Physics 1901

Experimental Astronomy – Graduate Course Autumn (Apr-May 2014)

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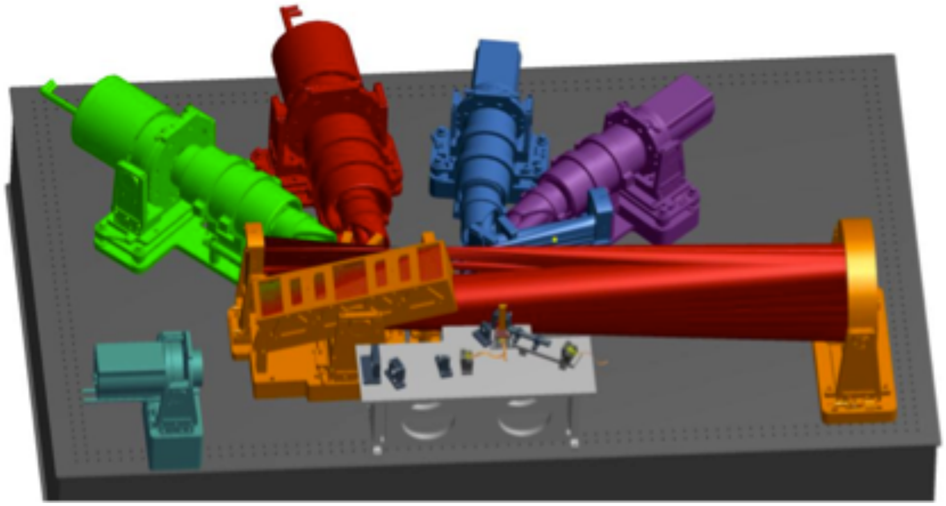
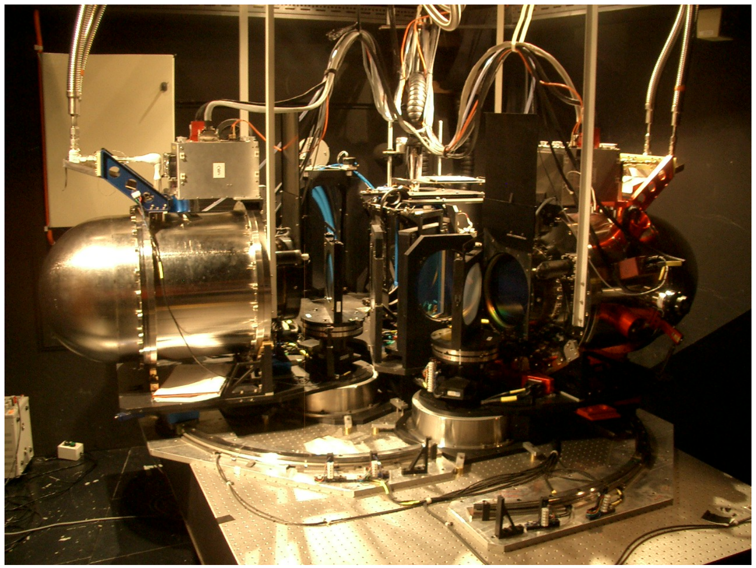
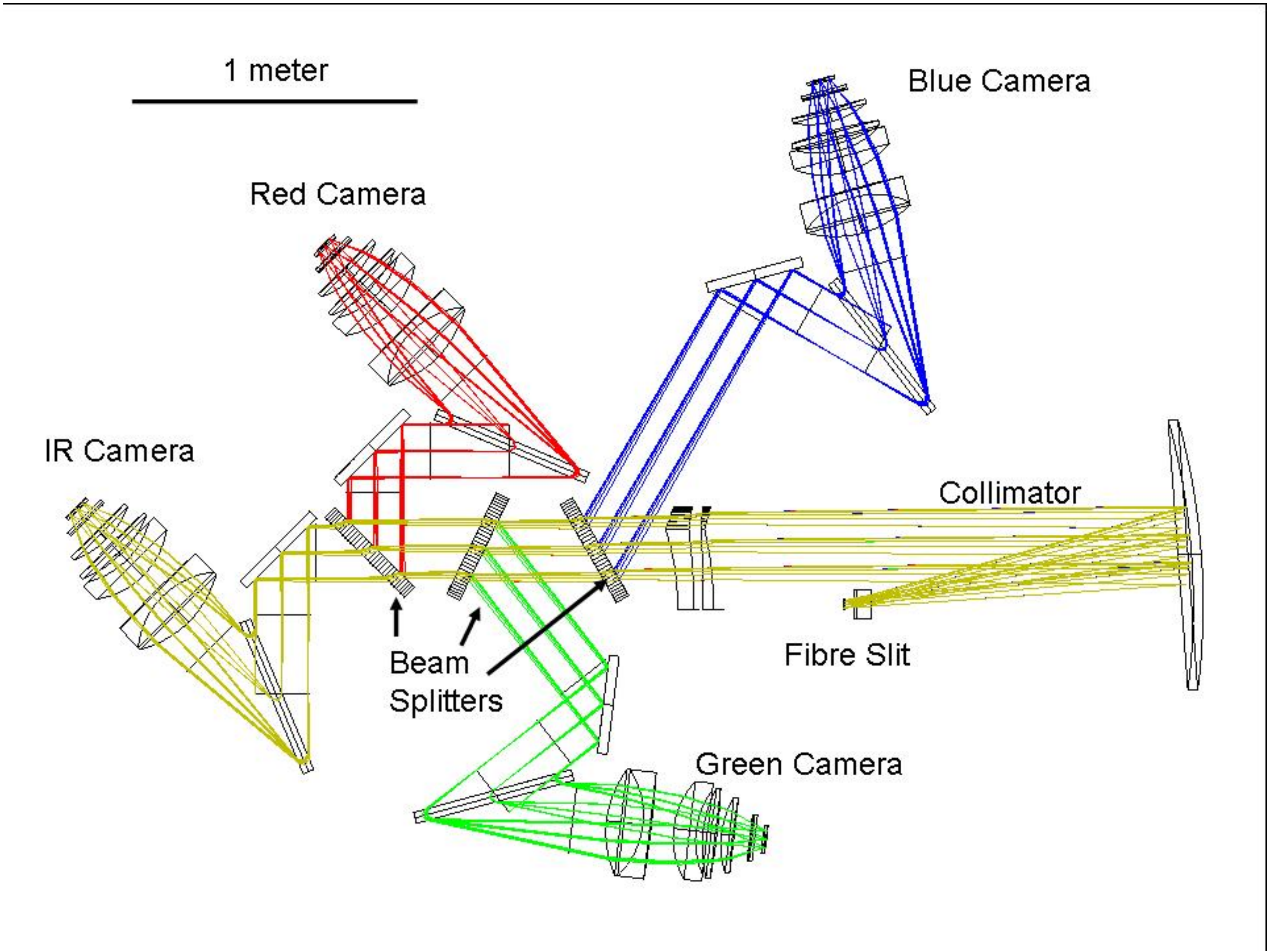
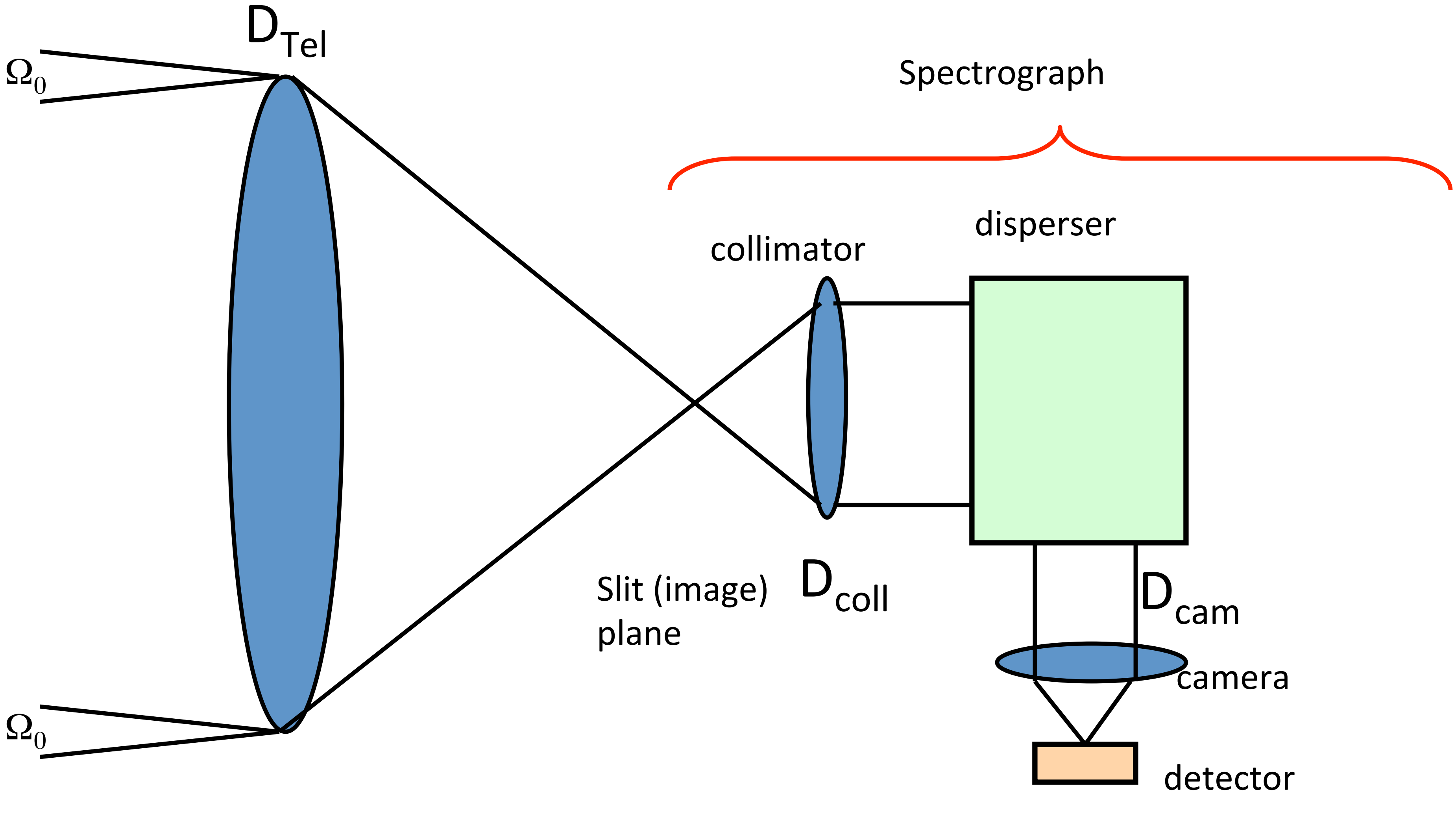
Some questions:
(which you should be able to answer at the end of the course)

- What are the parts of a spectrograph
- Why are spectrographs so big?
- What sets the sensitivity?
- How do I estimate the exposure time?






1 meter
Blue Camera
Red Camera
IR Camera
Collimator
Beam
Splitters
Fibre Slit
Green Camera



Telescope

Anamorphic factor,

$$r = D_{coll}/D_{cam}$$

Thought Experiment:

You observe the moon using an eyepiece attached to a 8 meter telescope. What is the relative brightness of the image compared to naked-eye viewing? (or will this blind you?) assume your eye has an 8mm diameter pupil.

A) Brightness=$(8e3/8)^2$=1,000,000 times
B) Brightness=(8e3/8)=1,000 times
C) The same, Brightness is conserved!

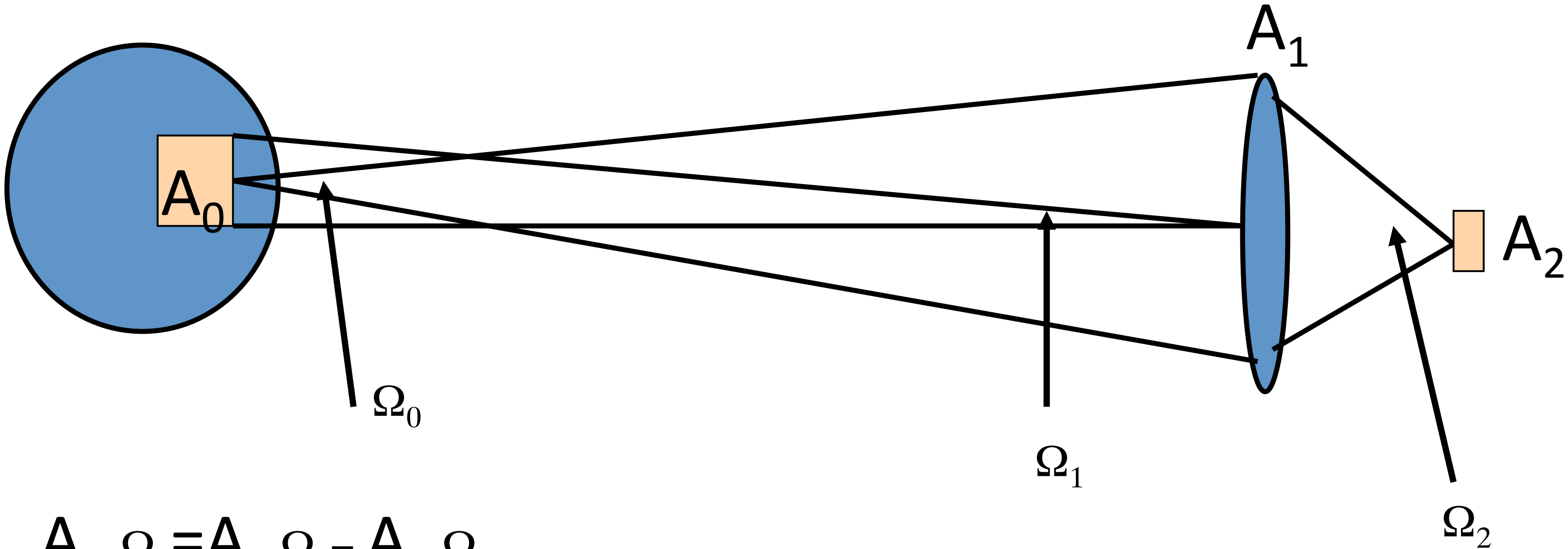
Surface Brightness

- Surface brightness is the energy per unit angle per unit area falling on (or passing through) a surface.
- Conserved for Finite size source (subtends a real angle)
- Also called
  - Specific intensity
  - Brightness, surface brightness
  - Specific brightness
- Units: (Jy sr^{-1}) or (W m^{-2} $Hz^{-1}sr^{-1}$) or (erg cm^{-2} Hz^{-1}) or (m $arcsec^{-2}$)

Surface Brightness

Rybicki and Lightman,

Radiative Processes in Astrophysics (1979), Ch1



$$A_0\,\Omega_0 = A_1\,\Omega_1 = A_2\,\Omega_2$$

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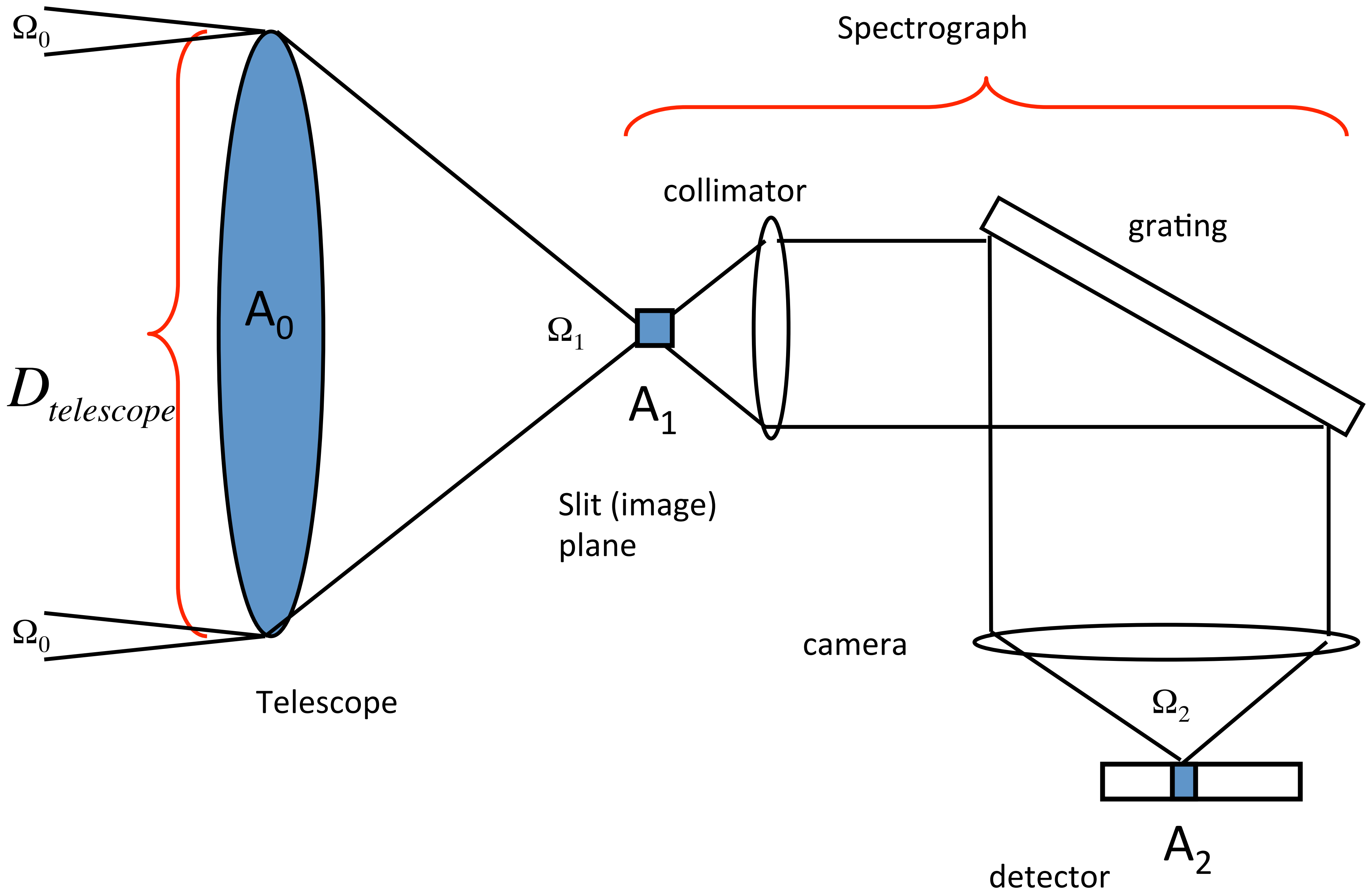

Spectrograph
Ω_0
Ω_0
$D_{telescope}$
A_0
Telescope
Ω_1
A_1
Slit (image) plane
collimator
grating
camera
Ω_2
A_2
detector

Surface Brightness

$$dE = S\,dA\,d\Omega$$

- Omega = solid angle of incoming beam measured at any surface
- I.e. Measured in RA and Dec on the sky (from the pupil)
- Measured in square degrees at the image plane
- dA = area measured in square meters
- For a resolved object, *Surface Brightness is Conserved*!
- Includes lenses treated as objects, very important for NIR
- For an unresolved object SB is decreases (2nd Law of Thermo)
- Fiber optics do not conserve SB.
- Diffraction does not conserve surface brightness.

Surface Brightness

$$dE = S \bullet losses \bullet emission \bullet dA\, d\Omega$$

- Omega = solid angle of incoming beam measured at any surface
- I.e. Measured in RA and Dec on the sky (from the pupil)
- Measured in square degrees at the image plane
- dA = area measured in square meters
- For a resolved object, *Surface Brightness is Conserved*!
- In the thermal IR, the SB is only a function of T and emissivity.
- Includes lenses treated as objects, very important for NIR
- For an unresolved object SB is decreases by diffraction (2nd Law of Thermo)
- Fiber optics do not conserve SB.
- Diffraction does not conserve SB.

Energy Collected

$$E = S A \Omega$$

- S is a constant (modulo absorption)
- Energy collected by an optical system is proportional to AΩ
- Conservation of energy implies AΩ is a constant.
- Also called Optical invariant, etendu
- S is also called the phase space density: Louivilles theorm
- *In order maximize energy collected you want to maximize A Omega and minimize absorption!*

How do I calculate the number of photo electrons/s on my detector?

- For an extended object in the IR that is easy: You just need the temperature of the source, the system losses (absorption, QE etc), resolution and etendu of a pixel. No telescope aperture or F/#, no slit size, no optical train!
- For an extended object in the visible: You just need the surface brightness of the source, the system losses (absorption, QE etc), resolution and etendu of a pixel. No telescope aperture or F/#, no slit size, no optical train!
- For an unresolved object, you need the source magnitude, telescope aperture, plate scale, resolution, and pixel size.

Ex 1: Thermal Imaging

R=5000

Pixel size= 10 microns

Final focal ratio at detector = F/3

Source temperature=5000K

Operating near 2 microns

SB from Planck=1,157,314 watts/(m^2 sr micron)

$\Delta\lambda$=2 microns/5000=0.0004

$$Solid\ Angle = \frac{\pi/4}{(F/\#)^2}$$

$$E = I_\lambda A\Omega\ QE\ \Delta\lambda = (1157314)(10\cdot 10^{-6})^2(\pi/4)(1/3)^2(0.0004)$$

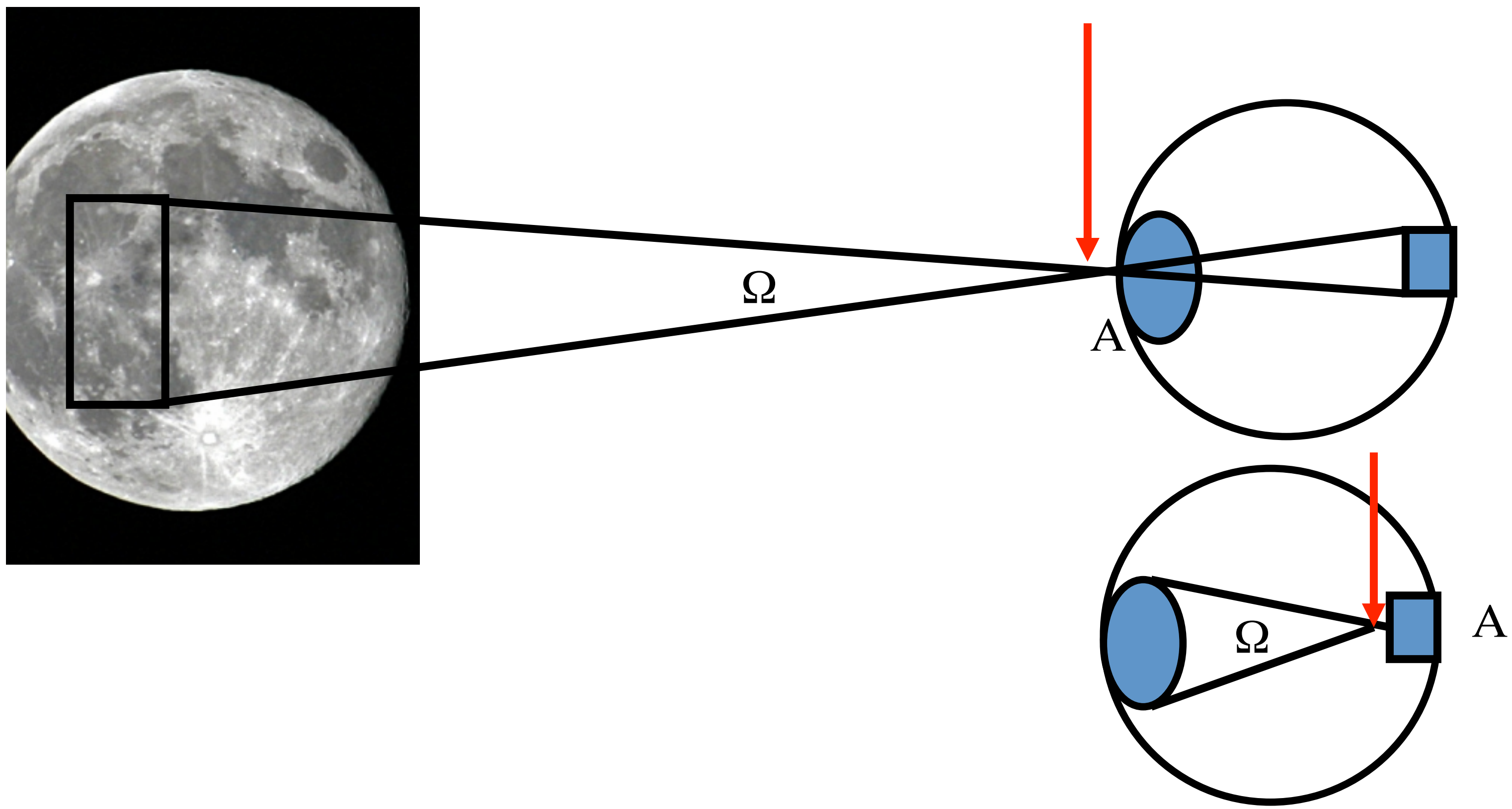
$$= 4.039\cdot 10^{-9}\,watts$$

$$Nphots = E/h\nu$$

$$= \frac{(4.039\cdot 10^{-9})\cdot(2\cdot 10^{-6})}{6.62\cdot 10^{-34}\cdot 3\cdot 10^{8}} = 4.06\cdot 10^{10}$$

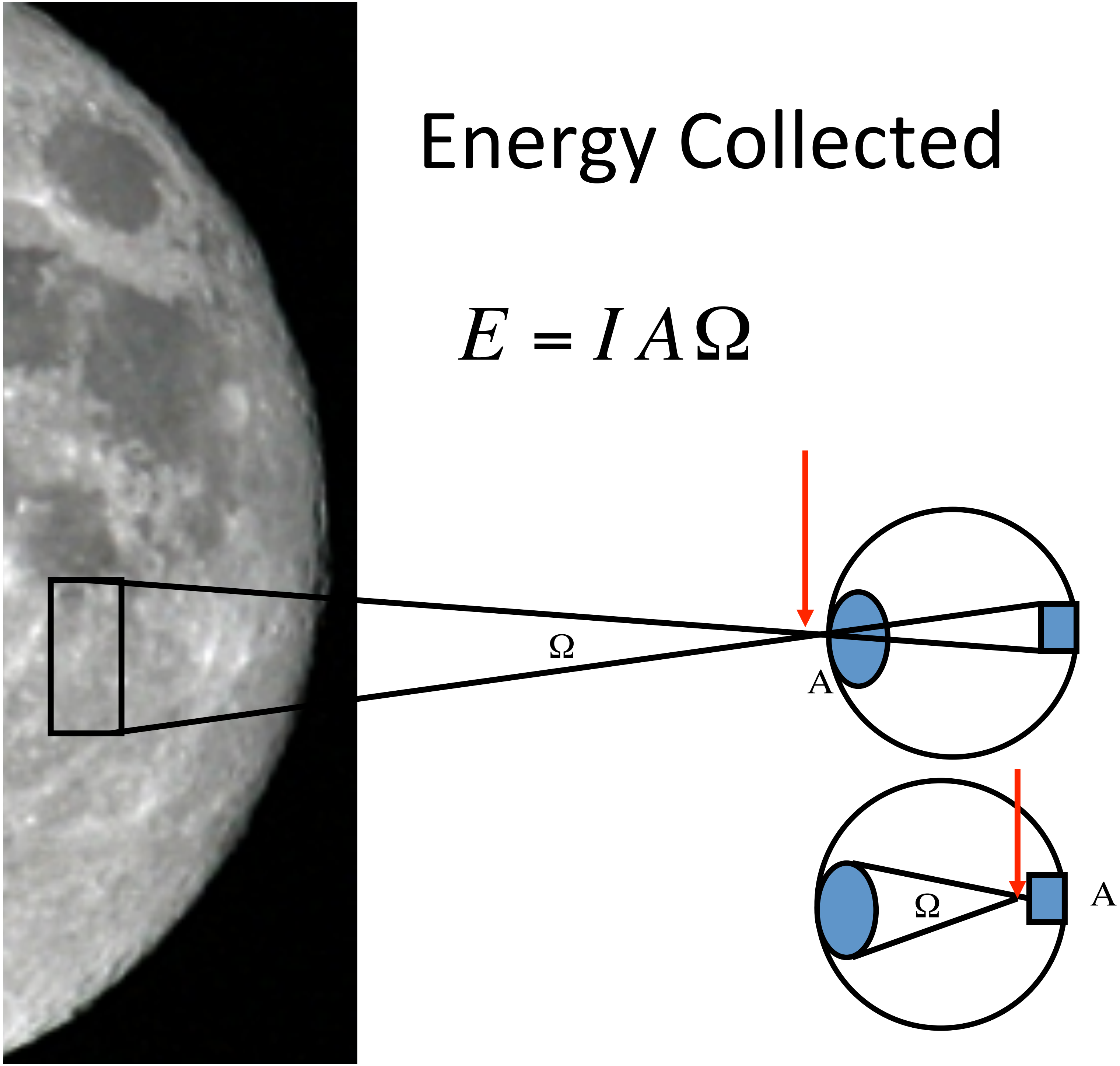
Energy Collected

$$E = I A \Omega$$



Energy Collected

$$E = I\,A\,\Omega$$



Specific Surface Brightness

$$dE = I_{\upsilon}(\Omega, \upsilon, t, p)\, d\Omega\, d\upsilon\, dt\, dA$$

Where I will depend on:

- Omega measured in RA and Dec
- v= frequency
- t= Integration time
- P=polarization
- Location where you are receiving the light.

Observation

$$E = \int S_{\upsilon}(\Omega, \upsilon, t, p) r(\Omega) F(\upsilon) d\Omega \, d\upsilon \, dt \, dA$$

$$E = A \delta t \int S_{\upsilon}(\Omega, \upsilon, t, p) r(\Omega) F(\upsilon) d\Omega \, d\upsilon$$

- E=energy received during measurement
- R=energy from the sky
- F= filter function

Do not confuse Surface Brightness with Flux

Flux is total energy incident on some area dA from a source (resolved or not).
Flux is not conserved and falls of as R^{-2}.

$$f_\upsilon = \iint I_\nu \, d\Omega$$

$$dE = f_\upsilon dA \, d\upsilon \, dt$$

$$dE = f_\upsilon (4\pi R^2) \, d\upsilon \, dt = L_\upsilon d\upsilon \, dt$$

$$\therefore \quad f_\upsilon = \frac{L_\upsilon}{(4\pi R^2)}$$

Flux: apparent magnitude

$$m_1 - m_2 = -2.5\log_{10}\left(\frac{f_1}{f_2}\right)$$

$$m = -2.5\log_{10}\left(\frac{f_1}{f_0}\right)$$

- m1 and m2 are the observed magnitudes of two objects (stars…etc) of the measured surface brightness in magnitudes per arcsecond.
- I find magnitudes confusing and generally convert to flux units for throughput calculations etc….

Flux: absolute magnitude

$$m_\lambda - M_\lambda = 5\log_{10} d - 5 + A(\lambda)$$

$$\because \frac{f_1}{f_2} = \left(\frac{d_2}{d_1}\right)^2$$

- M is the absolute magnitude. It is the apparent magnitude that would be observed at 10 pc.
- A is the total extinction due to interstellar dust in magnitudes

Surface brightness in magnitudes/ arcsecond^2

$$S = m_\lambda + 2.5\log_{10} A$$

- m is the apparent magnitude of object subtending A square arc seconds

Ex Surface brightness of the moon

M=-12.6 (V-band apparent magnitude

Diameter=30 arcminutes

$$m = -2.5\log_{10}\left(\frac{f_1}{f_0}\right)$$

$$f_1 = f_0 \cdot 10^{-m/2.5} = 3.63 \cdot 10^{-23} \cdot 10^{12.6/2.5}$$

$$= 3.96 \cdot 10^{-18} w/m^2/Hz$$

$$A = \pi(15 \cdot 60)^2 \sec^2$$

$$= 2.54 \cdot 10^6 \sec^2$$

$$I_\nu = \frac{(3.96 \cdot 10^{-18})}{2.54 \cdot 10^6} = 1.55 \cdot 10^{-24} W/M^2/Hz/\sec^2$$

Standard choices for reference flux

- Vega system: apparent Magnitude of Vega = 0 in all bands.
- Convenient, but non-physical
- A-B magnitude system:
- F_0=3.63e10^{-23} W m^{-2} Hz^{-1}, flat spectrum
- Agrees with Vega at 548nm (center of V-band)

Interesting magnitudes (V-band)

- Sun: m=-26.7
- Full moon: m=-12.6
- Sirius: m=-1.5
- Naked eye limit: m=6
- Brightest stars in Andromeda: m=19
- Present day limit: m~29
- Night sky: m=21.5 (best sites, dark time)
- Night sky: m=18 (bright time)

Total E/t= Luminosity, L

$$dE = L(t)dt$$

$$dE = L_{\upsilon}(t)d\upsilon\, dt$$

$$dE = L_{\lambda}(t)d\lambda\, dt$$

Ln = specific luminosity

What does this have to do with spectrograph design?

- Surface brightness of object conserved.
- A Ω is conserved through optical system!
- Must consider both A and Ω!
- In general Ω set by slit and A set by telescope
- Absorption reduces SB, emission increases noise.
- Need to use surface brightness analysis in calculating background (especially in the IR!)
- Fibers, diffraction, non-linear effects all decrease SB.

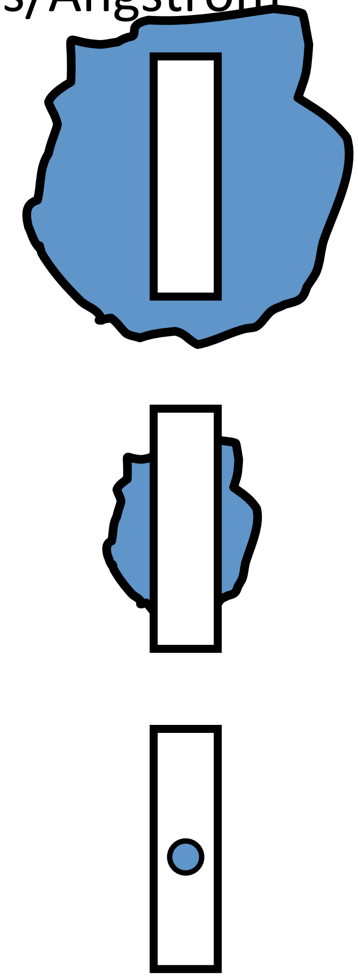
Spectrograph Speed

Speed=# of counts/s/Angstrom

I. Slit-limited

II. Intermediate

III. Image-limited



Bowen, I.S., “Spectrographs,” in Astronomical Techniques, ed. by W.A. Hiltner, (U. of Chicago Press, 1962), pp. 34-62.

Spectrograph Speed

Schroeder 12.2e, Ira Bowen (1962)

I. Slit-limited

$$Speed \propto D_{Tel}^{0} \bullet W_{grating}^{2}$$

II. Intermediate

$$Speed \propto D_{Tel}^{1} \bullet W_{grating}^{1}$$

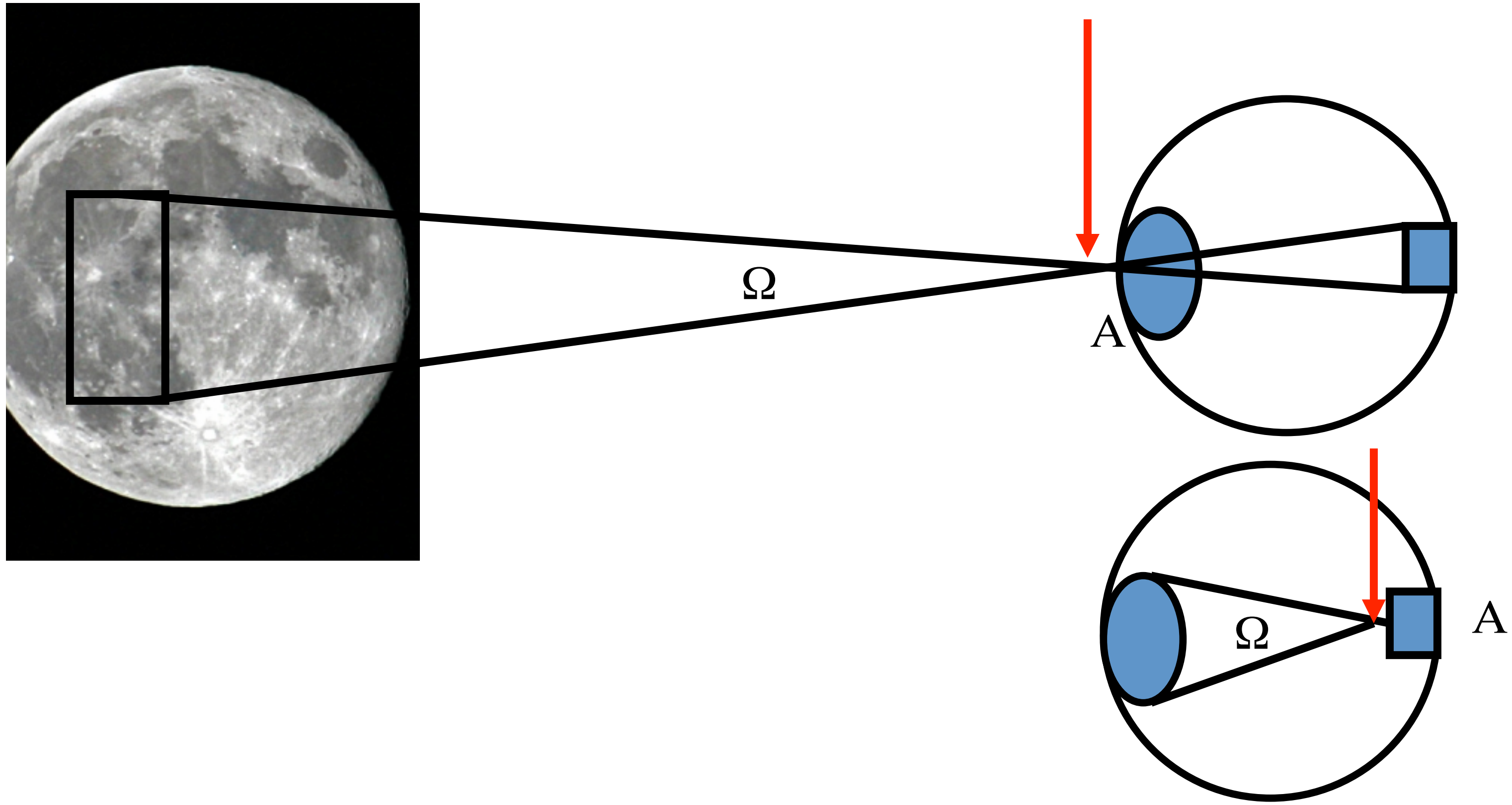
III. Image-limited

$$Speed \propto D_{Tel}^{2} \bullet W_{grating}^{0}$$

Speed=# of counts/s/Angstrom, W= illuminated grating length

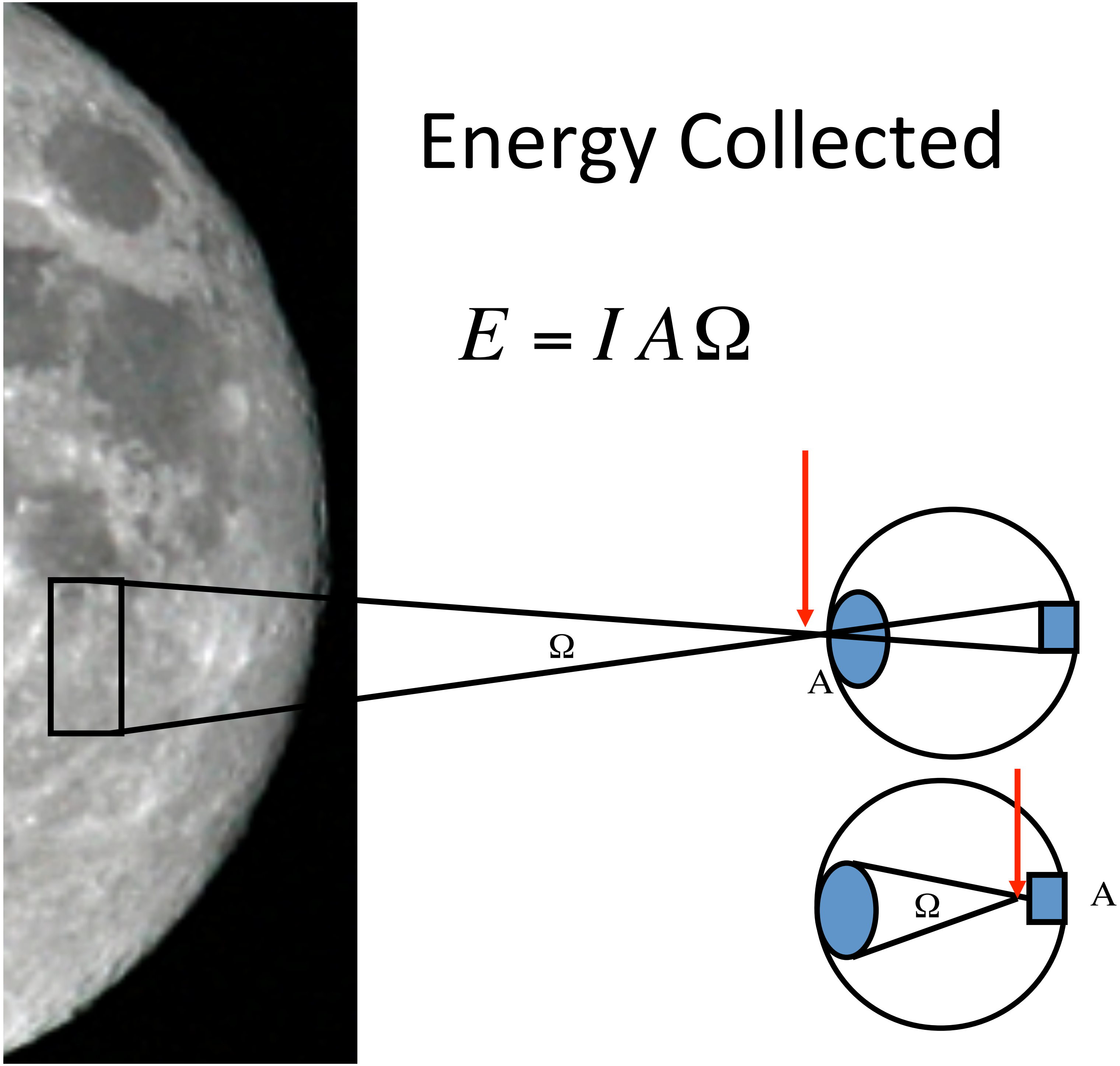
Energy Collected

$$E = I\,A\,\Omega$$



Energy Collected

$$E = I A \Omega$$



Some questions:

- What are the parts of a spectrograph
- Why are spectrographs so big?
- What sets the sensitivity?
- How do I estimate the exposure time?

S/N for object measured in aperture with radius r: n_{pix}=# of pixels in the aperture= πr^2

Noise from the dark current in aperture

Signal ⟷

$$\frac{R_* t}{\left[R_* \cdot t + R_{sky} \cdot t \cdot n_{pix} + \left((RN)^2 + \left(\frac{\text{gain}}{2}\right)^2\right) \cdot n_{pix} + \text{Dark} \cdot t \cdot n_{pix}\right]^{\frac{1}{2}}}$$

Noise ⟷

$$\sqrt{(R_* \cdot t)^2}$$

Readnoise in aperture

Noise from sky *e*- in aperture

All the noise terms added in quadrature
Note: always calculate in *e*-

How do I calculate the number of photo electrons/s on my detector?

- Resolved source
  - We are measuring surface brightness
  - $E = A\Omega I_\nu$
- For an extended object in the IR that is easy: You just need the temperature of the source, the system losses (absorption, QE etc), resolution and etendu of a pixel. No telescope aperture or F/#, no slit size, no optical train!
- For an extended object in the visible: You just need the surface brightness of the source, the system losses (absorption, QE etc), resolution and etendu of a pixel. No telescope aperture or F/#, no slit size, no optical train!
- Point source
  - we are measuring flux
  - $E = Af_\nu dt$
- For an unresolved object, you need the source magnitude, telescope aperture, system losses and resolution.

Ex 1: Thermal Imaging

R=5000
Pixel size= 10 microns
Final focal ratio at detector = F/3
Source temperature=5000K
Operating near 2 microns

SB from Planck=1,157,314 watts/(m^2 sr micron)
$\Delta\lambda$=2 microns/5000=0.0004

$$Solid\ Angle = \frac{\pi/4}{(F/\#)^2}$$

$$E = I_\lambda A\Omega\ QE\ \Delta\lambda = (1157314)(10\cdot 10^{-6})^2(\pi/4)(1/3)^2(0.0004)$$

$$= 4.039\cdot 10^{-9}\, watts$$

$$Nphots = E/h\nu$$

$$= \frac{(4.039\cdot 10^{-9})\cdot(2\cdot 10^{-6})}{6.62\cdot 10^{-34}\cdot 3\cdot 10^{8}} = 4.06\cdot 10^{10}$$

Ex 2: Extended Object in the Visible

R=5000
Pixel size= 10 microns
Final focal ratio at detector = F/3
Moon (SB=1.81E-16 W/(m2 Sr Hz)
Operating near 1/2 micron
$\Delta\lambda$=0.5 microns/5000=1 Angstrom
$d\upsilon=(c/\lambda^2)d\lambda$=1.2E11 Hz
QE = 1

$$Solid\ Angle = \frac{\pi/4}{(F/\#)^2}$$

$$E = I_\upsilon A\Omega\, QE\, \Delta\nu = (1.81\cdot 10^{-16})(10\cdot 10^{-6})^2(\pi/4)(1/3)^2(12\cdot 10^{-11})$$

$$= 4.7\cdot 10^{-17}\, watts$$

$$Nphots = E/h\nu$$

$$= \frac{(4.7\cdot 10^{-17})\cdot(0.5\cdot 10^{-6})}{6.62\cdot 10^{-34}\cdot 3\cdot 10^{8}} = 118$$

Ex 2B: Unresolved Object in the Visible

R=5000

Pixel size= 10 microns

Final focal ratio at detector = F/3

Apparent brightness = Vmagnitude= 10

Operating near 1/2 micron

$\Delta\lambda$=0.5 microns/5000=1 Angstrom

$d\upsilon=(c/\lambda^2)d\lambda$=1.2E11 Hz

QE = 1

$$E = f_{\upsilon} A_{Tel}\ QE\ \Delta\nu \Delta t =$$

$$m = -2.5\log_{10}\left(\frac{f_{\upsilon}}{f_0}\right)$$

$$Nphots = E / h\nu$$

Ex 3: Surface brightness of the moon

M=-12.6 (V-band apparent magnitude

Diameter=30 arcminutes

$$m = -2.5\log_{10}\left(\frac{f_1}{f_0}\right)$$

$$f_1 = f_0 \cdot 10^{-m/2.5} = 3.63 \cdot 10^{-23} \cdot 10^{12.6/2.5}$$

$$= 3.96 \cdot 10^{-18} w/m^2/Hz$$

$$A = \pi(15 \cdot 60)^2 \sec^2$$

$$= 2.54 \cdot 10^6 \sec^2$$

$$S = m_\lambda + 2.5\log_{10} A$$

S=Surface brightness in magnitudes/arcsecond^2

$$I_\nu = \frac{(3.96 \cdot 10^{-18})}{2.54 \cdot 10^6} = 1.55 \cdot 10^{-24} W/M^2/Hz/\sec^2$$

- Noise Sources:

$\sqrt{R_* \cdot t}$ $\Rightarrow$ shot noise from source

$\sqrt{R_{sky} \cdot t \cdot \pi r^2}$ $\Rightarrow$ shot noise from sky in aperture

$\sqrt{RN^2 \cdot \pi r^2}$ $\Rightarrow$ readout noise in aperture

$\sqrt{\left[RN^2 + (0.5 \times \text{gain})^2\right]} \cdot \sqrt{\pi r^2}$ $\Rightarrow$ more general RN

$\sqrt{\text{Dark} \cdot t \cdot \pi r^2}$ $\Rightarrow$ shot noise in dark current in aperture

$R_* = \text{e}^-/\text{sec}$ from the source

$R_{sky} = \text{e}^-/\text{sec/pixel}$ from the sky

$RN =$ read noise (as if RN^2 e^- had been detected)

$\text{Dark} = \text{e}^-/\text{second/pixel}$

Sources of Background noise

- Relic Radiation from Big Bang
- Integrated light from unresolved extended sources
- Thermal emission from dust
- Starlight scattered from dust
- Solar light scattered from dust (ZL)
- Line emission from galactic Nebulae
- Line emission from upper atmosphere (Airglow)
- Thermal from atmosphere
- Sun/moonlight scattered by atmosphere
- Manmade light scattered into the beam
- Thermal or scatter from the telescope/dome/instrument

S/N for object measured in aperture with radius r: n_{pix}=# of pixels in the aperture= πr^2

Noise from the dark current in aperture

Signal ⟷ $R_* t$

Noise ⟷

$$\frac{R_* t}{\left[R_* \cdot t + R_{\text{sky}} \cdot t \cdot n_{\text{pix}} + \left((RN)^2 + \left(\frac{\text{gain}}{2}\right)^2 \right) \cdot n_{pix} + \text{Dark} \cdot t \cdot n_{\text{pix}}\right]^{\frac{1}{2}}}$$

$$\sqrt{(R_* \cdot t)^2}$$

Noise from sky *e*- in aperture

Readnoise in aperture

All the noise terms added in quadrature
Note: always calculate in *e*-

S/N - some limiting cases. Let's assume CCD with Dark=0, well sampled read noise.

$$\frac{R_* t}{\left[R_* \cdot t + R_{\rm sky} \cdot t \cdot n_{\rm pix} + (RN)^2 \cdot n_{pix}\right]^{\frac{1}{2}}}$$

<u>Bright Sources:</u>

$(R_* t)^{1/2}$ dominates noise term

$$S/N \approx \frac{R_* t}{\sqrt{R_* t}} = \sqrt{R_* t} \propto t^{\frac{1}{2}}$$

<u>Read-noise</u> Limited

$$(3\sqrt{R_{\rm sky} t} < RN):\ S/N \propto \frac{R_* t}{\sqrt{n_{\rm pix} RN^2}} \propto t$$

<u>Sky Limited</u>

$$(\sqrt{R_{\rm sky} t} > 3 \times RN):\ S/N \propto \frac{R_* t}{\sqrt{n_{\rm pix} R_{\rm sky} t}} \propto \sqrt{t}$$

Note: seeing comes in with $n_{\rm pix}$ term

End lecture 2