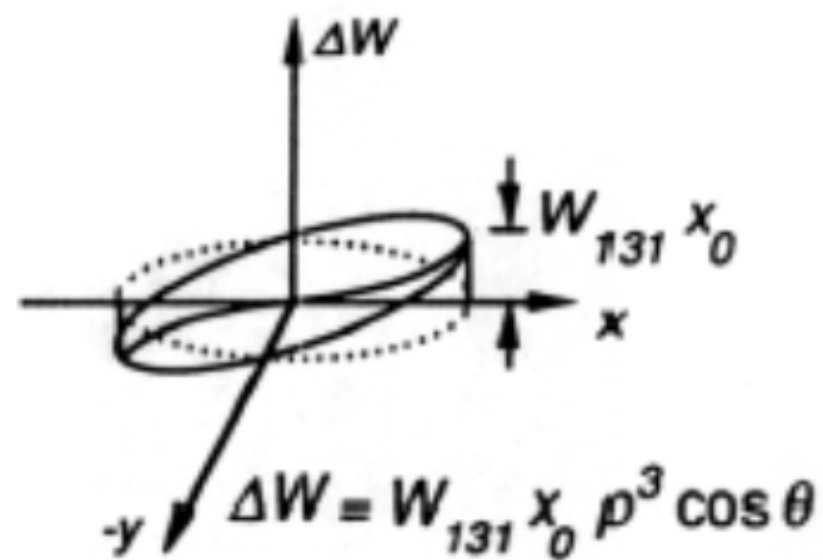
$$\Delta W = \rho^2 \cos^2 \theta$$



$$\Delta W = \rho^3 \cos \theta$$

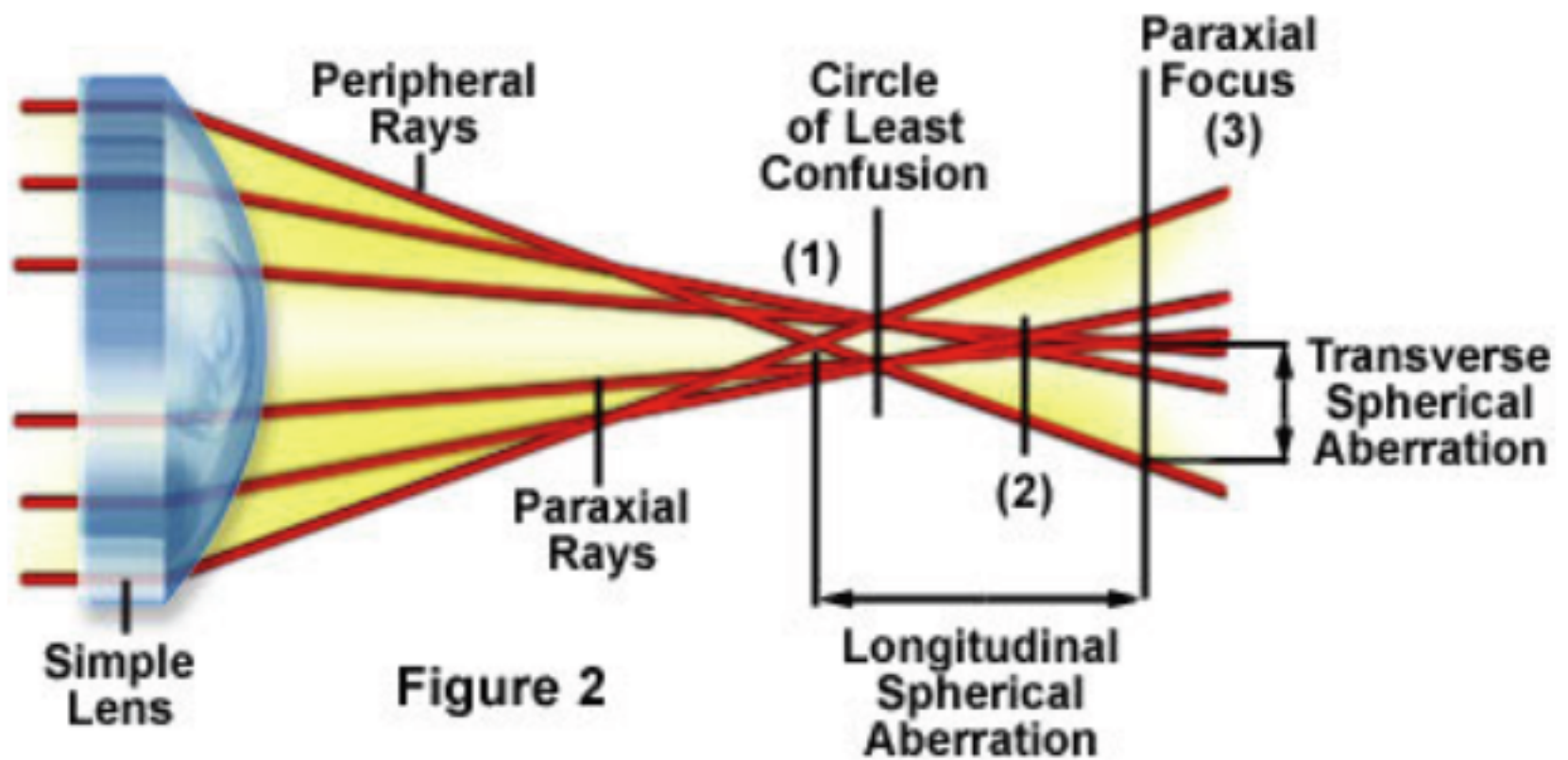


**Spherical Aberration**



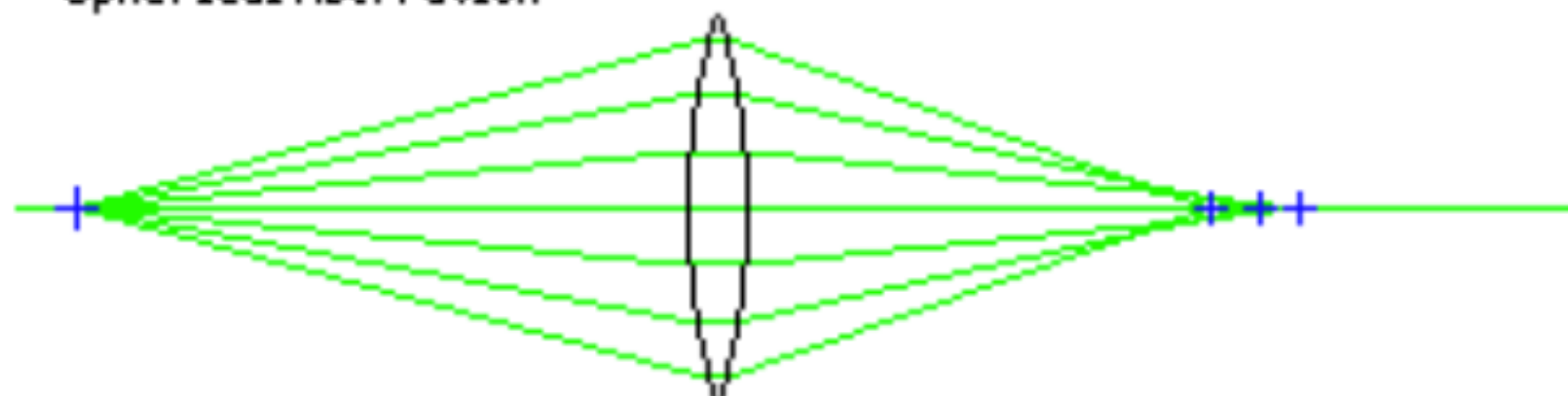
**Coma**

## Longitudinal and Transverse Spherical Aberration

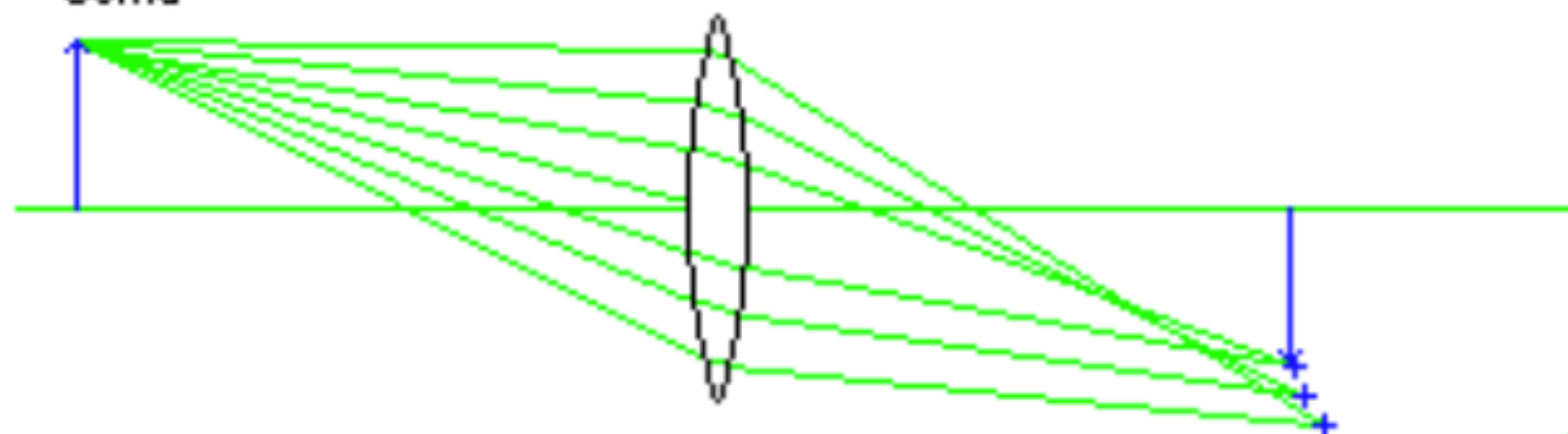


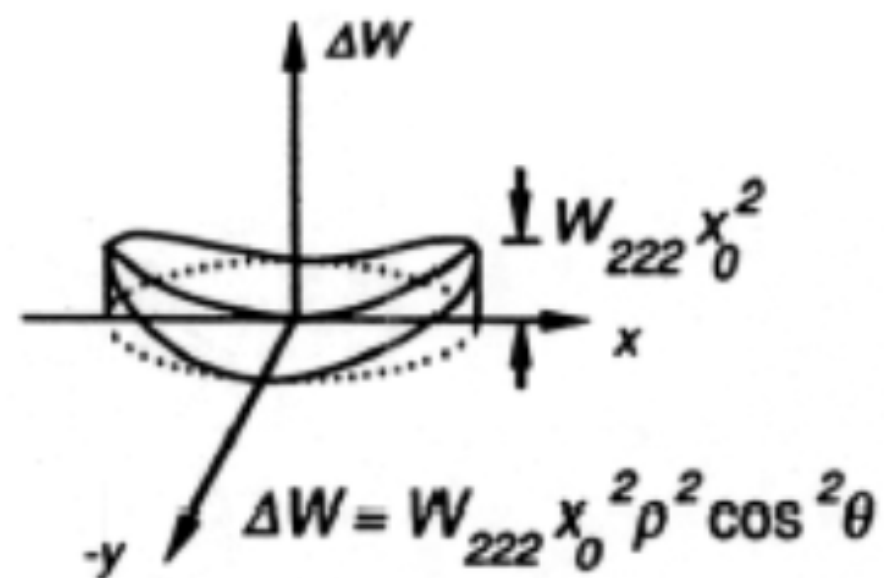


Spherical Aberration

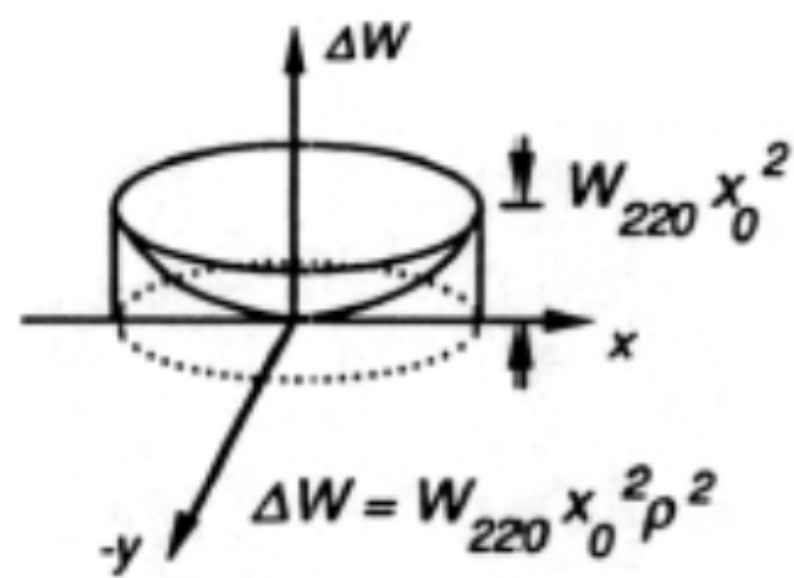


Coma

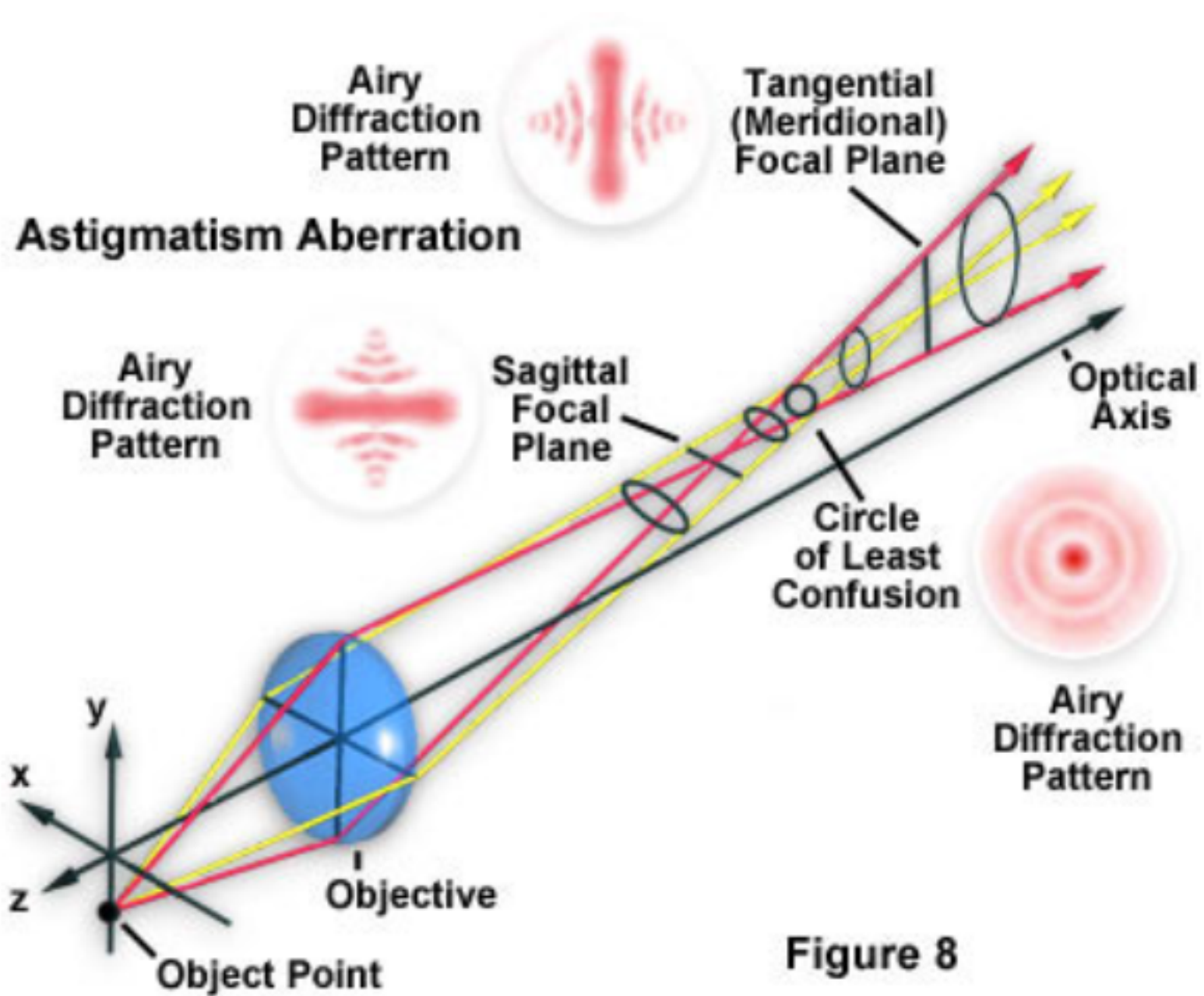




**Astigmatism**

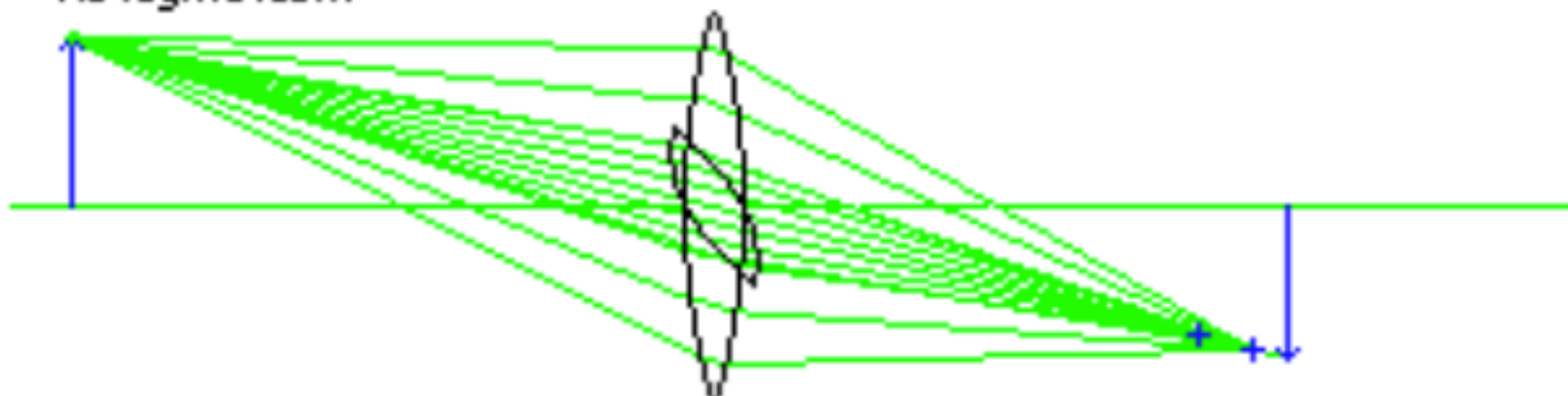


**Field Curvature**

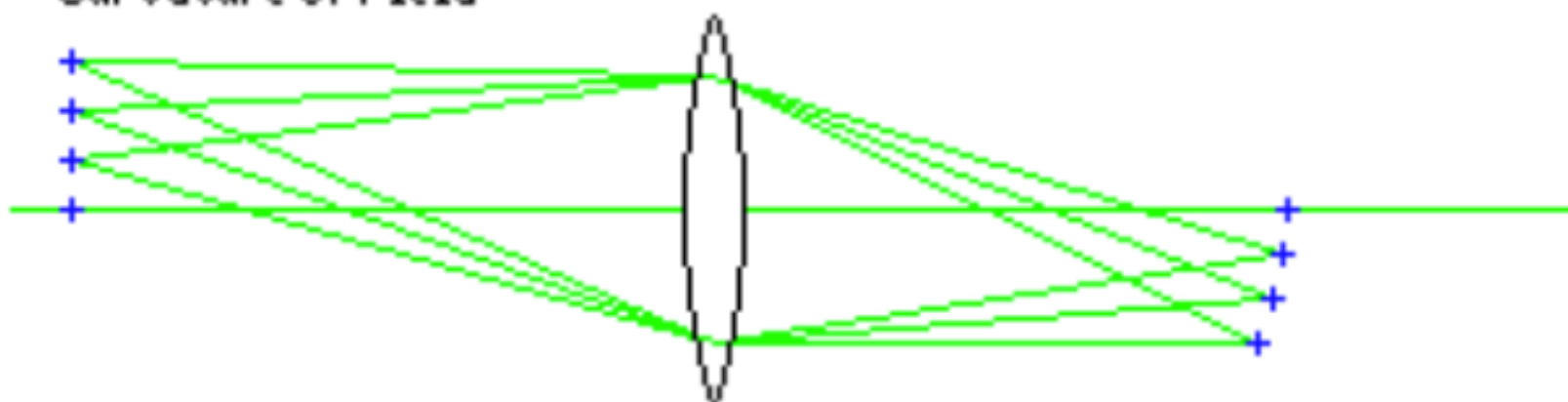


**Figure 8**

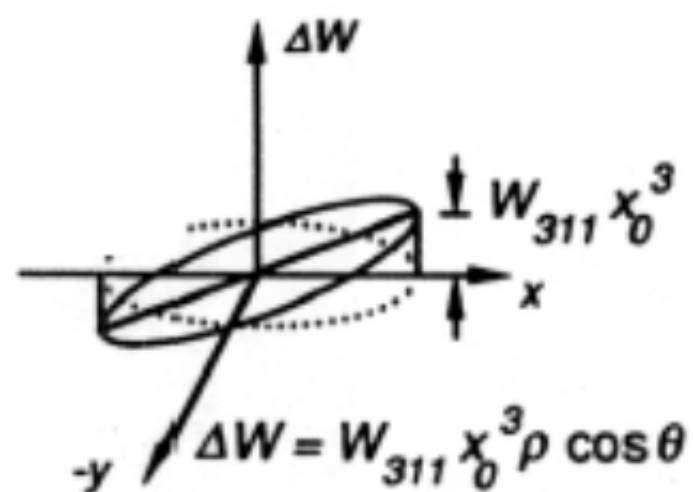
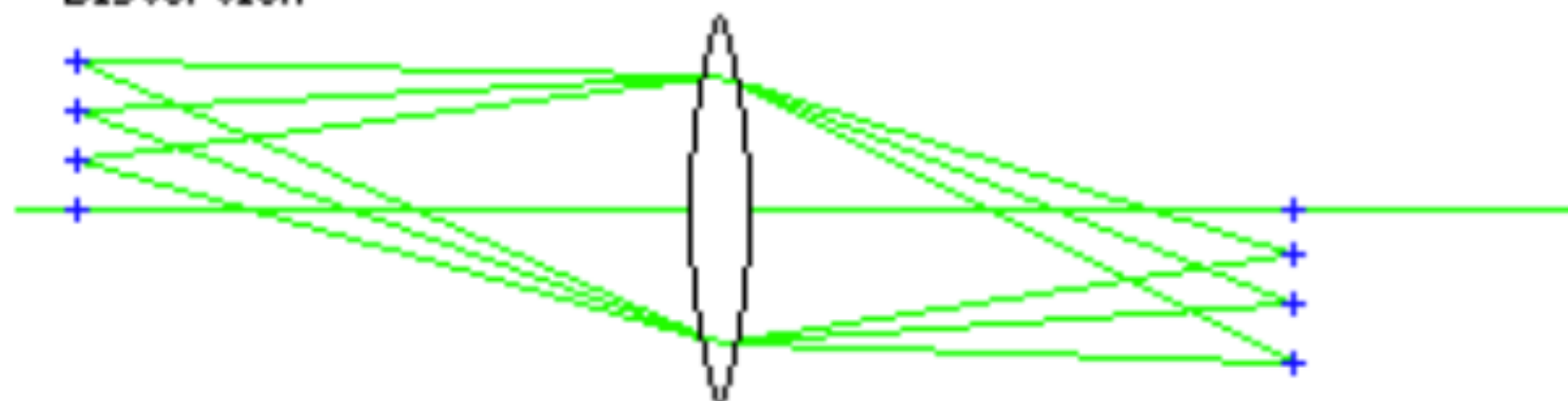
Astigmatism



Curvature of Field

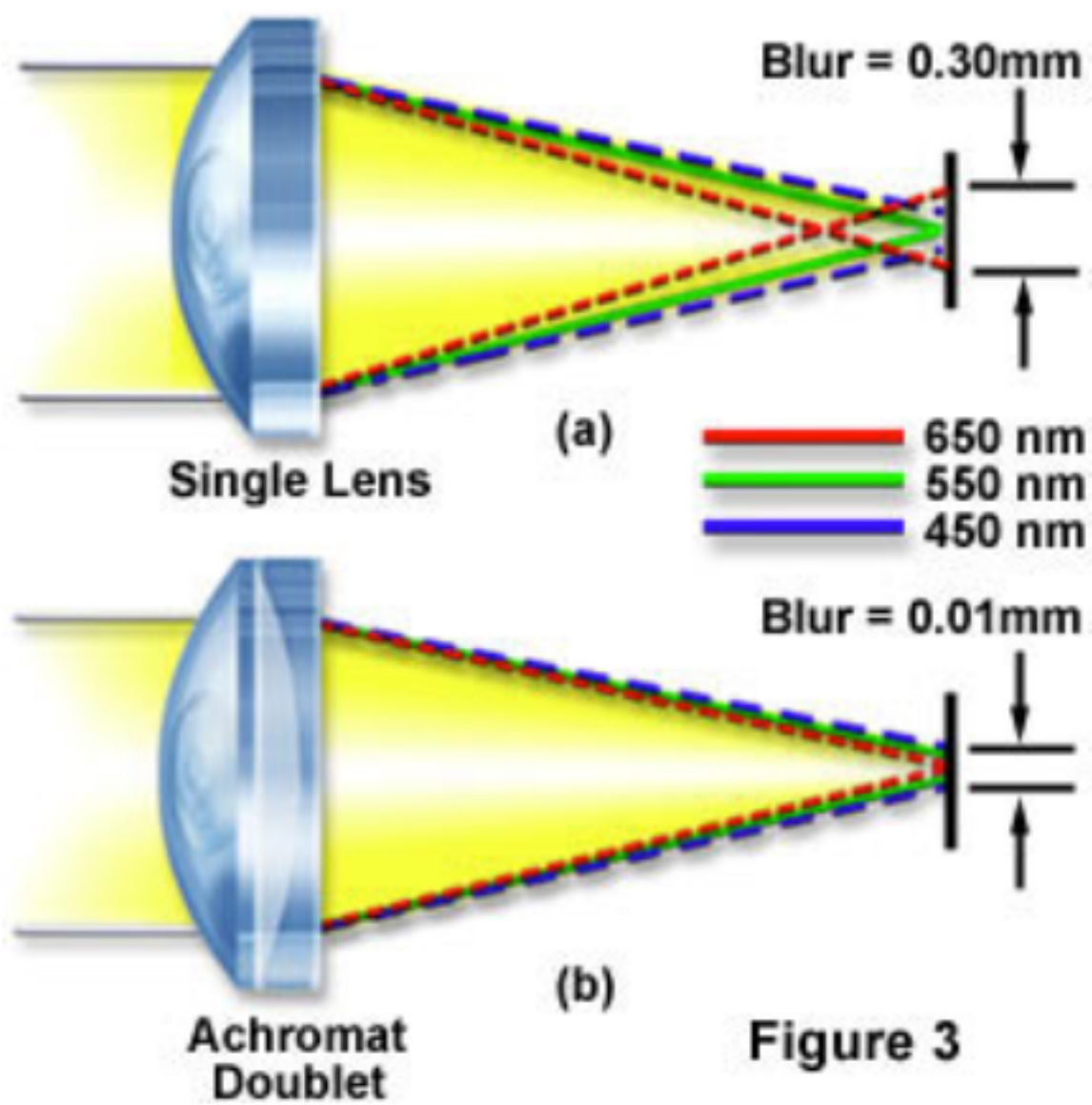


Distortion



Distortion

## Axial Chromatic Aberration



# How to “fix” aberrations

$$Z_{real} - Z_{ideal} = S\rho^4 - \cancel{CyY\rho^2} + \cancel{Ay^2Y^2} + \cancel{Ky^2\rho^2} + \cancel{Dy^3Y}$$

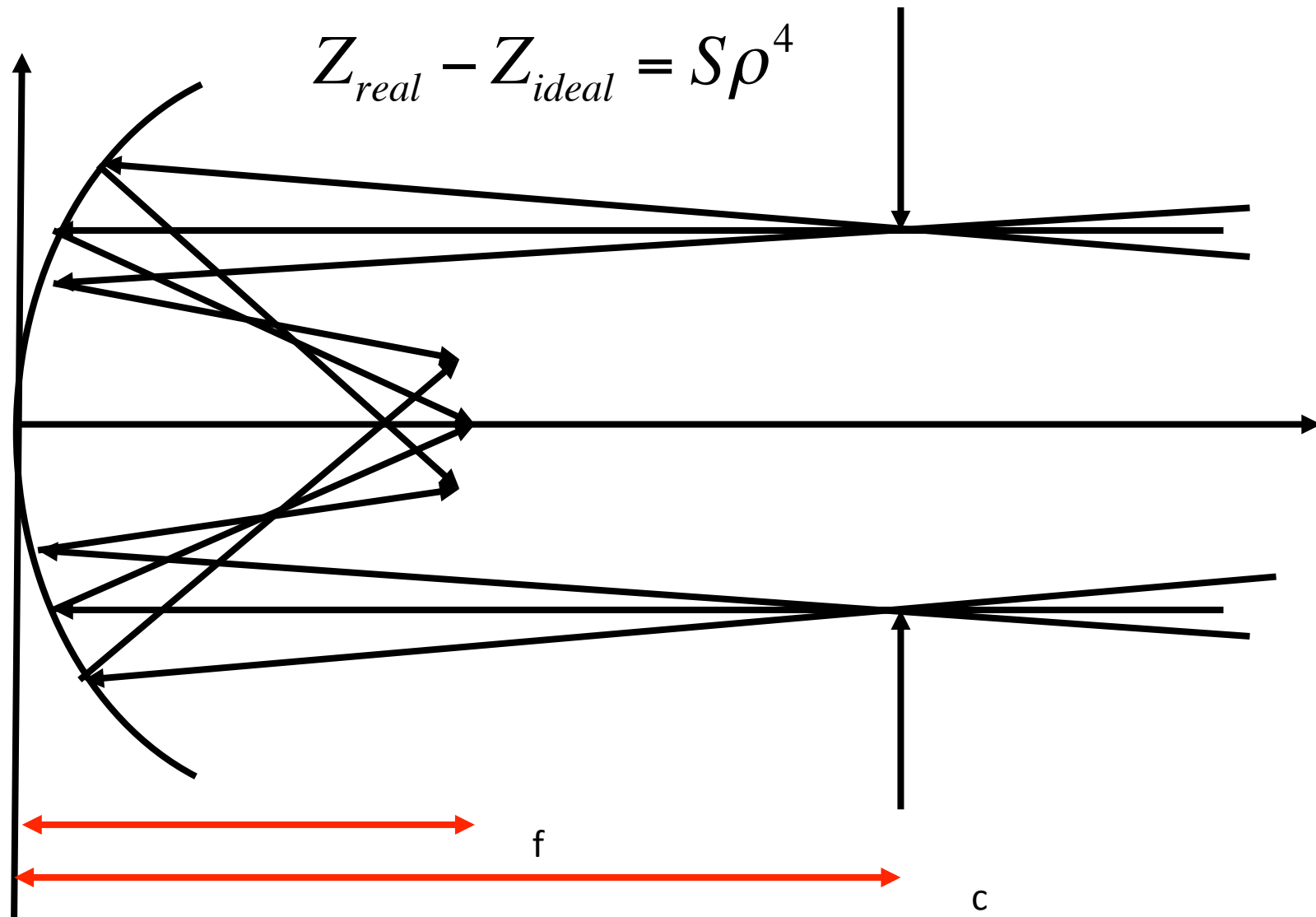
- On-axis >  $y=0$

$$Z_{real} - Z_{ideal} = S\rho^4$$

$$Z_{real}^2 - A_1 Z_{real} - A_2 \rho^2 = A_3$$

This is a parabola. Perfect image quality, zero field.

# How to “fix” aberrations

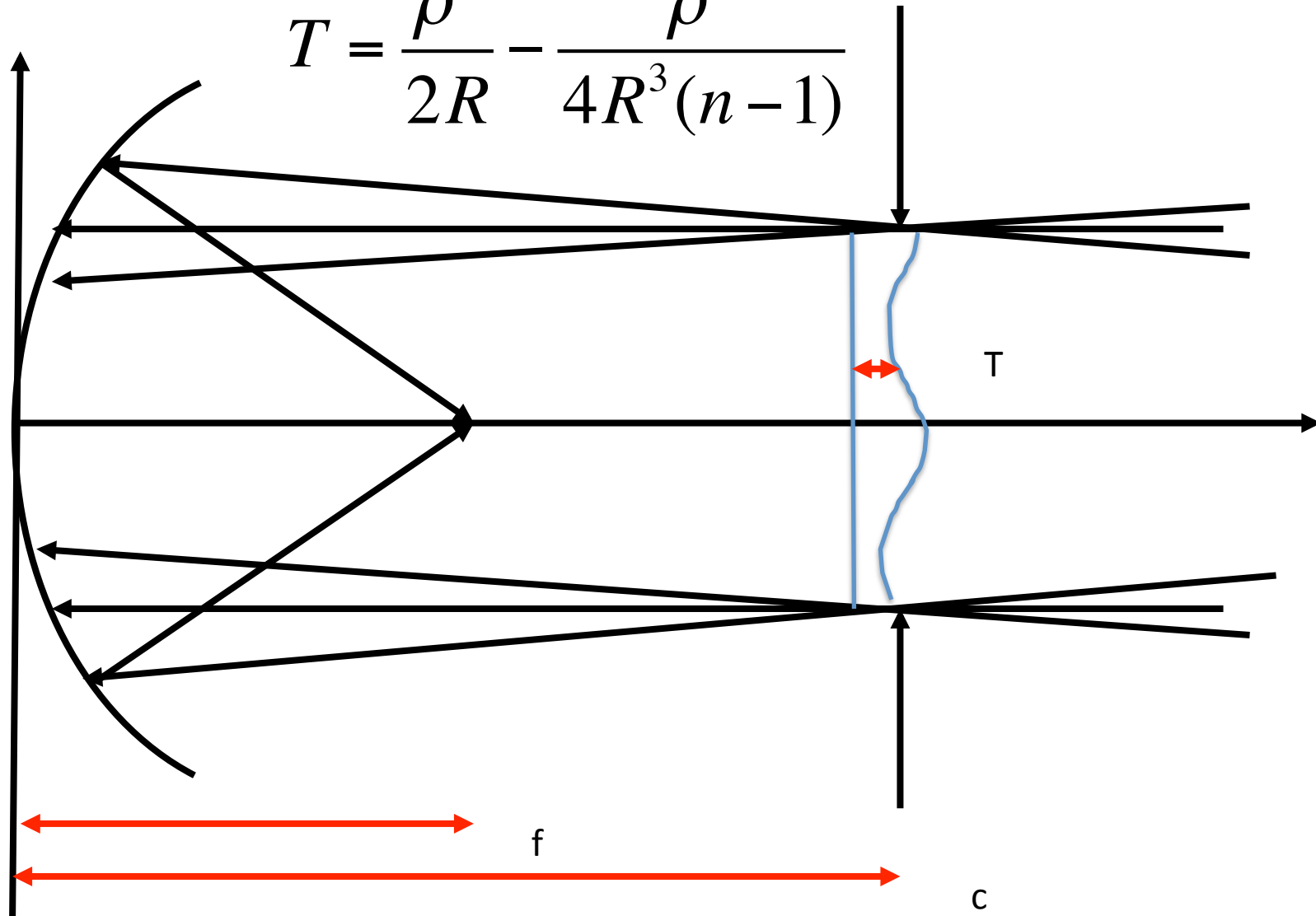


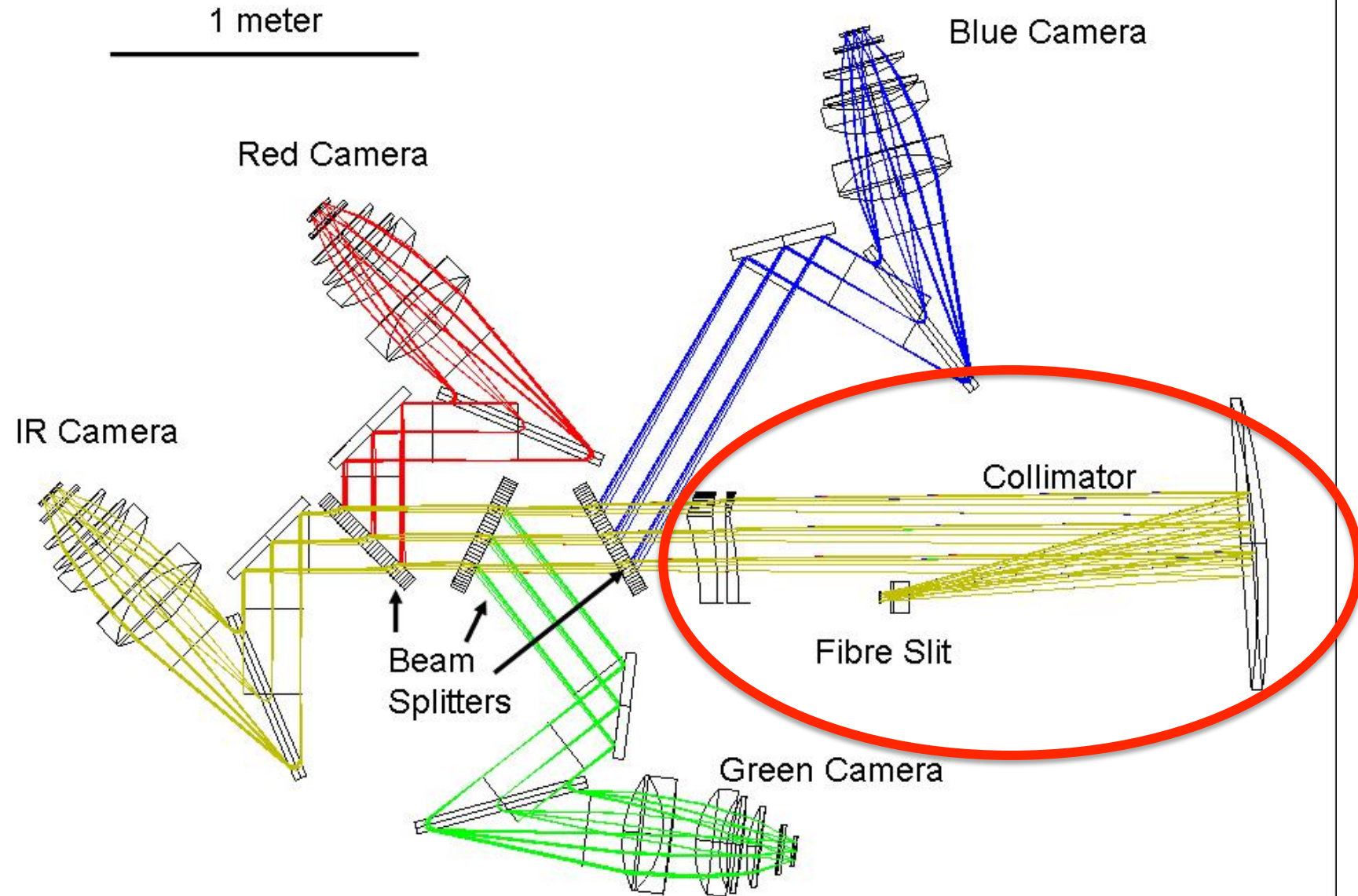


$$Z_{real} - Z_{ideal} = S\rho^4 = (n-1)T$$

$$T = \rho^4 / (S(n-1))$$

$$T = \frac{\rho^2}{2R} - \frac{\rho^4}{4R^3(n-1)}$$





End lecture 2